12.0 PLATE GIRDERS

12.1 General

Why do we use plate girders?

What proportions are most economical?

- Rolled sections:
  - (i) have limited moment capacity
  - (ii) have relatively thick webs to provide adequate shear resistance on short spans. The V_r/M_r ratio is high

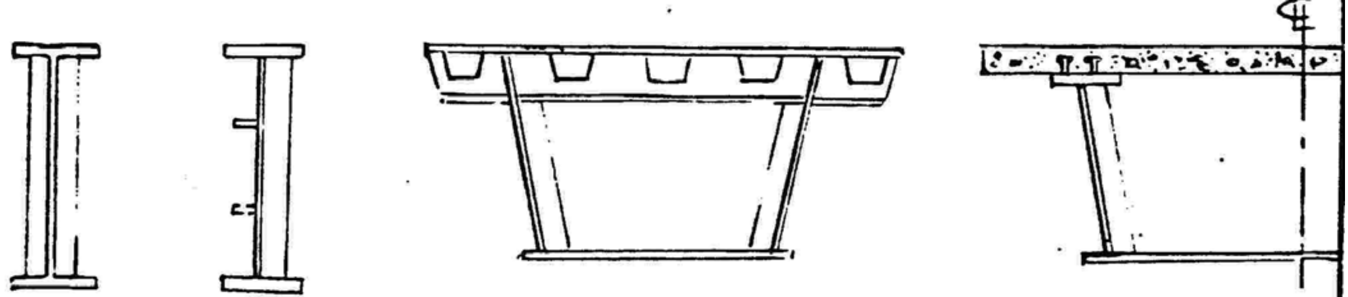
Plate girders:

- (i) are used on longer spans when rolled sections have insufficient capacity
- (ii) for the same M_r are more expensive than rolled sections
- (iii) can be manufactured so that M_r just equals M_f
- (iv) with longer spans, shear is less relatively and therefore reduce web thickness
- (v) have replaced trusses in many applications (especially bridges) — aesthetics, economics, reduced depth, handling equipment

(vi) can have various combinations of web thicknesses and stiffeners to give minimum cost

(vii) shop fabricated in sections and spliced in field - size dictated by handling and transportation equipment

(viii) can be made in a variety of forms and depth varied as $f(M)$



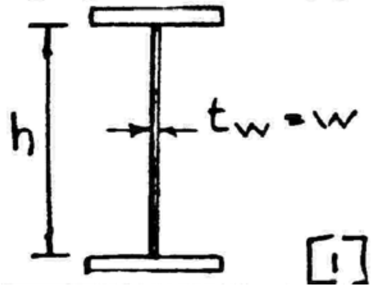
(ix) welding has made fabrication much simpler than when rivetted construction was used.

(x) field connections generally at least partially bolted

(xi) now define a plate girder as a built-up flexural member with a slender web.

12.2 Preliminary proportioning

12.2.1 Cross sectional Area



For a girder of constant depth and web thickness to mimize the cross sectional area

[1] $h = 540(M_f/F_y)^{1/3}$

where M_f is in kN.m
F_y is in MPa

then

[2] $$A_f = \frac{M_f}{\phi F_y h}$$

Flange is selected to meet b/t limits for a Class 3 section only because thin web precludes reaching M_p

Usually provide lateral bracing so that lateral-torsional buckling is not a problem.

As is the usual case it is assumed that the web carries all the transverse shear

$\therefore$ [3] $$A_w = wh = \frac{V_f}{\phi F_s}$$

where $F_s = f(h/w)$

2.2.2 Web thickness

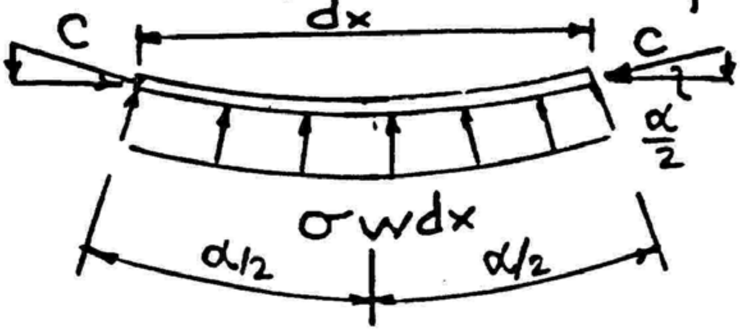
w_1 = minimum web thickness for corrosion
= 10 mm (Ontario Hwy Bridge Design Code)
= 4.5 mm (S16.1-M89 (buildings))

w_2 = absolute maximum web slenderness
$h/w = 83000/F_y$ (F_y in MPa)

w_3 = web slenderness if no reduction in moment capacity is . $h/w = 1900/\sqrt{F_y}$

Choose $w > w_1$; $w > w_2$; $w < w_3$ (economics)

