BEAM-COLUMNS

SUMMARY:

- Structural members subjected to axial compression and bending are known as *beam columns*.
- The interaction of normal force and bending may be treated elastically or plastically using equilibrium for the classification of cross-section.
- The behaviour and design of beam-columns are presented within the context of members subjected to *uniaxial bending*, i.e. deformation takes place only in the plane of the applied moments.
- In the case of beam-columns which are susceptible to *lateral-torsional buckling*, the out-of-plane flexural buckling of the column has to be combined with the lateral-torsional buckling of the beam using the relevant interaction formulae.
- For beam-columns with *biaxial bending*, the interaction formula is expanded by an additional term.

OBJECTIVES:

- Evaluate the in-plane bending and axial compression force for beam-columns.
- Calculate the lateral-torsional buckling of beam-columns.
- Calculate the biaxial bending and axial compression force for beam-columns.

REFERENCES:

- Eurocode 3: Design of steel structures Part 1.1 General rules and rules for buildings.
- Chen W F and Atsuta T: "Theory of Beam-Columns" Vols. 1 & 2, McGraw-Hill, 1976.
- Trahair N. & Bradford M: "Behaviour and Design of Steel Structures", 2nd edt., Chapman & Hall, 1988.
- Dowling P J, Owens G W & Knowles P: "Structural Steel Design", Butterworths, 1988.
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CONTENTS:

1. INTRODUCTION.

Beam-columns are defined as members subject to combined bending and compression.

In principle, all members in frame structures are actually beam-columns, with the particular cases of beams (N = 0) and columns (M = 0) simply being the two extremes.

Depending upon the exact way in which the applied loading is transferred into the member, the form of support provided and the member's cross-sectional shape, different forms of response will be possible.

The simplest of these involves bending applied about one principal axis only, with the member responding by bending solely in the plane of the applied moment.

2. IN-PLANE BEHAVIOUR OF BEAM-COLUMNS.

When a beam-column is subjected to in plane bending (figure 1a), its behaviour shows an interaction between beam bending and compression member buckling, as indicated in figure 1b.

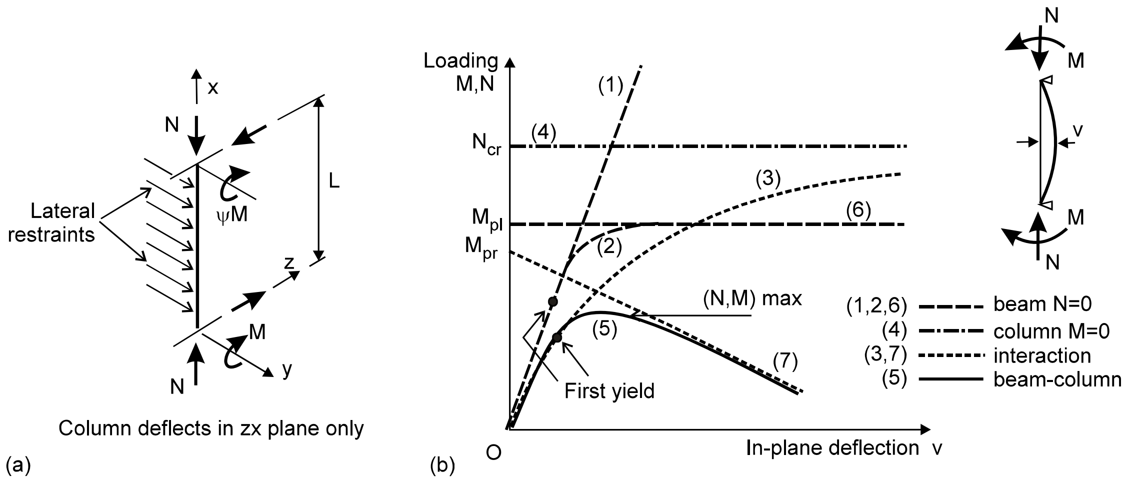


Figure 1 – In-plane behaviour of beam-columns.

Curve 1 shows the beam elastic linear behaviour.

Curve 6 shows the limiting behaviour of a rigid-plastic beam at the full plastic moment M_{pl}.

Curve 2 shows the transition of real elastic-plastic beams from curve 1 to curve 6.

The elastic buckling load of a concentrically loaded compression member, N_{cr} is shown in curve 4.

Curve 3 shows the interaction between bending and buckling in elastic members, and allows for the traditional moment $N\,v$ exerted by the axial load.

Curve 7 shows the interaction between bending moment and axial force causing the member to become fully plastic. This curve allows for the reduction from the full plastic moment M_{pl} to M_{pr} caused by the axial load, and for the additional moment Nv.

The actual behaviour of a beam-column is shown by curve 5 which provides a transition from curve 3 for elastic members to curve 7 for full plasticity.

2.1 CROSS-SECTIONAL BEHAVIOUR.

2.1.1 Bending and axial force for class 1 and 2 cross-sections.

If full plasticity is allowed to occur, then the failure condition will be as shown in figure 2 and the combination of axial load and moment giving this condition will be:

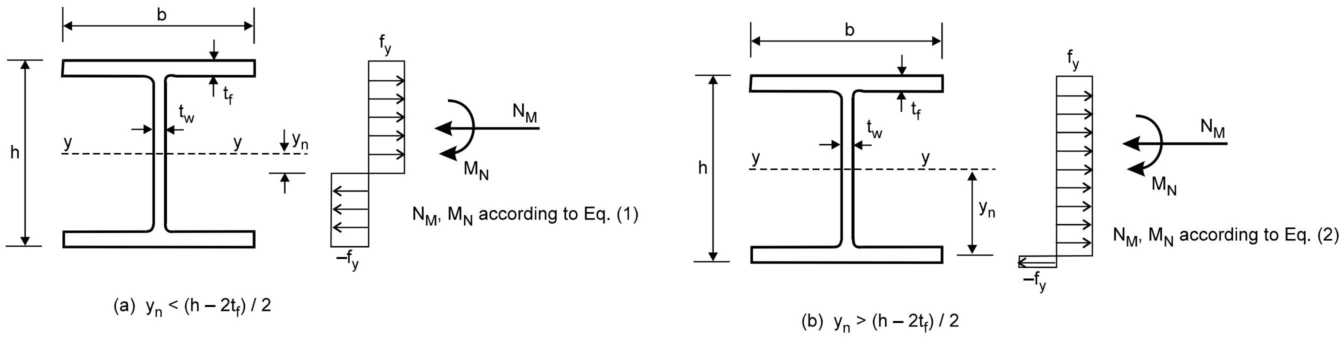


Figure 2 – Full plasticity under axial load and moment.

a. For $y_n \le (h - t_f)/2$ <u>neutral axis in web</u>

$$N_M = 2 f_y t_w y_n$$

$$M_N = f_y b t_f (h - t_f) + f_y \left[\left(\frac{h - 2t_f}{2} \right)^2 - y_n^2 \right] t_w \qquad (1)$$

b. For $y_n > (h - t_f)/2$ <u>neutral axis in flange</u>

$$N_M = f_y \left[t_w (h - 2t_f) + 2b \left(t_f - \frac{h}{2} + y_n \right) \right]$$

$$M_N = f_y b \left(\frac{h}{2} - y_n \right) (h - y_n) t_f \qquad (2)$$

Figure 3 compares Eqs. (1) and (2) with the approximation used in Eurocode 3 of:

$M_{Ny.Rd} = M_{pl.y}(1-n)/(1-0{,}5a)$ but $M_{Ny.Rd} \le M_{ply.Rd}$ (3)	*Eurocode 3* 5.4.8.1 (5.25) *or eq. 6.36*

in which $n = N_{Sd} / N_{pl.Rd}$ is the ratio of axial load to “squash” load ($f_y A$), and $a = (A - 2bt_f)/A \le 0{,}5$

For cross-sections without bolt holes, the following approximations may be used for *z* axis moments:

for $n > a$: $M_{Nz.Rd} = M_{pl.z.Rd}$ for $n > a$: $M_{Nz.Rd} = M_{pl.z.Rd} \cdot \left[1 - \left(\frac{n-a}{1-a} \right)^2 \right]$	*Eurocode 3* 5.4.8.1 (5.26) *or eq. 6.37 and 6.38*

where $n = N_{Sd} / N_{pl.Rd}$ and $a = (A - 2bt_f)/A$ but $a \le 0{,}5$.

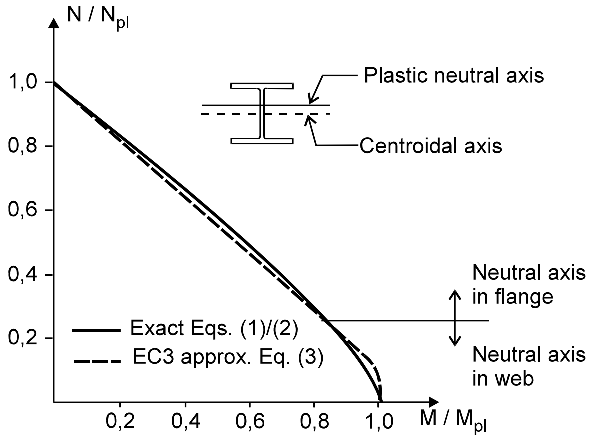


Figure 3 – Full plasticity interaction – major axis bending of HEA 450 section.

Eurocode 3

6.2.9 Bending and axial force

6.2.9.1 Class 1 and 2 cross-sections

(1) Where an axial force is present, allowance shall be made for its effect on the plastic moment resistance.

(2) For class 1 and 2 cross sections, the following criterion should be satisfied:

$$M_{Ed} \leq M_{N,Rd} \tag{6.31}$$

where $M_{N,Rd}$ is the design plastic moment resistance reduced due to the axial force N_{Ed}.

