# FRAME IDEALISATION & ANALYSIS

## **SUMMARY:**

- Introduce the various global frame modelling analysis approaches & basic concepts.
- Introduce the classification of frames as braced/unbraced, sway/non-sway.
- Introduce frame & member imperfections and when to account for them in the global analysis.
- The idea that the choice of the analytical tool is guided partly by the method of design to be used and partly by the user as a function of software availability is introduced.
- The different methods, "direct" and "indirect", for second-order analysis is outlined and the consequence that each has for the design of the frame and its components is explained.
- Use of elastic (linear & non-linear) methods of analysis and design are discussed.
- Investigated the influence of horizontal forces & imperfections on the frame modelling.
- The different sources of non-linear structural behaviour are identified.
- Second order effects are explained as well as the methods and limitations for a second order analysis.
- The assumptions and limitations of the various elastic methods of analysis are given.

## **OBJECTIVES:**

- Understand that the available tools for the practical analysis of structures have limitations due to the adopted assumptions and simplifications about material and member behaviour.
- Understand the notions of classification: braced/unbraced and sway/non-sway.
- Understand what and when imperfections have to be included in the global analysis.
- Understand how to obtain the elastic critical load for a frame.
- Understand the differences between the various methods of structural analysis.
- Understand the origin of second order effects and when they are important to consider.
- Understand the basis of and limitations of approximate approaches to second order analysis.
- Recognise there are cases where a given type or types of analyses are excluded, but that often there are a number of various methods which may be employed to predict the structural response.
- Appreciate the links between various analytical and design approaches.

#### **REFERENCES:**

- [1]. Annex H *Modelling of building structures for analysis*, ENV 1993-1-1:1992, Amendment A2, Eurocode 3.
- [2]. ENV 1998 Eurocode 8 Design provisions for earthquake resistance of structures.
- [3]. ENV 1993-1.3 Eurocode 3 General rules Supplementary rules for cold formed thin gauge members & sheeting.
- [4]. Maquoi R. and Chabrolin, B., *Frame design including joint behaviour*, Report N° EUR 18563 EN, 1998.
- [5]. Livesley, R.K., Matrix methods of structural analysis, Pergamon Press, 1969.
- [6]. Gachon, H., Galea, Y., Model of non-linear analysis for plane frames (french), Const. Mét., 2, 1978.
- [7]. Galea, Y., Non-linear elasto-plastic analysis programme for plane frames (french), Constr. Mét., 4,-1978.
- [8]. Chen, W.K., Goto, Y. and Liew, J.Y.R., *Stability design of semi-rigid frames*, Wiley & Sons, Inc. 1996.
- [9]. ECCS -Technical Committee 8- Structural Stability, Technical Working Group 8.1/8.2 Skeletal Structures, *Practical analysis of single-storey frames*, ECCS Publication N° 61, 1991.
- [10]. Clarke, M..J., Plastic zone analysis of frames in Advanced analysis of steel frames: Theory, Software and Applications, Chen, W.F. and Toma, S., eds., Boca Rotan, FL, CRC Press, pp 259-274, 1994.
- [11]. Neal, B.G., *Plastic methods of structural analysis*, Chapman and Hall, 1956.
- [12]. The Steel Construction Institute, *Steel Designers Manual*, 5th Edt., Blackwell Scientific Publications, 1992.
- [13] Wood, R.H., *Effective lengths of columns in multi-storey buildings*, The Struct. Eng., 52, N° 7,8 & 9, 1974.
- [14] Petersen, C., "Stahlbau", F.Vieweg & Sohn, Braunschweig, Wiesbaden, 1998.
- [15] King, C.M., Plastic design of single-storey pitched-roof portal frames to EC3, T. Rep., SCI. 147, 1995.
- [16] Merchant, W., Frame stability in the plastic range, Brit.Weld. Jour.; N°3, (366), 1956.
- [17] Kirby P.A., Nethercot D.A., *Design for structural stability*, Collins, London, 1988.

# **1. INTRODUCTION TO FRAME BEHAVIOUR.**

## 1.1 Scope.

Global frame analysis  $\rightarrow$  distribution of the internal forces/corresponding deformations to a specified loading.

Adoption of adequate models  $\rightarrow$  assumptions about the behaviour of the structure. component members and joints.

### **1.2 Load displacement relationship of frames**

Response of a structure to loading  $\rightarrow$  relationship between a load parameter and a significant displacement parameter.

An example of the behaviour of a typical sway frame under increasing load is shown in Figure 1.