12.2.3 Vertical buckling of web



From free-body diagram

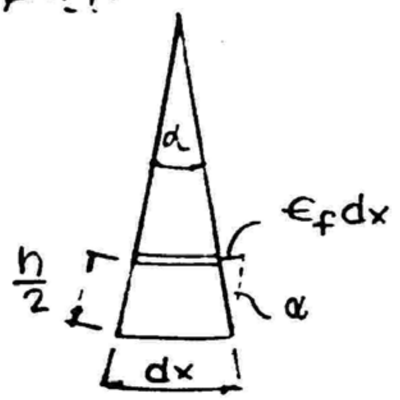
$$2C \sin\frac{\alpha}{2} = \sigma w\,dx$$

but for small angles $\frac{\alpha}{2} = \sin\frac{\alpha}{2}$

$$\therefore\; 2C\frac{\alpha}{2} = \sigma w\,dx$$

and $C = A_f\,\sigma_Y$

Also



The deformation $\epsilon_f dx = \frac{\alpha h}{2}$

or $\alpha = \frac{2\epsilon_f dx}{h}$

$$\therefore\; C\alpha = \frac{2A_f\,\sigma_Y\,\epsilon_f\,dx}{h} = \sigma w\,dx$$

and when buckling is about to occur

$$\sigma = \sigma_{cr} = \frac{k\pi^2 E}{12(1-\nu^2)\left(\frac{h}{w}\right)^2}$$

with $k = 1.00$ as web buckles as a column

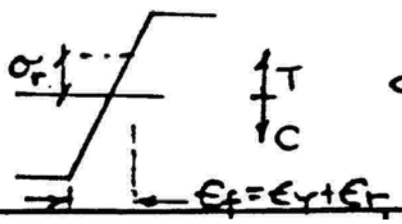
$$\therefore\; \frac{2A_f\,\sigma_Y\,\epsilon_f\,dx}{h} = \frac{\pi^2 E\,w\,dx}{12(1-\nu^2)\left(\frac{h}{w}\right)^2}$$

[4] $$\frac{h}{w} = \sqrt{\frac{\pi^2 E}{12(1-\nu^2)}\frac{A_w}{A_f}\cdot\frac{1}{2\sigma_Y\,\epsilon_f}}$$

where $A_w = h.w$

Consider flange strains, ϵ_f

residual stress pattern



To get compression yielding

$$\epsilon_f = \epsilon_Y + \epsilon_r$$

with $A_w/A_f \approx 0.5$

$\nu = 0.3$

$E = 200\,000$

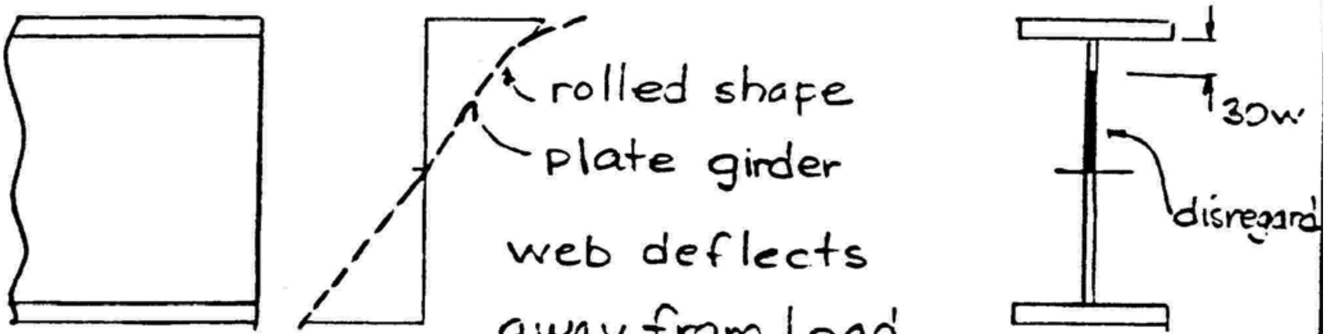
$\sigma_r \approx \sigma_Y/3$

[5] $$\frac{h}{w} = \sqrt{\frac{\pi^2 E^2 (0.5)}{12(1-\nu^2)} \frac{1}{2 \times \sigma_Y \times 1.33\sigma_Y}} = \frac{82\,400}{\sigma_Y}$$

Clause 12.4.3 of S16.1-M89 gives

[5a] $$\left(\frac{h}{w}\right)_{max} = \frac{83\,000}{\sigma_Y}$$

12.3 Design of cross-section for bending



Basler and Thurlimann (1961) suggest

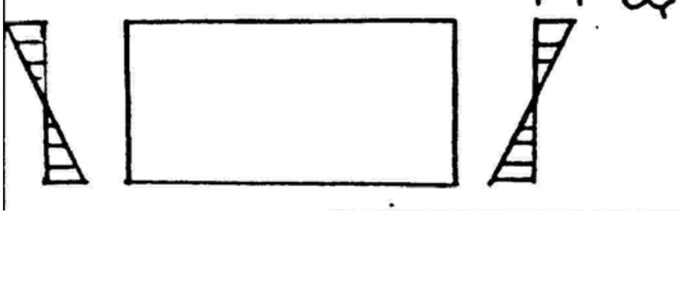
[6] $$\frac{M_u}{M_Y} = 1 - C\left[\left(\frac{h}{w}\right) - \left(\frac{h}{w}\right)_Y\right]$$

where C = a linear reduction constant

h/w = actual value provided

$(h/w)_Y$ = value that permits M_Y to be obtained

Consider



$$\sigma_{cr} = \frac{k\pi^2 E}{12(1-\nu^2)\left(\frac{h}{w}\right)^2}$$

for edges simply supported $k = 23$

$$\therefore \frac{h}{w} = 4.56\sqrt{\frac{E}{\sigma_Y}}$$

for edges fixed $k = 39.6$

$$\therefore \frac{h}{w} = 6.00\sqrt{\frac{E}{\sigma_Y}}$$

Basler and Thurlimann proposed $5.7\sqrt{\frac{E}{\sigma_Y}}$

In S16.1-M89 the expression is

$$[6a]\quad M_r' = M_r\left[1 - 0.0005\frac{A_w}{A_f}\left(\frac{h}{w} - \frac{1900}{\sqrt{\frac{M_r}{\phi S}}}\right)\right]$$