(3) For a rectangular solid section without bolt holes $M_{N,Rd}$ should be taken as:

$$M_{N,Rd} = M_{pl,Rd}\left[1-\left(N_{Ed} / N_{pl,Rd}\right)^2\right] \tag{6.32}$$

(4) For doubly symmetrical I- and H-sections or other flanges sections, allowance need not be made for the effect of the axial force on the plastic resistance moment about the y-y axis when both the following criteria are satisfied:

$$N_{Ed} \leq 0,25\, N_{pl,Rd} \text{ and} \tag{6.33}$$

$$N_{Ed} \leq \frac{0,5\, h_w\, t_w\, f_y}{\gamma_{M0}} \tag{6.34}$$

For doubly symmetrical I- and H-sections, allowance need not be made for the effect of the axial force on the plastic resistance moment about the z-z axis when:

$$N_{Ed} \leq \frac{h_w\, t_w\, f_y}{\gamma_{M0}} \tag{6.35}$$

(5) For cross-sections where bolt holes are not to be accounted for, the following approximations may be used for standard rolled I or H sections and for welded I or H sections with equal flanges:

$$M_{N,y,Rd} = M_{pl,y,Rd}(1-n)/(1-0,5a) \quad \text{but } M_{N,y,Rd} \leq M_{pl,y,Rd} \tag{6.36}$$

$$\text{for } n \leq a: \quad M_{N,z,Rd} = M_{pl,z,Rd} \tag{6.37}$$

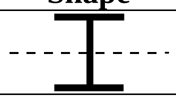
$$\text{for } n > a: \quad M_{N,z,Rd} = M_{pl,z,Rd}\left[1-\left(\frac{n-a}{1-a}\right)^2\right] \tag{6.38}$$

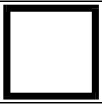
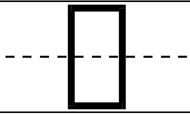
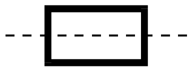
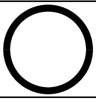
where $n = N_{Ed} / N_{pl.Rd}$

$a = (A-2bt_f)/A$ but $a \leq 0,5$

Further simplifications for a range of common cross-sectional shapes are provided in Table 1.

Table 1 – Expressions for reduced plastic moment resistance M_N (Notation: $n = N_{Sd} / N_{pl.Rd}$).

Cross-section	Shape	Expression for M_N	5.4.8.1
Rolled I or H		$M_{N,y} = 1{,}11 M_{pl.y}(1-n)$	(5.27)

		$M_{N,z} = 1{,}56 M_{pl.z}(1-n)(0{,}6+n)$	(5.28)
Square hollow section		$M_{N,y} = 1{,}26 M_{pl}(1-n)$	(5.31)
Rectangular hollow section		$M_{N,y} = 1{,}33 M_{pl.y}(1-n)$	(5.32)
		$M_{N,y} = M_{pl.z} \dfrac{1-n}{0{,}5+\dfrac{ht}{A}}$	(5.33)
Circular hollow section		$M_{N,y} = 1{,}04 M_{pl}(1-n^{1{,}7})$	(5.34)

For cross-sections where bolt holes are not to be accounted for, the following approximations may be used for rectangular structural hollow sections of uniform thickness and for welded box sections with equal flanges and equal webs:

$$M_{N,y,Rd} = M_{pl,y,Rd}(1 - n)/(1 - 0{,}5a_w) \quad \text{but } M_{N,y,Rd} \le M_{pl,y.Rd} \qquad (6.39)$$

$$M_{N,z,Rd} = M_{pl,z,Rd}(1 - n)/(1 - 0{,}5a_f) \quad \text{but } M_{N,z,Rd} \le M_{pl,z,Rd} \qquad (6.40)$$

where $a_w = (A - 2bt)/A$ but $a_w \le 0{,}5$ for hollow sections

$a_w = (A-2bt_f)/A$ but $a_w \le 0{,}5$ for welded box sections

$a_f = (A - 2ht)/A$ but $a_f \le 0{,}5$ for hollow sections

$a_f = (A-2ht_w)/A$ but $a_f \le 0{,}5$ for welded box sections

In all cases the value of M_N should, of course, not exceed that of M_{pl}.

2.1.2 Bending and axial force for Class 3 cross-sections.

Figure 4 shows a point somewhere along the length of an H-shape column where the applied compression and moment about the *y* axis produce the uniform and varying stress distribution shown in figures 4a and 4b.

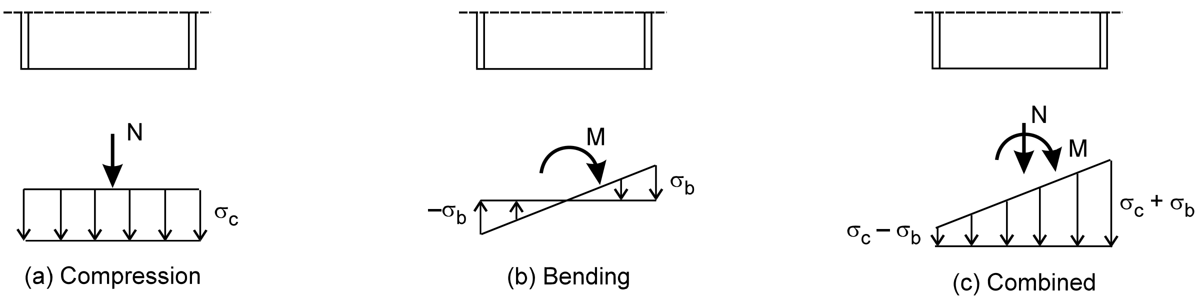


Figure 4 – Elastic behaviour of cross-section in compression and bending.

For <u>elastic</u> behaviour the principle of superposition may be used to simply add the two stress distributions as shown in figure 4c.

First yield will therefore develop at the edge where the maximum compressive bending stress occurs and will correspond to the condition:

$$f_y = \sigma_c + \sigma_b$$

where:

- f_y is the material yield stress, *h* is the overall depth of section and *I* is the second moment of area about the *y* axis.

- $\sigma_c = N / A$ is the stress due to the compressive load N
- $\sigma_b = \dfrac{Mh/2}{I}$ is the maximum compressive stress due to the moment M.

Class 3 cross-sections will be satisfactory if the maximum longitudinal stress $\sigma_{x.Ed}$ satisfies the criterion:

$\sigma_{x.Ed} \le f_{yd} \;\; ; \;\; f_{yd} = f_y / \gamma_{M0}$	*Eurocode 3* 5.4.8.2 (5.31) *or eq. 6.42*

2.1.3 Bending and axial force for class 4 cross-sections.

Class 4 cross-sections will be satisfactory if the maximum longitudinal stress $\sigma_{x.Ed}$ calculated using the effective widths of the compression elements (5.3.2.(2) of EC3) satisfies the criterion:

$\sigma_{x.Ed} \le f_{yd} \;\; ; \;\; f_{yd} = f_y / \gamma_{M0}$	*Eurocode 3* 5.4.8.32 (5.39) *or eq. 6.43*

2.2 OVERALL STABILITY.

The treatment of cross-sectional behaviour in the previous section took no account of the exact way in which the moment *M* at the particular cross-section under consideration was generated.

Figure 5 shows a beam-column undergoing lateral deflection as a result of the combination of compression and equal and opposite moments applied at the ends.

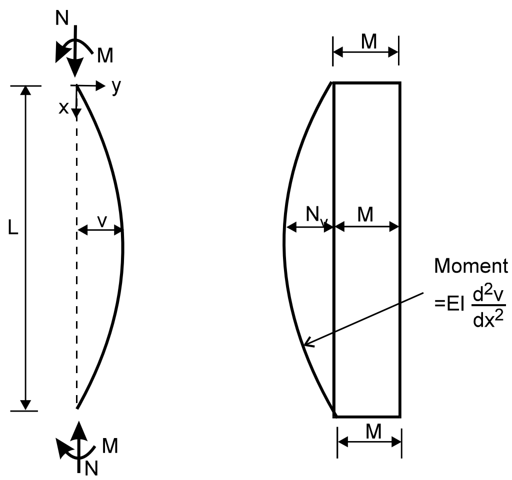


Figure 5 – Primary and secondary moments.

The moment at any point within the length may conveniently be regarded as being composed of:

- primary moment *M*
- secondary moment $N\,v$.

Using elastic strut theory gives the maximum deflection at the centre (Trahair & Bradford, 1988) as:

$$v_{\max} = \frac{M}{N} \sec \frac{\pi}{2} \sqrt{\frac{N}{P_{Ey}}} - 1 \tag{4}$$

where $P_{Ey} = \frac{\pi^2 EI_y}{L^2}$ is the Euler critical load for major axis buckling, and the maximum moment is:

$$M_{\max} = M \sec \frac{\pi}{2} \sqrt{\frac{N}{P_{Ey}}} \tag{5}$$

In both equations the secant term may be replaced by noting that the first order deflection (due only to the end moments) and the first order moment (ordinary beam theory) are approximately amplified by:

$$\frac{1}{1 - N / P_{Ey}} \tag{6}$$

as shown in figure 6.

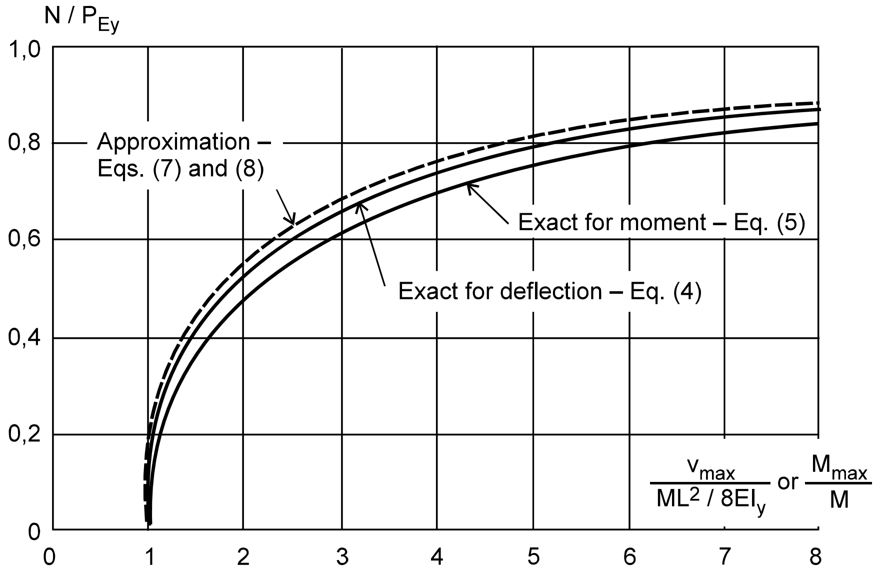


Figure 6 – Maximum deflection and moment in beam-columns with equal moments.