where $0.0005\frac{A_w}{A_f}$ is from Basler and Thurlimann

1900 = limit for Class 2 webs

M_r = basic moment capacity $\ngtr \phi M_Y$

M_r' = reduced moment capacity

when $M_r = \phi M_Y$

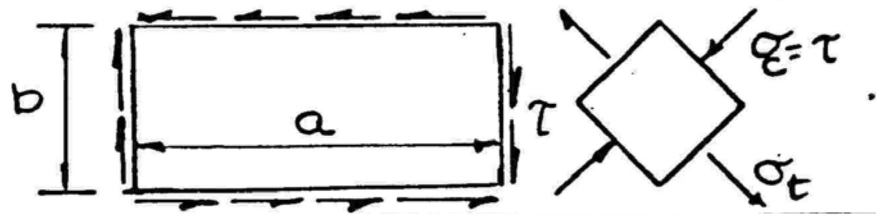
$$\sqrt{\frac{M_r}{\phi S}} = \sqrt{\frac{\phi M_Y}{\phi S}} = \sqrt{F_Y}$$

otherwise $\sqrt{\frac{M_r}{\phi S}} < \sqrt{F_Y}$

This reduction is generally relatively small

12.4 Design of cross-section for shear

12.4.1 Unstiffened web, shear buckling



occurs because of compressive stresses $\sigma_c = \tau$ at 45°

the critical shear buckling stress is

$$\tau_{cr} = \frac{k\pi^2 E}{12(1-\nu^2)\left(\frac{b}{w}\right)^2}$$

for $a/b > 1$ and simply supported edges

$$k = 5.34 + \frac{4}{(a/b)^2}$$

for $a/b > 1$ and fixed edges

$$k = 8.98 + \frac{5.6}{(a/b)^2}$$

for an unstiffened web $a/t \to \infty$ and assuming simply supported edges

[7] $$\tau_{cr} = \frac{180{,}800}{(h/w)^2} \times 5.34 = \frac{965\,000}{(h/w)^2}$$

S16.1-M89 gives [7a] $F_{cre} = \dfrac{180\,000\,k_v}{(h/w)^2}$

with $k_v = 5.34$

∴ For elastic buckling

[8] $$V_r = \phi A_w F_s = \phi A_w F_{cre}$$

12.4.2 Unstiffened web, shear yielding

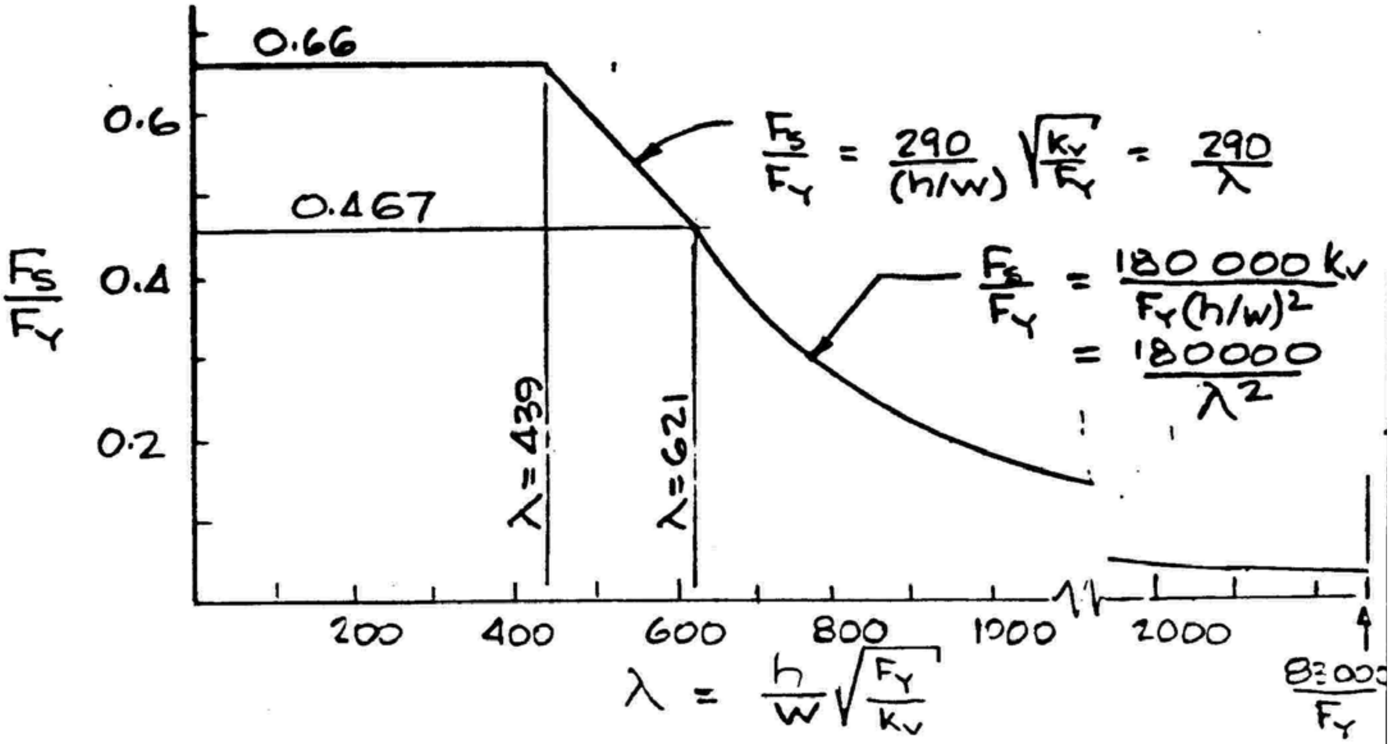
By von Mises-Hencky criterion

$\tau_Y = \sigma_Y/\sqrt{3} = 0.577\sigma_Y$

but use $0.66\sigma_Y$ (15 % increase)

because of beneficial effects of strain hardening

Therefore can construct the graph of (F_s/F_Y) versus (h/w)



Elastic buckling assumed valid up to $\sim 0.80\,\tau_Y$, therefore $\frac{180\,000 K_v}{\left(\frac{h}{w}\right)^2} = \frac{0.80 F_Y}{\sqrt{3}}$

$$h/w = 621\sqrt{\frac{K_v}{F_Y}}$$

Let $\lambda = \frac{h}{w}\sqrt{\frac{F_Y}{K_v}}$, then $\lambda_e = 621$

The inelastic curve is [9] $\frac{F_s}{F_Y} = \frac{290}{\lambda}$

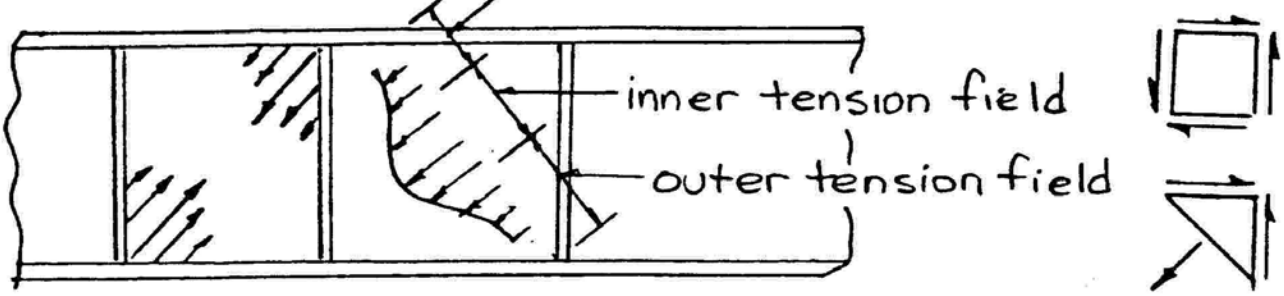
12.4.3 Stiffened webs

Shear yielding is not affected by presence of stiffeners; therefore upper limit is still $F_s/F_Y = 0.66$. For large values of λ buckling will precede shear yielding. but with stiffened web develop additional strength because of tension field action as developed by Wagner in aircraft industry

We do not depend on anchorage of tension field against flanges as they are considered to be too flexible.

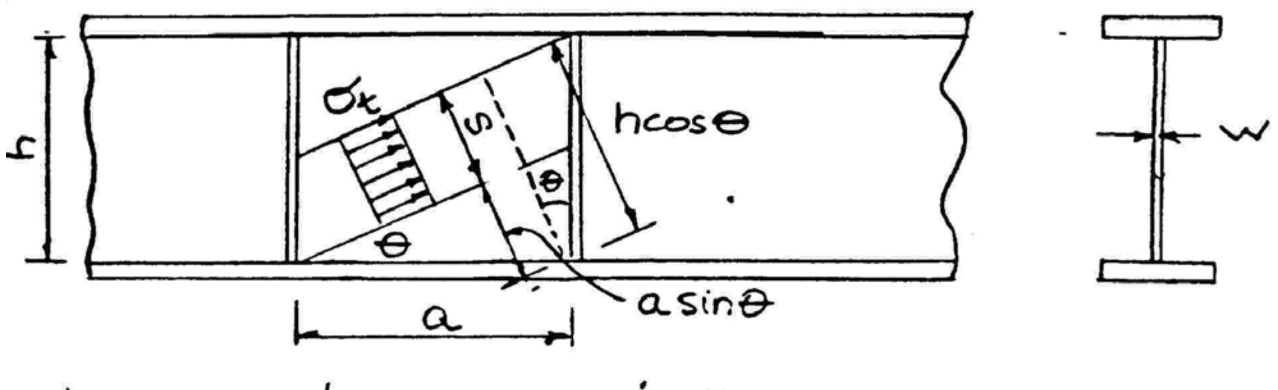
We therefore have shear capacity of the unstiffened web $V = \tau_{cr} \cdot h \cdot w$ plus the partial tension field action.

Consider



Basler and Thurlimann neglected outer tension field.

Consider



$$s = h\cos\theta - a\sin\theta$$

as θ increases s decreases

θ increases the vertical component of σ_t increases

V_t = shear due to tension field action

$= \sigma_t \cdot w \cdot s \cdot \sin\theta$

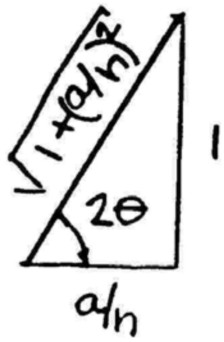
$$V_t = \sigma_t w \left(h \cos\theta \sin\theta - a \sin^2\theta \right)$$

$$= \sigma_t w \left(\frac{h}{2} \sin 2\theta - a \sin^2\theta \right)$$

$$\frac{dV_t}{d\theta} = \sigma_t w \left(h \cos 2\theta - 2a \sin\theta \cos\theta \right) = 0$$

or $\quad h \cos 2\theta - a \sin 2\theta = 0$

$$\tan 2\theta = \frac{h}{a} = \frac{1}{a/h}$$



$$\therefore \cos 2\theta = a/h \Big/ \sqrt{1+(a/h)^2}$$

$$\sin 2\theta = 1 \Big/ \sqrt{1+(a/h)^2}$$

also $\quad \sin^2\theta = \dfrac{1-\cos 2\theta}{2} = \dfrac{1}{2}\left[1 - \dfrac{a/h}{\sqrt{1+(a/h)^2}} \right]$