Thus:

$$v_{\max} = \frac{ML^2}{8EI_y} \frac{1}{1 - N / P_{Ey}} \tag{7}$$

$$M_{\max} = M \frac{1}{1 - N / P_{Ey}} \tag{8}$$

Since the maximum elastic stress will be:

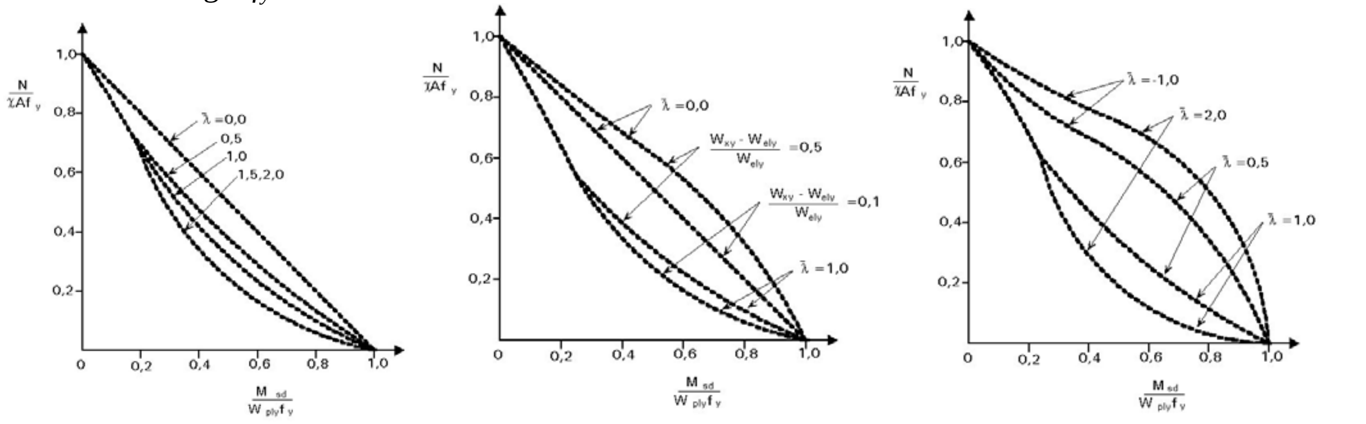
$$\sigma_{\max} = \sigma_c + \sigma_b \frac{M_{\max}}{M} \tag{9}$$

Eq. (9) may be rewritten as:

$$\frac{\sigma_c}{f_y} + \frac{\sigma_b}{f_y (1 - N / P_{Ey})} = 1{,}0 \tag{10}$$

Eq. (10) may be solved for values of σ_c and σ_b that just cause yield, taking different values of P_{Ey} (which is dependent on the slenderness L/r_y).

This gives a series of curves as shown in figure 7, which indicate that as $\sigma_b \rightarrow 0$, σ_c tends to the value of material strength f_y.



(a) (b) (c)

Figure 7 – Form of Eq. (10) effect of: (a) slenderness (b) cross sectional shape (c) moment gradient.

Eq. (10) does not recognise the possibility of buckling under pure axial load at a stress σ_{Ey} given by:

$$\sigma_{Ey} = \frac{P_{Ey}}{A} = \frac{\pi^2 EI_y}{AL^2} = \frac{\pi^2 E}{\lambda_y^2} \tag{11}$$

Use of both Eq. (10) and Eq. (11) ensures that both conditions are covered as shown in figure 8.

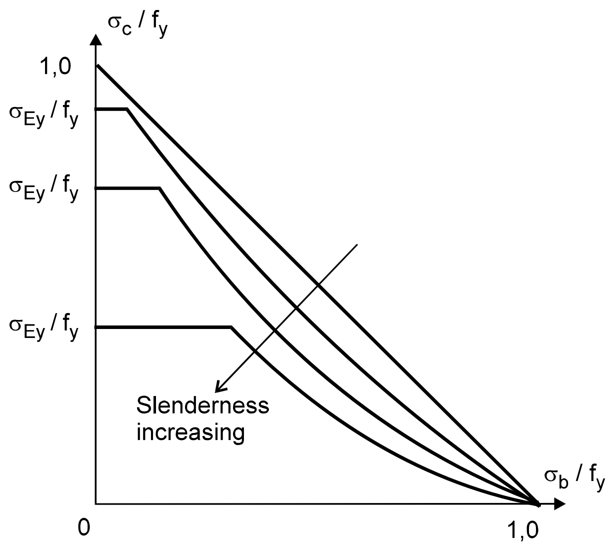


Figure 8 – Combination of Eqs. (10) and (11)

2.3 TREATMENT IN EUROCODE 3.

Eqs. (10) and (11) are written in terms of stresses and originate from the concept of “failure” being defined as either the attainment of first yield or elastic buckling of the perfect member.

Limit state design codes, as Eurocode 3, normally take ultimate load as the design criterion when considering resistance under static loading.

Thus these equations must be re-written in terms of forces and moments.

In doing this it is also necessary to make some allowance for those effects present in real structures that have not so far been explicitly allowed for, i.e. initial lack of straightness, residual stresses, etc.

For consistency in design it is essential that the interaction equation for combined loading reduces down to the column and beam design procedures as moment and axial load respectively reduce to zero.

2.3.1 Members with class 1 and 2 cross-sections.

The approach taken in Eurocode 3 (assuming bending about the *y* axis) is to use:

$\frac{N_{Sd}}{\chi_y A f_y} + \frac{k_y M_{y.Sd}}{W_{pl.y} f_y} \le 1$	(12)	*Eurocode 3* 5.5.4(1) (5.51) *or* *eq. 6.61 & 6.62*

in which χ_y is the reduction factor for column buckling, and

$$k_y = 1 - \frac{\mu_y N_{Sd}}{\chi_y A f_y} \quad \text{but} \quad k_y \le 1{,}5$$

where k_y is a modification factor discussed in section 2.3.4., and

$$\mu_y = \bar{\lambda}_y (2\beta_{My} - 4) + \frac{W_{pl,y}}{W_{el,y}} - 1 \quad \text{but} \quad \mu_y \le 0{,}90$$

where β_{My} is an equivalent uniform moment factor accounting for the non-uniformity of the moment diagram, see table 2 (moment diagram about *y* axis and restraints in the *z* direction).

2.3.2 Members with class 3 cross-sections.

Members with class 3 cross-sections subject to bending and axial load shall satisfy:

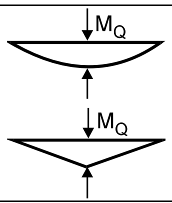
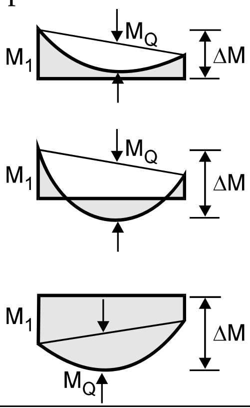
$\frac{N_{Sd}}{\chi_y A f_y} + \frac{k_y M_{y.Sd}}{W_{el.y} f_y} \le 1$	(13)	*Eurocode 3* 5.5.4(3) (5.53) *or* *eq. 6.61 & 6.62*

where k_y and χ_y is as in Eq. (12) with

$$\mu_y = \bar{\lambda}_y (2\beta_{My} - 4) \quad \text{but} \quad \mu_y \le 0{,}90$$

Table 2 – Equivalent uniform moment factors β_M. *(C_m)*

MOMENT DIAGRAM	EQUIVALENT UNIFORM MOMENT FACTOR β_M
End moments  $-1 \le \psi \le 1$	$\beta_{M,\psi} = 1{,}8 - 0{,}7\psi$
Moments due to in-plane lateral loads	For uniformly distributed load: $\beta_{M,Q} = 1{,}3$

	For concentrated load: $\beta_{M,Q} = 1{,}4$
Moments due to in-plane lateral loads plus end moments 	$\beta_M = \beta_{M,\psi} + \frac{M_Q}{\Delta M}(\beta_{M,Q} - \beta_{M,\psi})$ where: $M_Q = \lvert \max M \rvert$ due to lateral load only and $\Delta M = \lvert \max M \rvert$ for moment diagram without change of sign $\Delta M = \lvert \max M \rvert + \lvert \min M \rvert$ where sign of moment diagram changes

2.3.3 Members with class 4 cross-sections.

Members with class 3 cross-sections subject to bending and axial load shall satisfy:

$\frac{N_{Sd}}{\chi_y A_{eff} f_y} + \frac{k_y(M_{y.Sd} + N_{Sd}e_{N.z})}{W_{eff.y} f_y} \leq 1$ (14)	*Eurocode 3* 5.5.4(3) (5.56) *or eq. 6.61 & 6.62*

Where:

- k_y and χ_y is as in Eq. (12) with μ_y as in Eq. (13)
- $A_{eff.y}$ is the effective cross-sectional area for pure compression
- $W_{eff.y}$ is the effective cross-sectional modulus for pure bending
- $e_{N.z}$ is the shift in neutral axis comparing the full cross-section with the effective cross-section (calculated assuming pure compression) used to account for local buckling

2.3.4 The role of k_y.

The value of k_y, as shown by the equations explaining Eq. (12), depends in a rather complex way on:

- Level of axial load as measured by the ratio $\frac{N_{Sd}}{\chi_y A f_y}$
- Member slenderness λ_y
- Margin between the cross-section's plastic & elastic moduli W_{pl} & W_{el} (for class 1 & 2 only)
- Pattern of primary moments.

When all of this combine in the most severe way the safe value of k_y is 1,5.
The role of k_y is to allow for the secondary bending effect described earlier plus the effects of non-uniform moment and spread of yield.

Figure 5 showed how, for the particular cases of equal and opposite beam moments, the primary moments are amplified due to the effect of the axial load *N* acting through the lateral displacements *v*.

When the pattern of primary moments is different, the two effects will not be so directly additive since maximum primary and secondary moments will not necessarily occur at the same location.

Figure 9 illustrates the situation for end moments M and ψM, where ψ can adopt values between +1 (uniform single curvature) and –1 (double curvature).

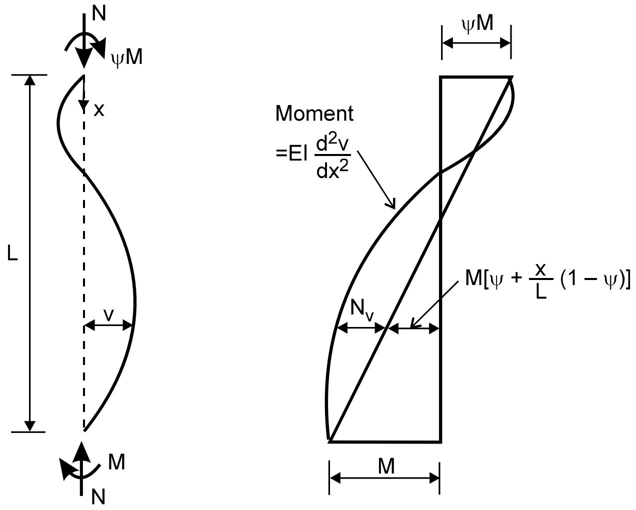


Figure 9 – Non-uniform moment case.