this then gives for V_t max

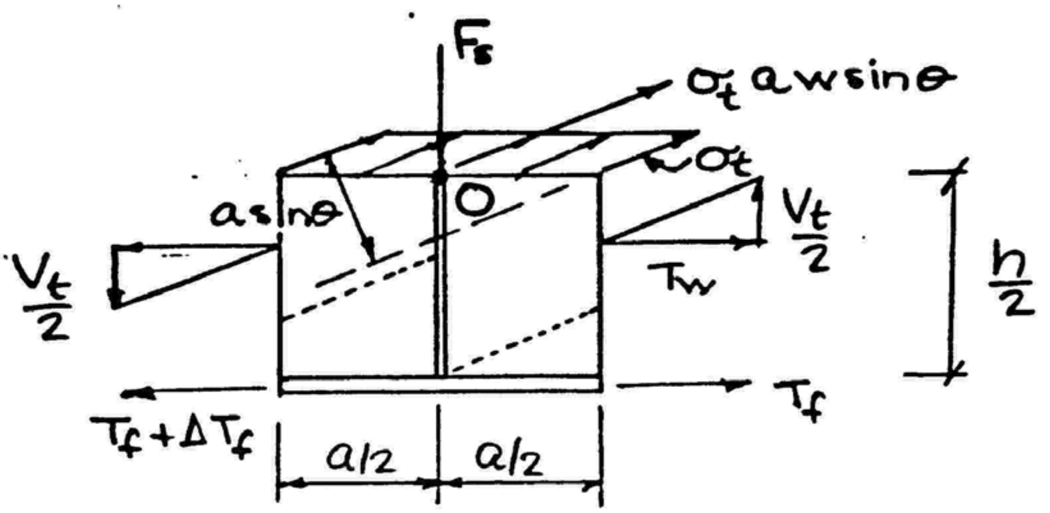
$$V_t = \frac{\sigma_t w}{2} \left[\frac{h}{\sqrt{1+(a/h)^2}} - a + \frac{a^2/h}{\sqrt{1+(a/h)^2}} \right]$$

$$= \frac{\sigma_t w h}{2} \left[\frac{1+(a/h)^2}{\sqrt{1+(a/h)^2}} - \frac{a}{h} \right]$$

[10] $\quad V_t = \dfrac{\sigma_t w h}{2} \left[\sqrt{1+(a/h)^2} - \dfrac{a}{h} \right]$

Note : This equation is not in Basler's work

Rather than substituting the optimum value of θ in the original equation for V_t he considered the free-body diagram following using the optimum value of θ



$$\Sigma F_x = 0 \quad \therefore \quad \Delta T_f = \sigma_t \, a w \sin\theta \cos\theta$$

$$\Sigma M_o = 0 = \Delta T_f \cdot \frac{h}{2} - \frac{V_t}{2} \cdot a$$

$$\therefore \quad V_t = \Delta T_f \, h/a$$

$$= \sigma_t \, w h \sin\theta \cos\theta = \sigma_t \, w h \frac{\sin 2\theta}{2}$$

$$[11] \qquad V_t = (\sigma_t \, w h)/2\sqrt{1 + (a/h)^2}$$

E.H. Gaylord, in discussion of Basler's work stated that [11] was equivalent to a full tension field not the partial field assumed. Basler, in reply, stated "It is not evident to the writer that this result is inconsistent with the assumptions made to derive it."

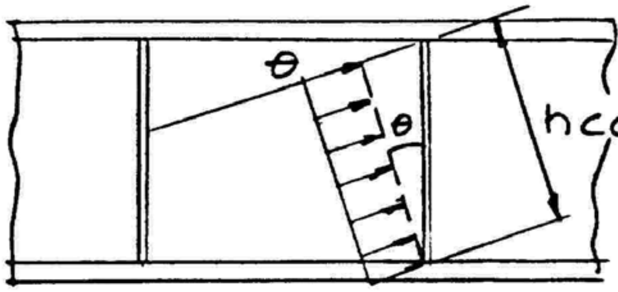
McGuire refers to the inconsistency established by Gaylord.

Salmon and Johnson, use the free-body diagram above, establish [11] and state that shear outside of the band s must be added. ! !

But how can we get 2 answers to the same problem?

Equation [11] is consistent with full or nearly full tension field action.

Consider

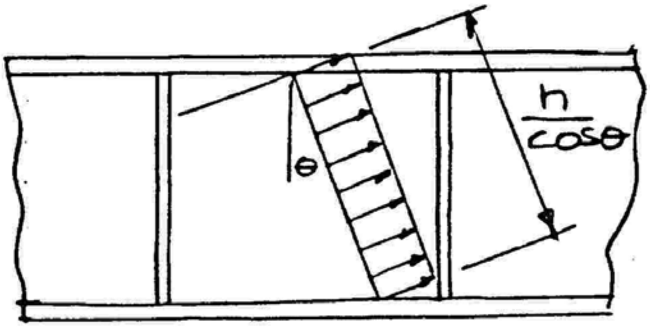


$V_t = \sigma_t wh \cos\theta \sin\theta$

the tensile stress is acting on a greater width than

$s = h\cos\theta - a\sin\theta$

Full tension field would be



Tensile force

$T = \sigma_t wh/\cos\theta$

$V_t = T\sin\theta$

$= \sigma_t wh \tan\theta$

We do not use this because this results in a significant pull on the flanges which they may not be able to provide.

Obviously Basler's solution requires more than the partial tension field he assumed. It is used in S16.1 because it correlates well with test results, indicating that there is a contribution from the outer tension fields.

Question: What is the error in Basler's free-body diagram?

Accepting that more than the width "s" of the strip is effective the total shear carried is

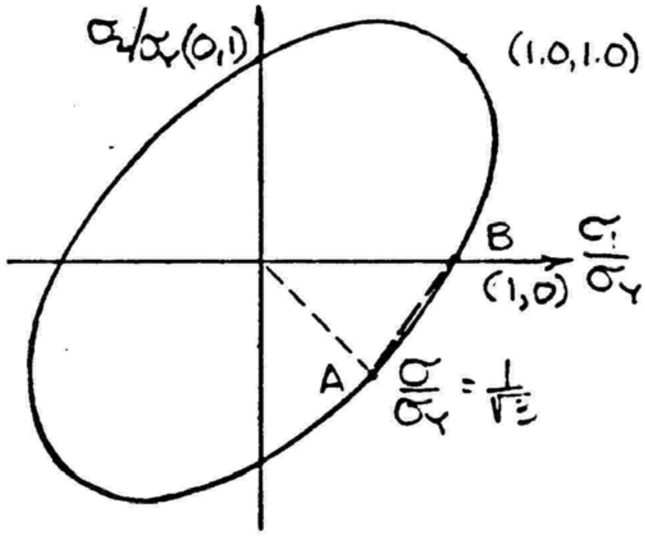
[12] $V_u = V_b + V_t$

$= V_{\text{elastic buckling}} + V_{\text{tension field}}$

$= \tau_{cr} wh + \sigma_t wh / 2\sqrt{1+(a/h)^2}$

When we have both τ and σ acting, use von Mises-Hencky failure theory to get combined stresses. In terms of principal stresses

$\sigma_1^2 + \sigma_2^2 - \sigma_1\sigma_2 = F_Y^2$



At A the stress state is $\sigma_1 = -\sigma_2$ or one of pure shear

$\tau_{max} = \tau_{cr} = \tau_Y = \sigma_Y/\sqrt{3}$

$\therefore$ Changing cordinate axes to σ_t/σ_Y and τ/τ_Y the coordinates of A are (0,1) and of B are (1,0). A straight line approximation A to B is

$\sigma_t/\sigma_Y = 1 - \tau/\tau_Y$ or $\sigma_t = \sigma_Y - \sqrt{3}\tau$

where τ is the shear stress due to elastic buckling $= \tau_{cr}$

[13] $\sigma_t = \sigma_Y - \sqrt{3}\,\tau_{cr}$

Now $\tau_{cr} = \dfrac{k_v \pi^2 E}{12(1-\nu^2)\left(\frac{h}{w}\right)^2}$ and $\tau_Y = \dfrac{\sigma_Y}{\sqrt{3}}$

$\therefore$ substituting in [12]

$$V_u = \frac{k_v \pi^2 E}{12(1-\nu^2)\left(\frac{h}{w}\right)^2} \cdot wh + \frac{wh}{2\sqrt{1+\left(\frac{a}{h}\right)^2}}\left[\sigma_Y - \frac{\sqrt{3}\,k_v \pi^2 E}{12(1-\nu^2)\left(\frac{h}{w}\right)^2}\right]$$

in terms of stresses dividing by wh and evaluating $\dfrac{\pi^2 E}{12(1-\nu^2)} = 180{,}800$

[14] $F_s = \dfrac{180800\,k_v}{(h/w)^2} + \dfrac{0.50\,\sigma_Y}{\sqrt{1+\left(\frac{a}{h}\right)^2}} - \dfrac{156{,}500\,k_v}{\left(\frac{h}{w}\right)^2\sqrt{1+\left(\frac{a}{h}\right)^2}}$

In S16.1 this is written as

$F_s = F_{cre} + F_t$

where $F_{cre} = \dfrac{180\,000\,k_v}{(h/w)^2}$

[15] $F_t = \left(0.50\,F_Y - 0.87\,F_{cre}\right)\left(\dfrac{1}{\sqrt{1+\left(\frac{a}{h}\right)^2}}\right)$

and is considered to be applicable when $\dfrac{h}{w} > 621\sqrt{\dfrac{k_v}{F_Y}}$ or $\dfrac{h}{w}\sqrt{\dfrac{F_Y}{k_v}} = \lambda > 621$

An inelastic transition is provided when $502\sqrt{\dfrac{k_v}{F_Y}} < \dfrac{h}{w} \leq 621\sqrt{\dfrac{k_v}{F_Y}}$

or $\quad 502 < \lambda \leq 621$:

$$F_s = F_{cri} + F_t$$

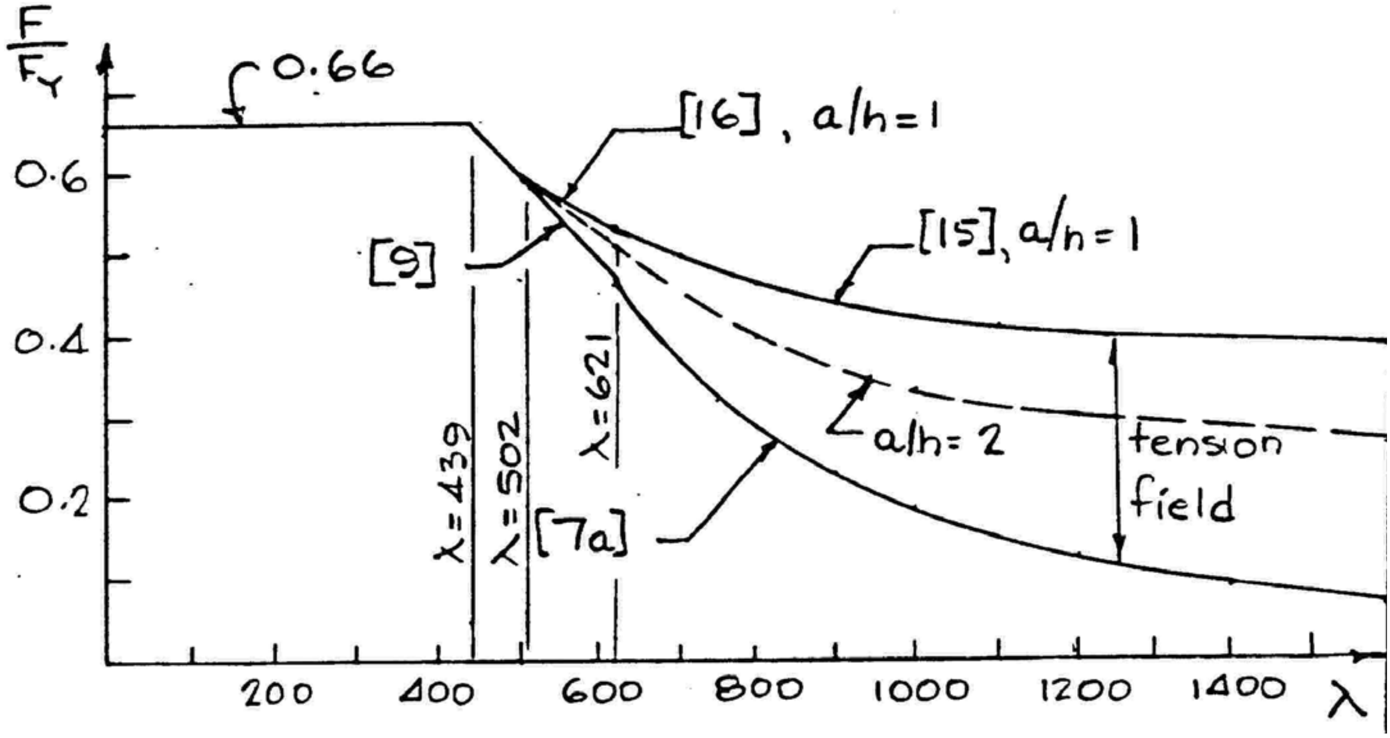
$$F_{cri} = \frac{290\sqrt{F_y k_v}}{h/w} \quad \text{or} \quad \frac{F_{cri}}{F_y} = \frac{290}{\lambda}$$

and $$F_t = \frac{0.50 F_y}{\sqrt{1+\left(\frac{a}{h}\right)^2}} - 0.87 F_{cri} \frac{1}{\sqrt{1+(a/h)^2}},$$

or

[16] $$\frac{F_t}{F_y} = \frac{0.50}{\sqrt{1+\left(\frac{a}{h}\right)^2}} - \frac{252}{\lambda} \cdot \frac{1}{\sqrt{1+(a/h)^2}}$$

Thus the curve for stiffened webs for a particular value of a/h can be plotted on the same diagram as that for unstiffened webs



12.5 Stiffener Requirements

12.5.1 Bearing stiffeners are required where the web cannot sustain the concentrated loads or reactions. See S16.1-M89 Clauses 15.6 and 15.9.

12.5.2 Intermediate stiffeners

From p. PG 11 $F_s = \sigma_t w a \sin^2\theta$

$$\therefore F_s = \sigma_t \frac{wa}{2}(1 - \cos 2\theta)$$

$$= \sigma_t \frac{wa}{2}\left[1 - \frac{(a/h)}{\sqrt{1+(\frac{a}{h})^2}}\right]$$

but $\sigma_t = \sigma_Y\left(1 - \frac{\tau_{cr}}{\tau_Y}\right)$

and with $F_s = A_s \sigma_Y$

$$[17] \quad A_s \geqslant \frac{F_s}{\sigma_Y} = \frac{aw}{2}\left[1 - \frac{a/h}{\sqrt{1+(a/h)^2}}\right]\left[1 - \frac{310\,000 k_v}{F_Y(\frac{h}{w})^2}\right]\frac{F_Y}{F_{YS}}$$

the last term accounts for the case when the yield stress of the stiffener is different from that of the web

[17] as given in S16.1-M89 is for pairs of stiffeners placed symmetrically about the centre-line of the web and

A_s by [17] is multiplied by

1.8, for angle stiffener on one side only

2.0 for plate stiffener on one side only

The stiffener area may be reduced in the ratio V_f/V_r when $V_f < V_r$.

Two other requirements are

(a) the force to be transferred from the stiffener into the web shall be not less than

$$v = 1\times10^{-4}\, h\, (F_y)^{3/2} \quad N/mm.$$

and the moment of inertia of the stiffener about an axis in the plane of the web shall be not less than

$$I_s \geq (h/50)^4$$

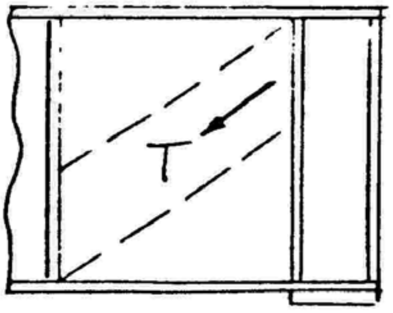
12.5.3 Limits on stiffener spacing

(a) for $\frac{h}{w} \leq 150$; $\frac{a}{r} \leq 3$

(b) for $\frac{h}{w} > 150$ $\quad a/h \leq \frac{67\,500}{(h/w)^2}$

(tension field contribution is reduced and also avoids handling and erection problems)

12.5.4 Tension field at girder ends or adjacent to large openings

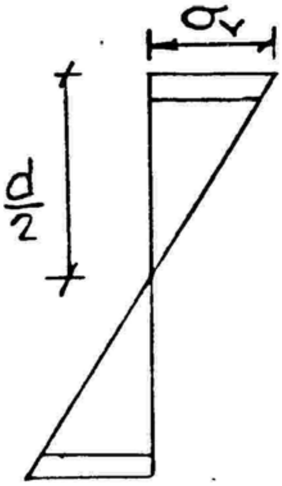


unless means are provided to anchor tension field (to resist horizontal component of T) take tension field component = 0 thus only have F_{cri} or F_{cre}.