The particular case shown corresponds to a value $\psi \cong -0{,}5$.

For the case illustrated the maximum moment still occurs within the member length but the situation is clearly less severe than that of figure 5 assuming all conditions to be identical apart from the value of ψ.

It is customary to recognise this in design by reducing the contribution of the moment term to the interaction relationship. Thus in Eurocode 3 k_y in Eq. (12) depends upon the ratio ψ.

Since the case of uniform single curvature moment is the most severe, it follows that a safe simplification is always to use the procedure for $\psi = 1{,}0$.

Returning to figure 9, it is possible for the point of maximum moment to be at the end at which the larger primary moment is applied.

This would usually occur if the axial load was small and/or slenderness was low so that secondary bending effects were relatively slight.

In such cases design will be controlled by ensuring adequate cross-sectional resistance at this end. The formula, table 2, for the particular shape of cross-section being used, should therefore be employed.

In cases where only the uniform moment ($\psi = 1.0$) arrangement is being considered, the overall buckling check of Eq. (12) will always be more severe than (or in the limit equal to) the cross-sectional check which, and therefore this latter check need not be performed separately.

6.3.3 Uniform members in bending and axial compression

(1) Unless second order analysis is carried out using the imperfections as given in 5.3.2, the stability of uniform members with double symmetric cross sections for sections not susceptible to distortional deformations should be checked as given in the following clauses, where a distinction is made for:

- members that are not susceptible to torsional deformations, e.g. circular hollow sections or sections restraint from torsion
- members that are susceptible to torsional deformations, e.g. members with open cross-sections and not restraint from torsion.

(2) In addition, the resistance of the cross-sections at each end of the member should satisfy the requirements given in 6.2.

NOTE 1 The interaction formulae are based on the modelling of simply supported single span members with end fork conditions and with or without continuous lateral restraints, which are subjected to compression forces, end moments and/or transverse loads.

NOTE 2 In case the conditions of application expressed in (1) and (2) are not fulfilled, see 6.3.4.

(3) For members of structural systems the resistance check may be carried out on the basis of the individual single span members regarded as cut out of the system. Second order effects of the sway system (P-Δ-effects) have to be taken into account, either by the end moments of the member or by means of appropriate buckling lengths respectively.

(4) Members which are subjected to combined bending and axial compression should satisfy:

$$\frac{N_{Ed}}{\frac{\chi_y N_{Rk}}{\gamma_{M1}}} + k_{yy} \frac{M_{y,Ed} + \Delta M_{y,Ed}}{\chi_{LT} \frac{M_{y,Rk}}{\gamma_{M1}}} + k_{yz} \frac{M_{z,Ed} + \Delta M_{z,Ed}}{\frac{M_{z,Rk}}{\gamma_{M1}}} \leq 1 \qquad (6.61)$$

$$\frac{N_{Ed}}{\frac{\chi_z N_{Rk}}{\gamma_{M1}}} + k_{zy} \frac{M_{y,Ed} + \Delta M_{y,Ed}}{\chi_{LT} \frac{M_{y,Rk}}{\gamma_{M1}}} + k_{zz} \frac{M_{z,Ed} + \Delta M_{z,Ed}}{\frac{M_{z,Rk}}{\gamma_{M1}}} \leq 1 \qquad (6.62)$$

where N_{Ed}, $M_{y,Ed}$ and $M_{z,Ed}$ are the design values of the compression force and the maximum moments about the y-y and z-z axis along the member, respectively

$\Delta M_{y,Ed}$, $\Delta M_{z,Ed}$ are the moments due to the shift of the centroidal axis according to 6.2.9.3 for class 4 sections, see Table 6.7,

χ_y and χ_z are the reduction factors due to flexural buckling from 6.3.1

χ_{LT} is the reduction factor due to lateral torsional buckling from 6.3.2

k_{yy}, k_{yz}, k_{zy}, k_{zz} are the interaction factors

Table 6.7: Values for $N_{Rk} = f_y A_i$, $M_{i,Rk} = f_y W_i$ and $\Delta M_{i,Ed}$

Class	1	2	3	4
A_i	A	A	A	A_{eff}
W_y	$W_{pl,y}$	$W_{pl,y}$	$W_{el,y}$	$W_{eff,y}$
W_z	$W_{pl,z}$	$W_{pl,z}$	$W_{el,z}$	$W_{eff,z}$
$\Delta M_{y,Ed}$	0	0	0	$e_{N,y} N_{Ed}$
$\Delta M_{z,Ed}$	0	0	0	$e_{N,z} N_{Ed}$

NOTE For members not susceptible to torsional deformation χ_{LT} would be $\chi_{LT} = 1{,}0$.

(5) The interaction factors k_{yy} , k_{yz} , k_{zy} , k_{zz} depend on the method which is chosen.

NOTE 1 The interaction factors k_{yy}, k_{yz}, k_{zy} and k_{zz} have been derived from two alternative approaches. Values of these factors may be obtained from Annex A (alternative method 1) or from Annex B (alternative method 2).

NOTE 2 The National Annex may give a choice from alternative method 1 or alternative method 2.

NOTE 3 For simplicity verifications may be performed in the elastic range only.

Annex A [informative] – Method 1: Interaction factors k_{ij} for interaction formula in 6.3.3(4)

Table A.1: Interaction factors k_{ij} (6.3.3(4))

Interaction factors	Design assumptions: elastic cross-sectional properties class 3, class 4	Design assumptions: plastic cross-sectional properties class 1, class 2
k_{yy}	$C_{my} C_{mLT} \dfrac{\mu_y}{1 - \dfrac{N_{Ed}}{N_{cr,y}}}$	$C_{my} C_{mLT} \dfrac{\mu_y}{1 - \dfrac{N_{Ed}}{N_{cr,y}}} \dfrac{1}{C_{yy}}$
k_{yz}	$C_{mz} \dfrac{\mu_y}{1 - \dfrac{N_{Ed}}{N_{cr,z}}}$	$C_{mz} \dfrac{\mu_y}{1 - \dfrac{N_{Ed}}{N_{cr,z}}} \dfrac{1}{C_{yz}} 0{,}6 \sqrt{\dfrac{w_z}{w_y}}$
k_{zy}	$C_{my} C_{mLT} \dfrac{\mu_z}{1 - \dfrac{N_{Ed}}{N_{cr,y}}}$	$C_{my} C_{mLT} \dfrac{\mu_z}{1 - \dfrac{N_{Ed}}{N_{cr,y}}} \dfrac{1}{C_{zy}} 0{,}6 \sqrt{\dfrac{w_y}{w_z}}$
k_{zz}	$C_{mz} \dfrac{\mu_z}{1 - \dfrac{N_{Ed}}{N_{cr,z}}}$	$C_{mz} \dfrac{\mu_z}{1 - \dfrac{N_{Ed}}{N_{cr,z}}} \dfrac{1}{C_{zz}}$

Auxiliary terms:

$$\mu_y = \frac{1 - \dfrac{N_{Ed}}{N_{cr,y}}}{1 - \chi_y \dfrac{N_{Ed}}{N_{cr,y}}}$$

$$\mu_z = \frac{1 - \dfrac{N_{Ed}}{N_{cr,z}}}{1 - \chi_z \dfrac{N_{Ed}}{N_{cr,z}}}$$

$$w_y = \frac{W_{pl,y}}{W_{el,y}} \leq 1{,}5$$

$$w_z = \frac{W_{pl,z}}{W_{el,z}} \leq 1{,}5$$

$$n_{pl} = \frac{N_{Ed}}{N_{Rk} / \gamma_{M1}}$$

C_{my} see Table A.2

$$a_{LT} = 1 - \frac{I_T}{I_y} \geq 0$$

$$C_{yy} = 1 + (w_y - 1)\left[\left(2 - \frac{1{,}6}{w_y} C_{my}^2 \bar{\lambda}_{max} - \frac{1{,}6}{w_y} C_{my}^2 \bar{\lambda}_{max}^2\right) n_{pl} - b_{LT}\right] \geq \frac{W_{el,y}}{W_{pl,y}}$$

with $b_{LT} = 0{,}5\, a_{LT}\, \bar{\lambda}_0^2 \dfrac{M_{y,Ed}}{\chi_{LT} M_{pl,y,Rd}} \dfrac{M_{z,Ed}}{M_{pl,z,Rd}}$

$$C_{yz} = 1 + (w_z - 1)\left[\left(2 - 14 \frac{C_{mz}^2 \bar{\lambda}_{max}^2}{w_z^5}\right) n_{pl} - c_{LT}\right] \geq 0{,}6 \sqrt{\frac{w_z}{w_y}} \frac{W_{el,z}}{W_{pl,z}}$$

with $c_{LT} = 10\, a_{LT} \dfrac{\bar{\lambda}_0^2}{5 + \bar{\lambda}_z^4} \dfrac{M_{y,Ed}}{C_{my}\, \chi_{LT}\, M_{pl,y,Rd}}$

$$C_{zy} = 1 + (w_y - 1)\left[\left(2 - 14 \frac{C_{my}^2 \bar{\lambda}_{max}^2}{w_y^5}\right) n_{pl} - d_{LT}\right] \geq 0{,}6 \sqrt{\frac{w_y}{w_z}} \frac{W_{el,y}}{W_{pl,y}}$$

with $d_{LT} = 2\, a_{LT} \dfrac{\bar{\lambda}_0}{0{,}1 + \bar{\lambda}_z^4} \dfrac{M_{y,Ed}}{C_{my}\, \chi_{LT}\, M_{pl,y,Rd}} \dfrac{M_{z,Ed}}{C_{mz}\, M_{pl,z,Rd}}$

$$C_{zz} = 1 + (w_z - 1)\left[\left(2 - \frac{1{,}6}{w_z} C_{mz}^2 \bar{\lambda}_{max} - \frac{1{,}6}{w_z} C_{mz}^2 \bar{\lambda}_{max}^2\right) n_{pl} - e_{LT}\right] \geq \frac{W_{el,z}}{W_{pl,z}}$$

with $e_{LT} = 1{,}7\, a_{LT} \dfrac{\bar{\lambda}_0}{0{,}1 + \bar{\lambda}_z^4} \dfrac{M_{y,Ed}}{C_{my}\, \chi_{LT}\, M_{pl,y,Rd}}$

Table A.1 (continued)

$$\bar{\lambda}_{max} = \max\begin{cases}\bar{\lambda}_y \\ \bar{\lambda}_z\end{cases}$$

$\bar{\lambda}_0$ = non-dimensional slenderness for lateral-torsional buckling due to uniform bending moment, i.e. $\psi_y = 1{,}0$ in Table A.2

$\bar{\lambda}_{LT}$ = non-dimensional slenderness for lateral-torsional buckling

For $\bar{\lambda}_0 = 0$:

$$C_{my} = C_{my,0}$$
$$C_{mz} = C_{mz,0}$$
$$C_{mLT} = 1{,}0$$

For $\bar{\lambda}_0 > 0$:

$$C_{my} = C_{my,0} + (1 - C_{my,0})\frac{\sqrt{\varepsilon_y}\, a_{LT}}{1 + \sqrt{\varepsilon_y}\, a_{LT}}$$

$$C_{mz} = C_{mz,0}$$

$$C_{mLT} = C_{my}^2 \frac{a_{LT}}{\sqrt{\left(1 - \frac{N_{Ed}}{N_{cr,z}}\right)\left(1 - \frac{N_{Ed}}{N_{cr,T}}\right)}}$$

$\varepsilon_y = \dfrac{M_{y,Ed}}{N_{Ed}} \dfrac{A}{W_{el,y}}$ for class 1, 2 and 3 cross-sections

$\varepsilon_y = \dfrac{M_{y,Ed}}{N_{Ed}} \dfrac{A_{eff}}{W_{eff,y}}$ for class 4 cross-sections

$N_{cr,y}$ = elastic flexural buckling force about the y-y axis

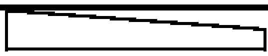
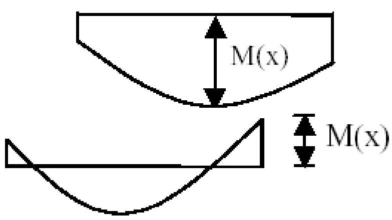
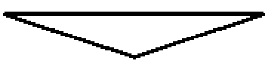
$N_{cr,z}$ = elastic flexural buckling force about the z-z axis

$N_{cr,T}$ = elastic torsional buckling force

I_T = St. Venant torsional constant

I_y = second moment of area about y-y axis

Table A.2: Equivalent uniform moment factors $C_{mi,0}$

Moment diagram	$C_{mi,0}$
M_1  ψM_1 $-1 \le \psi \le 1$	$C_{mi,0} = 0{,}79 + 0{,}21\psi_i + 0{,}36(\psi_i - 0{,}33)\frac{N_{Ed}}{N_{cr,i}}$
	$C_{mi,0} = 1 + \left(\frac{\pi^2 E I_i \lvert\delta_x\rvert}{L^2 \lvert M_{i,Ed}(x)\rvert} - 1\right)\frac{N_{Ed}}{N_{cr,i}}$ $M_{i,Ed}(x)$ is the maximum moment $M_{y,Ed}$ or $M_{z,Ed}$ $\lvert\delta_x\rvert$ is the maximum member displacement along the member
 	$C_{mi,0} = 1 - 0{,}18\frac{N_{Ed}}{N_{cr,i}}$ $C_{mi,0} = 1 + 0{,}03\frac{N_{Ed}}{N_{cr,i}}$

Annex B [informative] – Method 2: Interaction factors k_{ij} for interaction formula in 6.3.3(4)

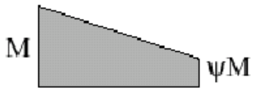
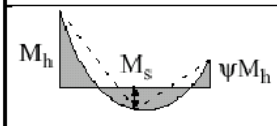
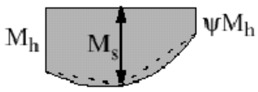
Table B.1: Interaction factors k_{ij} for members not susceptible to torsional deformations

Interaction factors	Type of sections	Design assumption: elastic cross-sectional properties class 3, class 4	Design assumption: plastic cross-sectional properties class 1, class 2
k_{yy}	I-sections RHS-sections	$C_{my}\left(1+0,6\bar{\lambda}_y \frac{N_{Ed}}{\chi_y N_{Rk}/\gamma_{M1}}\right) \le C_{my}\left(1+0,6\frac{N_{Ed}}{\chi_y N_{Rk}/\gamma_{M1}}\right)$	$C_{my}\left(1+\left(\bar{\lambda}_y-0,2\right)\frac{N_{Ed}}{\chi_y N_{Rk}/\gamma_{M1}}\right) \le C_{my}\left(1+0,8\frac{N_{Ed}}{\chi_y N_{Rk}/\gamma_{M1}}\right)$
k_{yz}	I-sections RHS-sections	k_{zz}	$0,6\ k_{zz}$
k_{zy}	I-sections RHS-sections	$0,8\ k_{yy}$	$0,6\ k_{yy}$
k_{zz}	I-sections	$C_{mz}\left(1+0,6\bar{\lambda}_z \frac{N_{Ed}}{\chi_z N_{Rk}/\gamma_{M1}}\right) \le C_{mz}\left(1+0,6\frac{N_{Ed}}{\chi_z N_{Rk}/\gamma_{M1}}\right)$	$C_{mz}\left(1+\left(2\bar{\lambda}_z-0,6\right)\frac{N_{Ed}}{\chi_z N_{Rk}/\gamma_{M1}}\right) \le C_{mz}\left(1+1,4\frac{N_{Ed}}{\chi_z N_{Rk}/\gamma_{M1}}\right)$
	RHS-sections		$C_{mz}\left(1+\left(\bar{\lambda}_z-0,2\right)\frac{N_{Ed}}{\chi_z N_{Rk}/\gamma_{M1}}\right) \le C_{mz}\left(1+0,8\frac{N_{Ed}}{\chi_z N_{Rk}/\gamma_{M1}}\right)$
For I- and H-sections and rectangular hollow sections under axial compression and uniaxial bending $M_{y,Ed}$ the coefficient k_{zy} may be $k_{zy} = 0$.			

Table B.2: Interaction factors k_{ij} for members susceptible to torsional deformations

Interaction factors	Design assumptions: elastic cross-sectional properties class 3, class 4	Design assumptions: plastic cross-sectional properties class 1, class 2
k_{yy}	k_{yy} from Table B.1	k_{yy} from Table B.1
k_{yz}	k_{yz} from Table B.1	k_{yz} from Table B.1
k_{zy}	$\left[1-\frac{0,05\bar{\lambda}_z}{\left(C_{mLT}-0,25\right)}\frac{N_{Ed}}{\chi_z N_{Rk}/\gamma_{M1}}\right] \ge \left[1-\frac{0,05}{\left(C_{mLT}-0,25\right)}\frac{N_{Ed}}{\chi_z N_{Rk}/\gamma_{M1}}\right]$	$\left[1-\frac{0,1\bar{\lambda}_z}{\left(C_{mLT}-0,25\right)}\frac{N_{Ed}}{\chi_z N_{Rk}/\gamma_{M1}}\right] \ge \left[1-\frac{0,1}{\left(C_{mLT}-0,25\right)}\frac{N_{Ed}}{\chi_z N_{Rk}/\gamma_{M1}}\right]$ for $\bar{\lambda}_z < 0,4$: $k_{zy} = 0,6+\bar{\lambda}_z \le 1-\frac{0,1\bar{\lambda}_z}{\left(C_{mLT}-0,25\right)}\frac{N_{Ed}}{\chi_z N_{Rk}/\gamma_{M1}}$
k_{zz}	k_{zz} from Table B.1	k_{zz} from Table B.1

Table B.3: Equivalent uniform moment factors C_m in Tables B.1 and B.2

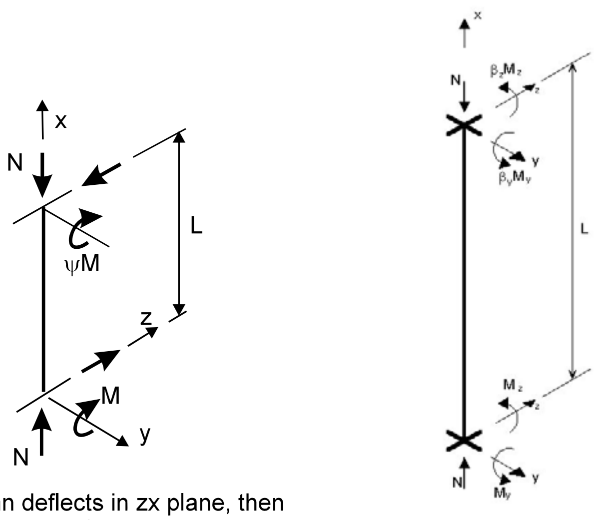
Moment diagram	range		C_{my} and C_{mz} and C_{mLT} uniform loading	C_{my} and C_{mz} and C_{mLT} concentrated load
	$-1 \le \psi \le 1$		$0,6 + 0,4\psi \ge 0,4$	
	$0 \le \alpha_s \le 1$	$-1 \le \psi \le 1$	$0,2 + 0,8\alpha_s \ge 0,4$	$0,2 + 0,8\alpha_s \ge 0,4$
	$-1 \le \alpha_s < 0$	$0 \le \psi \le 1$	$0,1 - 0,8\alpha_s \ge 0,4$	$-0,8\alpha_s \ge 0,4$
$\alpha_s = M_s/M_h$		$-1 \le \psi < 0$	$0,1(1-\psi) - 0,8\alpha_s \ge 0,4$	$0,2(-\psi) - 0,8\alpha_s \ge 0,4$
	$0 \le \alpha_h \le 1$	$-1 \le \psi \le 1$	$0,95 + 0,05\alpha_h$	$0,90 + 0,10\alpha_h$
	$-1 \le \alpha_h < 0$	$0 \le \psi \le 1$	$0,95 + 0,05\alpha_h$	$0,90 + 0,10\alpha_h$
$\alpha_h = M_h/M_s$		$-1 \le \psi < 0$	$0,95 + 0,05\alpha_h(1+2\psi)$	$0,90 - 0,10\alpha_h(1+2\psi)$
For members with sway buckling mode the equivalent uniform moment factor should be taken $C_{my} = 0,9$ or $C_{Mz} = 0,9$ respectively.				
C_{my}, C_{mz} and C_{mLT} shall be obtained according to the bending moment diagram between the relevant braced points as follows:				

moment factor	bending axis	points braced in direction
C_{my}	y-y	z-z
C_{mz}	z-z	y-y
C_{mLT}	y-y	y-y

3. LATERAL-TORSIONAL BEHAVIOUR OF BEAM-COLUMNS.

When an unrestrained beam-column is bent about its major axis (figure 10a), it may buckle by deflecting laterally & twisting at a load significantly less than the maximum load predicted by an in-plane analysis.