12.6 Combined shear and bending

May be of concern where high moments and shears co-exist (   )

Define M_n = moment carried by flanges alone



$\approx \sigma_Y d A_f$

also:

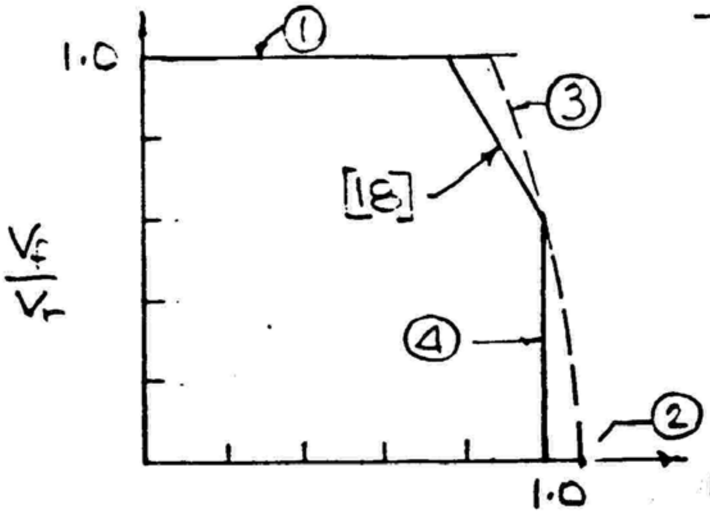
$$M_Y = \sigma_Y d A_f + \frac{\sigma_Y d}{2} \cdot w \cdot \frac{1}{2} \cdot \frac{2}{3} d$$

$$= \sigma_Y d \left(A_f + A_w/6 \right)$$

If it were possible to reach M_p without web buckling then

$$M_p = \sigma_Y d + \frac{\sigma_Y d}{2} \cdot w \frac{d}{2} = \sigma_Y d \left(A_f + Aw/4 \right)$$

In developing the shear capacity of the web no contribution was assumed from the flanges. Thus in regions where the flanges alone have sufficient moment capacity the shear strength is not reduced.



Thus for 1

$$0 < \frac{M_f}{M_y} < \frac{M_n}{M_Y} \therefore \frac{V_f}{V_r} = 1$$

$$\frac{M_n}{M_Y} = A_f/(A_f + Aw/6)$$

if $Aw = A_f$, $\frac{M_n}{M_Y} = 0.86$

At ② if no shear and web did not buckle then reach M_f :

$$\frac{M_f}{M_Y} = \frac{M_p}{M_Y} = \frac{\sigma_Y d\,(A_f + A_w/4)}{\sigma_Y d\,(A_f + A_w/6)}$$

and for $A_w = A_f$ (more likely)

$$\frac{M_f}{M_Y} = 1.25/1.17 = 1.07$$

Tests show the interaction curve ③ is parabolic

$$\left(\frac{V_f}{V_r}\right)^2 + \frac{M_f - M_n}{M_f - M_r} = 1.0$$

Now cut off at M_y by vertical line ④ which intersects the parabola at

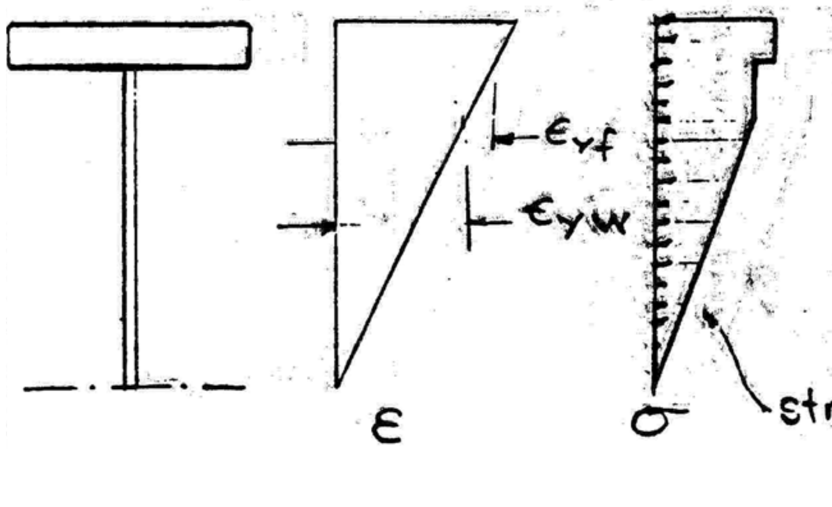
$$\frac{V_f}{V_r} = 0.60$$

and replace the remaining portion of the parabola by the straight line

[18] $$0.727\frac{M_f}{M_r} + 0.455\frac{V_f}{V_r} \leq 1.0$$

which is seen to be conservative

12.7 Hybrid plate girders



- Use lower F_y in web
- flexural strength not reduced much.
- not used much in Canada