The most general situation is illustrated in Figure 10b. When bending is applied about both principal axes the member's response will be 3-dimensional in nature, involving biaxial bending and twisting.



Column deflects in zx plane, then buckles by deflecting in yx plane and twisting about x axis

(a)

Column deflects in zx and yx planes and twists about x - axis

(b)

Figure 10 – Lateral-torsional behaviour.

This lateral-torsional buckling may occur while the member is still elastic (curve 1 of figure 11), or after some yielding (curve 2) due to in-plane bending and compression has occurred.

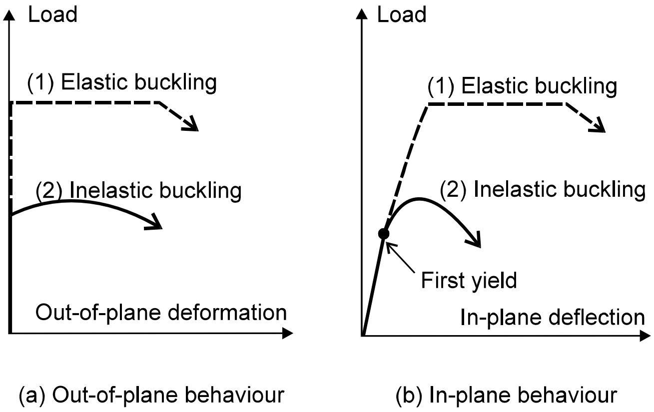


Figure 11 – Lateral-torsional buckling of beam-columns.

3.1 LATERAL-TORSIONAL BUCKLING.

Considering the lateral-torsional behaviour of an unrestrained I section beam-column bent about its major axis it can be assumed the elastic behaviour and the arrangement of applied loading and support conditions given in figure 12.

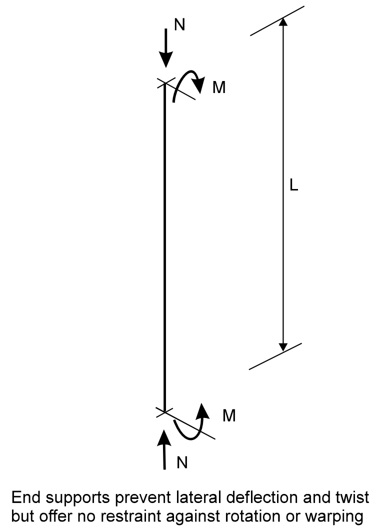


Figure 12 – Basic case for lateral-torsional buckling.

The critical combinations of N and M may be obtained from the solution of (Chen & Atsuta, 1976):

$$\frac{M^2}{i_0^2 P_{Ez} P_{E0}} = \left(1 - \frac{N}{P_{Ez}}\right)\left(1 - \frac{N}{P_{E0}}\right) \tag{15}$$

in which $i_0 = \sqrt{\frac{I_y + I_z}{A}}$ is the polar radius of gyration

$P_{Ez} = \frac{\pi^2 EI_z}{L^2}$ is the minor axis critical load

$P_{E0} = \frac{GI_t}{i_0^2}\left(1 + \frac{\pi^2 EI_w}{GI_t L^2}\right)$ is the torsional buckling load.

Eq. (15) reduces to the buckling of a beam when $N \to 0$ and to the buckling of a column in either flexure (P_{Ez}) or torsion (P_{E0}) as $M \to 0$. In the first case the critical value of M will be given by:

$$M_{cr} = \frac{\pi}{L}\sqrt{EI_z GI_t}\sqrt{1+\frac{\pi^2 EI_w}{L^2 GI_t}} \tag{16}$$

in which: EI_z is the minor axis flexural rigidity
GI_t is the torsional rigidity
EI_w is the warping rigidity.

In deriving Eq. (15) no allowance was made for the amplification of the in-plane moments M by the axial load acting through the in-plane deflections.

This may be approximated as $\dfrac{M}{1 - N/P_{Ey}}$. Eq. (15) can, therefore, be modified to:

$$\frac{M^2}{i_0^2 P_{Ez} P_{E0}} = \left(1-\frac{N}{P_{Ey}}\right)\left(1-\frac{N}{P_{Ez}}\right)\left(1-\frac{N}{P_{E0}}\right) \tag{17}$$

Noting the relative magnitudes of P_{Ey}, P_{Ez} and P_{E0}, and re-arranging gives the following approximation:

$$\frac{N}{P_{Ez}} + \frac{1}{1-N/P_{Ey}}\frac{M}{i_0\sqrt{P_{Ez}P_{E0}}} = 1 \tag{18}$$

or

$$\frac{N}{P_{Ez}} + \frac{1}{1-N/P_{Ey}}\frac{M}{M_{cr}} = 1 \tag{19}$$

3.2 THE DESIGN PROCESS IN EUROCODE 3.

For design purposes it is necessary to make suitable allowances for effects such as initial lack of straightness, partial yielding, residual stresses, etc., as has been fully discussed in earlier lectures in the context of columns and beams.

Thus some modification to Eq. (19) is necessary to make it suitable for design.

In particular, the end points (corresponding to the cases of M = 0 and N = 0) must conform to the established procedures for columns and beams

3.2.1 Members with class 1 and 2 cross-sections.

Eurocode 3 uses the interaction equation:

$\dfrac{N_{Sd}}{\chi_z A f_y} + \dfrac{k_{LT} M_{y.Sd}}{\chi_{LT} W_{pl.y} f_y} \le 1$ (20)	*Eurocode 3* 5.5.4(2) (5.52) *or eq. 6.61 & 6.62*

in which χ_z is the reduction factor for column buckling around the minor axis, χ_{LT} is the reduction factor for lateral-torsional beam buckling, and

$$k_{LT} = 1 - \frac{\mu_{LT} N_{Sd}}{\chi_z A f_y} \quad \text{but} \quad k_{LT} \leq 1{,}0$$

with

$$\mu_{LT} = 0{,}15(\bar{\lambda}_z 2\beta_{M,LT} - 1) \quad \text{but} \quad \mu_{LT} \leq 0{,}90$$

where $\beta_{M,LT}$ is a factor accounting for the non-uniformity of the moment diagram, see Table 2 (moment diagram about *y* axis and restrains in the *y* direction).

3.2.2 Members with class 3 cross-sections.

Members with class 3 cross-section should satisfy the following criterion:

$\frac{N_{Sd}}{\chi_z A f_y} + \frac{k_{LT} M_{y.Sd}}{\chi_{LT} W_{el.y} f_y} \leq 1$	(21)	*Eurocode 3* 5.5.4(4) (5.54) *or* *eq. 6.61 & 6.62*

3.2.3 Members with class 4 cross-sections.

Members with class 3 cross-section should satisfy the following criterion:

$\frac{N_{Sd}}{\chi_z A f_y} + \frac{k_{LT} M_{y.Sd} + N_{Sd} e_{N,z}}{\chi_{LT} W_{eff.y} f_y} \leq 1$	(22)	*Eurocode 3* 5.5.4(5) (5.57) *or* *eq. 6.61 & 6.62*

3.3 The role of k_{LT}.

The value of k_{LT}, as shown by the equations explaining Eq. (20), depends on:

- the level of axial load as measured by the ratio $\frac{N_{Sd}}{\chi_z A f_y}$
- the member slenderness λ_z
- the pattern of primary moments.

For the most severe combination k_{LT} adopts the value of unity, corresponding to a linear combination of the compressive and bending terms.

This reflects the reduced scope for amplification effects in this case, since the value of N_{Sd} cannot exceed $\chi_z A f_y$, which will, in turn, be significantly less than the elastic critical load for in-plane buckling P_{Ey}.

It is, of course, also necessary to ensure against the possibility of in-plane failure by excessive deflection in the plane of the web at a lower load than that given by Eq. (20).

This might occur, for example, in situations where different bracing and/or support conditions are provided in the *xy* and *xz* planes as illustrated in figure 13.

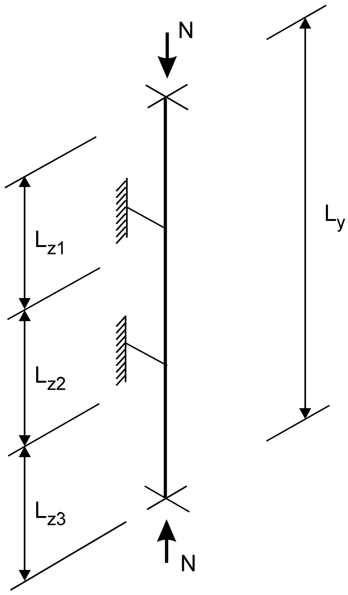


Figure 13 – Column with different support conditions in *xy* and *xz* planes.

Such cases should be treated by checking, in addition to Eq. (20), an in-plane equation of the form:

$$\frac{N_{Sd}}{\chi_{\min} A f_y} + \frac{k_y M_{y.Sd}}{W_{pl.y} f_y} \leq 1 \qquad (23)$$

in which χ_{min} depends on the in-plane conditions. Usually, however, Eq. (20) will govern.

4. BIAXIAL BENDING OF BEAM-COLUMNS.

Analysis for the full three-dimensional case, even for the simple elastic version, is extremely complex and closed-form solutions are not available.

Rather than starting analytically it is more convenient to approach the question of a suitable design approach from considerations of behaviour and the use of the methods already derived for the simpler cases of figure 14.

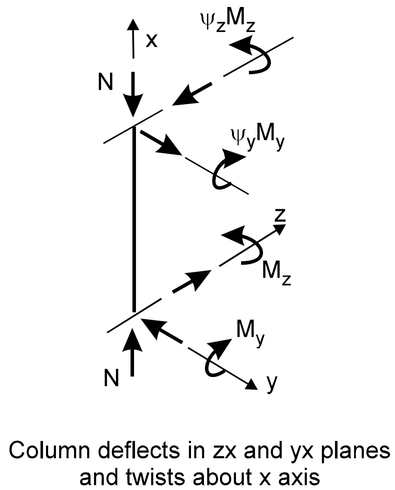


Figure 14 – Biaxial bending.

Figure 15 presents a diagrammatic version of the design requirement.

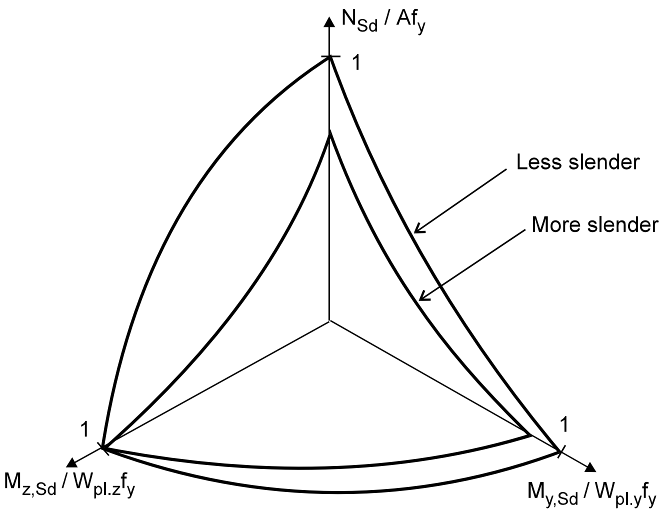


Figure 15 – Interaction diagram for biaxial bending.

The N-M_z and N-M_y axes correspond to the two uniaxial cases already examined.

Interaction between the two moments M_z and M_y corresponds to the horizontal plane.

When all the three load components N, M_y and M_z are present the resulting interaction plots somewhere in the three-dimensional space represented by the diagram.

Any point falling within the boundary corresponds to a safe combination of loads.

Assuming proportional loading, any combination may be regarded as a straight line starting at the origin, the orientation of which depends upon the relative sizes of the three load components.

Increasing the loads extends this line from the origin until it just reaches and exceeds the boundary.

In each case the axes have been taken as the ratio of the applied component to the member's resistance under the load component alone, e.g. $N_{Sd}/\chi_{min} A f_y$ in the case of the compressive loading.

Thus figure 15 actually represents the situation for one particular example with particular values of cross-sectional properties, slenderness and load arrangement.

4.1 DESIGN FOR BIAXIAL BENDING AND COMPRESSION.

Members with class 1 and 2 cross-sections subject to combined biaxial bending and axial compression should satisfy the following criterion:

$\dfrac{N_{Sd}}{\chi_{\min} A f_y} + \dfrac{k_y M_{y,Sd}}{W_{pl,y} f_y} + \dfrac{k_z M_{z,Sd}}{W_{pl,z} f_y} \le 1$	(24)	*Eurocode 3* 5.5.4(1) (5.51) *or eq. 6.61 & 6.62*

where k_z is a factor similar to k_y, see Eq. (12).

Members with class 1 and 2 cross-sections subject to combined biaxial bending and axial compression where lateral-torsional buckling is relevant, should also satisfy the following criterion:

$$\frac{N_{Sd}}{\chi_z A f_y} + \frac{k_{LT} M_{y,Sd}}{\chi_{LT} W_{pl,y} f_y} + \frac{k_z M_{z,Sd}}{W_{pl,z} f_y} \leq 1 \qquad (25)$$	*Eurocode 3* 5.5.4(2) (5.52) *or eq. 6.61 & 6.62*

Members with class 3 cross-sections subject to combined biaxial bending and axial compression:

$$\frac{N_{Sd}}{\chi_{\min} A f_y} + \frac{k_y M_{y,Sd}}{W_{el,y} f_y} + \frac{k_z M_{z,Sd}}{W_{el,z} f_y} \leq 1 \qquad (26)$$	*Eurocode 3* 5.5.4(3) (5.53) *or eq. 6.61 & 6.62*

Members with class 3 cross-sections subject to combined biaxial bending and axial compression where lateral-torsional buckling is relevant, should also satisfy the following criterion:

$$\frac{N_{Sd}}{\chi_z A f_y} + \frac{k_{LT} M_{y,Sd}}{\chi_{LT} W_{el,y} f_y} + \frac{k_z M_{z,Sd}}{W_{el,z} f_y} \leq 1 \qquad (27)$$	*Eurocode 3* 5.5.4(4) (5.54) *or eq. 6.61 & 6.62*

Members with class 4 cross-sections subject to combined biaxial bending and axial compression:

$$\frac{N_{Sd}}{\chi_{\min} A_{eff} f_y} + \frac{k_y (M_{y,Sd} + N_{Sd} e_{Nz})}{W_{eff,y} f_y} + \frac{k_z (M_{z,Sd} + N_{Sd} e_{Ny})}{W_{eff,z} f_y} \leq 1 \qquad (28)$$	*Eurocode 3* 5.5.4(5) (5.56) *or eq.6.44, 6.61 & 6.62*

Members with class 4 cross-sections subject to combined biaxial bending and axial compression where lateral-torsional buckling is relevant, should also satisfy the following criterion:

$$\frac{N_{Sd}}{\chi_z A_{eff} f_y} + \frac{k_{LT} M_{y,Sd} + N_{Sd} e_{Ny}}{\chi_{LT} W_{eff,y} f_y} + \frac{k_z (M_{z,Sd} + N_{Sd} e_{Nz})}{W_{eff,z} f_y} \leq 1 \qquad (29)$$	*Eurocode 3* 5.5.4(6) (5.57) *or eq.6.44, 6.61 & 6.62*

An important point to note from the definition of A_{eff} and W_{eff} above is that the calculation of cross-sectional properties, and thus also cross-sectional classification, should be undertaken on a separate basis for each of the three load components N, M_y and M_z.

This does, of course, mean that the same member may be classified as (say) class 1 for major axis bending, class 2 for minor axis bending and class 3 for compression.

The safe design approach is to check all beam-columns using the least favourable class procedures.

(2) The following criterion should be met:

$$\frac{N_{Ed}}{A_{eff}\, f_y / \gamma_{M0}} + \frac{M_{y,Ed} + N_{Ed}\, e_{Ny}}{W_{eff,y,min}\, f_y / \gamma_{M0}} + \frac{M_{z,Ed} + N_{Ed}\, e_{Nz}}{W_{eff,z,min}\, f_y / \gamma_{M0}} \leq 1 \qquad (6.44)$$

where A_{eff} is the effective area of the cross-section when subjected to uniform compression

$W_{eff,min}$ is the effective section modulus (corresponding to the fibre with the maximum elastic stress) of the cross-section when subjected only to moment about the relevant axis

e_N is the shift of the relevant centroidal axis when the cross-section is subjected to compression only, see 6.2.2.5(4)

NOTE The signs of N_{Ed}, $M_{y,Ed}$, $M_{z,Ed}$ and $\Delta M_i = N_{Ed}\, e_{Ni}$ depend on the combination of the respective direct stresses.

Class 4

4.2 CROSS-SECTION CHECKS.

If allowance has been made when determining the *k* factors (through the use of β_M) for the less severe effect of patterns of moment other than the uniform single curvature bending, it is necessary further to check that the cross-section is everywhere capable of locally restraining the combination of compression and primary moment(s) present at any point.

Expressions for checking several types of cross-section under compression plus uniaxial bending were given in section 1.1. For biaxial bending Eurocode 3 uses:

$\left(\dfrac{M_{y.Sd}}{M_{Ny.Rd}}\right)^{\alpha}+\left(\dfrac{M_{z.Sd}}{M_{Nz.Rd}}\right)^{\beta}\leq 1$	(30)	*Eurocode 3* 5.5.8.1 (5.35) *or eq.6.41*

in which the values of α and β depend upon the type of cross-section as indicated in table 3.

Table 3 – Values of α and β for use in Eq. (30) (Notation: $n = N_{Sd} / N_{pl.Rd}$).

Type of cross-section	α	β
I and H sections	2	$5n$ but ≥ 1
Circular tubes	2	2
Rectangular hollow sections	$\dfrac{1{,}66}{1-1{,}33n^2}$ but ≤ 6	$\dfrac{1{,}66}{1-1{,}33n^2}$ but ≤ 6
Solid rectangles and plates	$1{,}73 + 1{,}8n^3$	$1{,}73 + 1{,}8n^3$

(6) For bi-axial bending the following criterion may be used:

$$\left[\frac{M_{y,Ed}}{M_{N,y,Rd}}\right]^{\alpha}+\left[\frac{M_{z,Ed}}{M_{N,z,Rd}}\right]^{\beta}\leq 1 \qquad (6.41)$$

in which α and β are constants, which may conservatively be taken as unity, otherwise as follows:

- I and H sections:

  $\alpha = 2\ ;\ \beta = 5n \quad \text{but}\ \beta \geq 1$

- circular hollow sections:

  $\alpha = 2\ ;\ \beta = 2$

- rectangular hollow sections:

  $\alpha = \beta = \dfrac{1{,}66}{1-1{,}13n^2} \qquad \text{but}\ \alpha = \beta \leq 6$

  where $n = N_{Ed} / N_{pl,Rd}$.

A simpler but conservative alternative is:

$\dfrac{N_{Sd}}{N_{pl.Rd}}+\dfrac{M_{y.Sd}}{M_{Ny.Rd}}+\dfrac{M_{z.Sd}}{M_{Nz.Rd}}\leq 1$	(31)	5.5.8.1 (5.36)

5. VERIFICATION METHODS FOR ISOLATED MEMBERS AND WHOLE FRAMES.

Normally, the design of an individual member in a frame is done by separating it from the frame and dealing with it as an isolated substructure.

The end conditions of the member should then comply with its deformation conditions, in the spatial frame, in a conservative way, e.g. by assuming a nominally pinned end condition, and the internal action effects, at the ends of the members, should be considered by applying equivalent external end moments and end forces, Figure 16. Methods of verification for these members are given in Section 5.1.

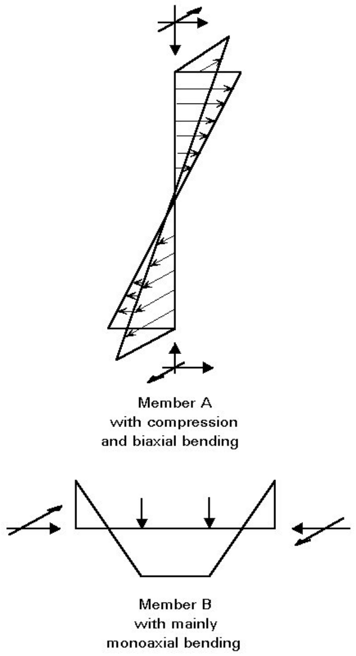


Figure 16 – Isolated members A and B from the plane frame analysis.

A more general procedure is given in Section 5.2, for the case where members cannot be isolated from the frame structure in the way described above.

5.1. Methods of verification for isolated members.

For the design of beam-columns, with mono-axial bending only, two checks must be carried out:

- the in-plane buckling check taking into account the in-plane imperfections.
- the out-of-plane buckling check, including the lateral-torsional buckling verification that takes account of the out-of-plane imperfections (Figure 17).

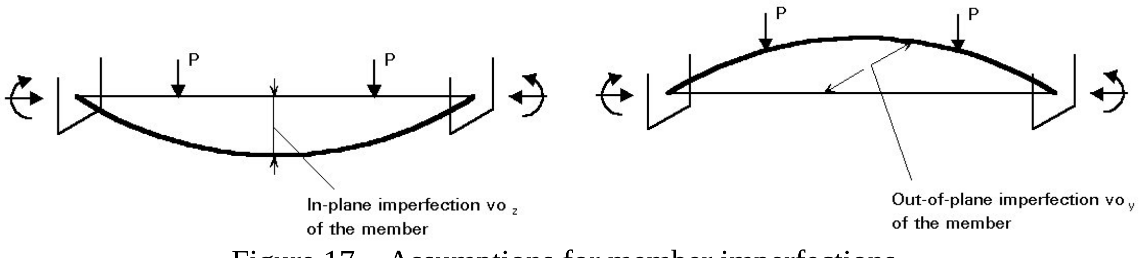


Figure 17 – Assumptions for member imperfections.

It has been found by test calculations that twist imperfections, ρ, of beam-columns that are susceptible to lateral-torsional buckling, can be substituted by flexural imperfections, see Figure 18.

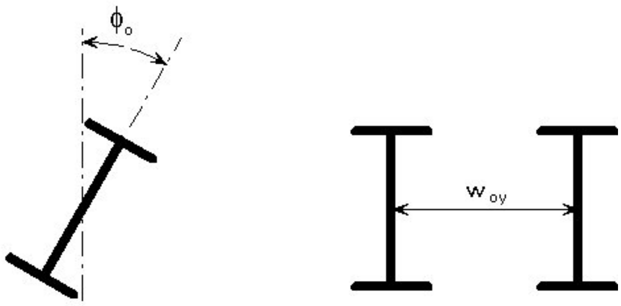


Figure 18 – Twist imperfections ϕ_0 and flexural imperfection W_{0y}.

Members with sufficient torsional stiffness, i.e. hollow section members, need not be verified for lateral-torsional bucking.

When the non-dimensional slenderness $\bar{\lambda}_{LT} \leq 0{,}4$, the reduction coefficient χ_{LT} need not be taken into account. This rule may be used for spacing the lateral restraints to resist lateral-torsional buckling.

5.2. Method of verification of whole frames.

Figure 19 gives an example of a portal frame with tapered columns and beams, the external flanges of which are laterally supported by the purlins which, due to their flexural stiffness, also provide torsional restraint; the beams and columns may, however, be subject to distortion of the cross-section, due to the flexibility of the web.

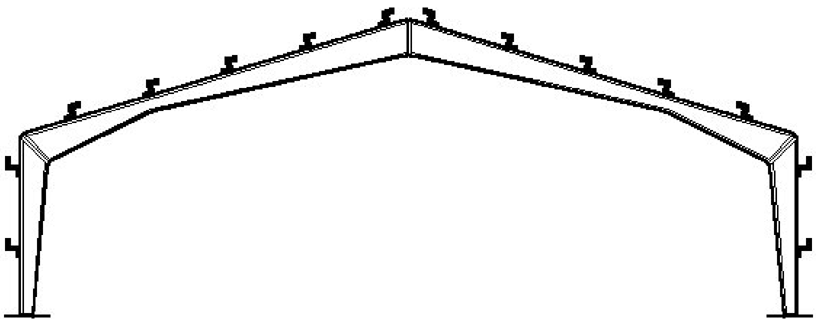


Figure 19 – Portal frame with tapered beams and columns, with elastic torsional restraints and displacements restraints by the purlins. The cross-section is susceptible to distortion.

An accurate verification of this arrangement should be based on a finite element model which takes the above effects into account.

The basic assumptions made regarding the imperfections in this model, would be such that the standard verification given previously would produce equally favourable results since the standard procedure has been calibrated against test results.

A more simplified procedure is, therefore, given here which is related to the verification of columns for flexural buckling, and beams for lateral-torsional buckling.

The basic principles governing the standard verification of columns for flexural buckling, and beams for lateral-torsional bucking, are as follows:

1. The non-dimensional slenderness $\bar{\lambda}$ is defined by:

$$\bar{\lambda}_{FB} = \sqrt{N_{pl}/N_{cr}}\;;\;\bar{\lambda}_{LT} = \sqrt{M_{pl}/M_{cr}}$$

Where N_{pl}, M_{pl} are the characteristic values of the elastic/plastic resistances of the column or beam neglecting any out-of-plane effects; and N_{cr}, M_{cr} are the critical bifurcation values for the column resistance, or the beam resistance, when considering out-of-plane deflections and hyperelastic behaviour in the equilibrium state.

2. Using the non-dimensional slenderness, $\bar{\lambda}$, a reduction factor χ can be determined from the European buckling curves that allows the design value of the resistance of the column or beam to be defined by:

$$N_{bd} = \chi\, N_{pl} / \gamma_{M1} \text{ for the column}$$

$$M_{bd} = \chi\, M_{pl} / \gamma_{M1} \text{ for the beam.}$$

In applying this principle to any loaded structure, see Figure 20, the procedure is as follows:

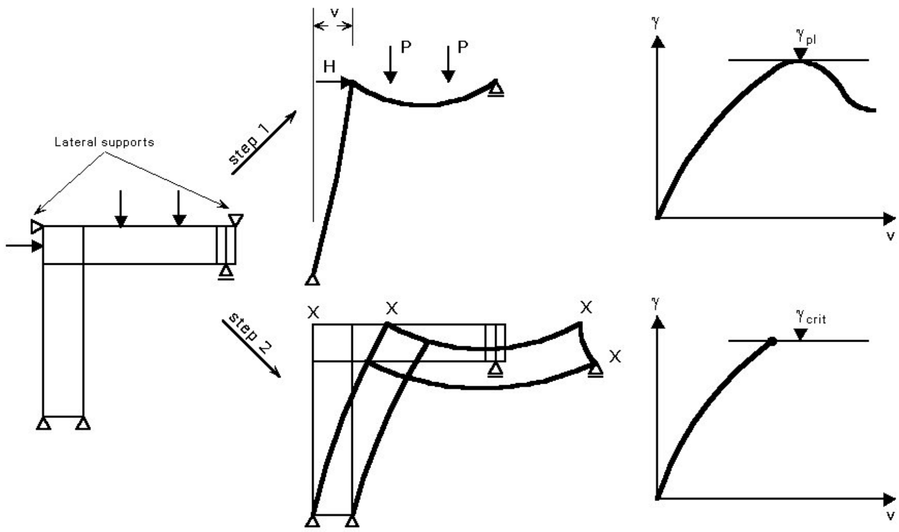


Figure 20 – Stepwise verification of a structure, assuming in step 1: elastic-plastic in-plane behaviour and no lateral deflections, and in step 2: hyperelastic behaviour and lateral deflection.

1. As a first step the structure is analysed for a given load case with an elastic or plastic analysis assuming that any out of plane deflections are prevented. By this analysis a multiplier, γ_{pl}, of the given loads is found that represents the ultimate resistance of the structure.
2. The structure is then checked assuming hyperelastic material behaviour allowing for lateral and torsional deflections. This leads to a multiplier γ_{crit} of the given loads that represents the critical elastic resistance of the structure to lateral buckling or lateral-torsional bucking.
3. The overall slenderness, $\bar{\lambda}$, of the structure can then be defined by:

$$\bar{\lambda} = \sqrt{\frac{\gamma_{pl}}{\gamma_{crit}}}$$

And by using the reduction coefficient χ from the relevant European buckling curve, e.g. curve c, the final safety factor γ can be derived.:

$$\gamma = \chi\, \gamma_{pl}$$

This procedure is analogous to the Merchant-Rankine procedure for the frames non- elastic verification.

In general the procedure described earlier needs a computer program that performs a planar elastic-plastic analysis of the frame and determines the elastic bifurcation load of the structure for lateral and torsional deflections, including distortion.

Such a program, for calculating the elastic bifurcation loads, can either be based on finite elements or on a grid model where the flanges and stiffeners are considered as beams and the web is represented by an equivalent lattice system that allows for second order effects; such programs are available on PC's.

6. CONCLUDING SUMMARY.

- Beam-columns are structural members subjected to axial compression and bending about one or both axes of the cross-section.
- The behaviour of beam-columns can be understood in three stages:

(a) behaviour of the restrained beam-column;

(b) uniaxial bending and compression of the unrestrained beam-column;

(c) biaxial bending and compression of the unrestrained beam-column.

- Stage (a) is governed by the behaviour of the cross-section.
- Stage (b) is governed by an interaction of the cross-section behaviour with in-plane column buckling and/or lateral-torsional buckling.
- Stage (c) is governed by the same factors as stage (b), but the moment about the other axis must be incorporated into the design equation.
- For the cross-section, the interaction of normal force and bending may be treated elastically using the principle of superposition or plastically using equilibrium and the concept of stress blocks.
- When considering the member as a whole, secondary-bending effects must be allowed for.
- Strut analysis may be used as a basis for examining the role of the main controlling parameters.
- Design is normally based on the use of an interaction equation, an essential feature of which is the resistance of the component as a beam and as a column.
- The class of cross-section will affect some of the values used in the interaction equations.
- The biaxial bending case is the most general and includes the two others as simpler and more restricted component cases.
- A three dimensional frame may generally be analysed by separating it into plane frames and analysing these on the assumption of no imperfections; the individual members of the frame should then be checked with the imperfection effects taken into account.
- The isolated members in general represent beam-columns with either in plane or biaxial bending.
- In certain cases the standard procedure for the verification of a beam- column is not applicable and more accurate models must be used.
- As a non-linear spatial analysis including the effect of imperfections is difficult, an alternative procedure is provided by which the overall slenderness of a frame is defined; this allows verification of the frame using the European buckling curves which take account of lateral- torsional buckling.

ADDITIONAL READING

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