Figure 1 - Load displacement response of a framed structure. The load parameter  $\lambda$  is a multiplier (or load factor) applied to all the load components to produce monotonic increase in the loading on the structure

Displacement parameter  $\rightarrow$  top floor lateral displacement

Curve Slope  $\rightarrow$  measure of the frame structure lateral stiffness

Once the linear limit is reached  $\rightarrow$  slope gradually reduces  $\rightarrow$  three kinds of non-linearity:

-geometrical non-linearity

-joint non-linearity

-material non-linearity

Joint non-linearity usually manifests  $\rightarrow$  low levels of load.

Geometrical non-linearity  $\rightarrow$  influence of the structure actual deformed shape on the internal force distribution  $\rightarrow$  evident well before the onset of material yielding

Maximum load  $\rightarrow$  equilibrium  $\rightarrow$  load  $\downarrow$  as deformations  $\uparrow$ 

Slope (stiffness) is zero  $\rightarrow$  peak load and then  $\rightarrow$  negative  $\rightarrow$  structure is unstable

Peak load, ultimate load,  $\rightarrow$  point of imminent structural collapse (in the absence of load shedding)

# 2. FRAME IDEALISATION & STRUCTURAL ANALYSIS BASIC CONCEPTS.

#### **2.1 Modelling of building structures for analysis**

Global analysis of frames  $\rightarrow$  structural model, structure geometric behaviour and the section/joint behaviour

Later design checks of the frame (members & joints)

Checks  $\rightarrow$  type of analysis performed and cross-section verification (ultimate limit state criteria)

2.1.1 Structural concept.

Layout of the structure  $\rightarrow$  requirements for the intended use of the building, + resistance to the actions that are likely to occur.

Identify the structural elements categories:

- 1.main structural elements: main frames, joints & foundations
- 2.secondary structural elements: secondary beams or purlins;
- 3.other elements: i.e. sheeting, roofing and partitions. 4.
- 2.1.2 Spatial behaviour.

Alternative  $\rightarrow$  3D framework analysis  $\rightarrow$  two series of independent plane frames in horizontal directions at right angles  $\rightarrow$  each plane frame has sufficient out-of-plane restraint to ensure its lateral stability



Fig. 2 - Reduction of a 3D framework to plane frames

#### **2.1.3 Resistance to horizontal forces**

#### I)FRAME CLASSIFICATION.

#### a) Braced and unbraced.

Bracing  $\rightarrow$  used to prevent/restrict, sway in multi-storey frames. Common bracing systems  $\rightarrow$  trusses or shear walls



Figure 3 - Common bracing systems.

Frame to be classified as a **braced frame**,  $\rightarrow$  adequately stiff bracing system

Frame  $\rightarrow$  braced  $\rightarrow$  analyse, separately, frame & bracing

- The frame without the bracing system  $\rightarrow$  fully supported laterally resisting only to vertical loads.
- The bracing system  $\rightarrow$  all the horizontal loads, any vertical loads applied to the bracing system and the effects of the initial sway imperfections.

Frames without a bracing system and frames with a bracing system not sufficiently stiff  $\rightarrow$  **unbraced**.

Unbraced frames, a single structural system, consisting of the frame/bracing when present  $\rightarrow$  both vertical/horizontal loads acting together as well as for imperfections effects.

#### b) Braced and unbraced classification criteria.

The existence of a bracing system does not guarantee that the frame structure  $\rightarrow$  classified as braced but only when it reduces the horizontal displacements by at least 80%.

- no bracing system is provided  $\rightarrow$  *unbraced*.
- bracing system is provided: when  $\Psi_{br} > 0.2 \quad \Psi_{unbr} : \rightarrow unbraced$ ,
  - when  $\Psi_{br} \leq 0,2 \ \Psi_{unbr}$ :  $\rightarrow braced$ ,  $\Psi_{br}$  structure lateral flexibility with bracing system  $\Psi_{unbr}$  structure lateral flexibility without bracing system

#### c) Sway and non-sway frames.

**Non-sway frame**  $\rightarrow$  frame response to in-plane horizontal forces is sufficiently stiff to neglect any additional forces or moments arising from horizontal node displacements.

Global second-order effects (*P*-  $\Delta$  sway effects) may be neglected for a non-sway frame.

If global  $2^{nd}$ -order effects are not negligible  $\rightarrow$  sway frame

Normally a frame with bracing is classified as *non-sway*, while an unbraced frame is classified as *sway*.

It is possible for an unbraced frame  $\rightarrow$  non-sway (often the case of one storey portal frame buildings) while a frame with bracing  $\rightarrow$  sway (possible for multi-storey buildings)



Figure 4 - Braced and unbraced frame.

*Non-sway frame*  $\rightarrow$  first-order analysis may always be used.

*Sway frame*  $\rightarrow$  second-order analysis shall be used  $\rightarrow$  iterations on a 1<sup>st</sup>-order elastic analysis is usually adequate

Sructure that meets certain conditions, enables a 1<sup>st</sup>-order analysis (without any iteration process) to be used either by:

-nominal correction to member end forces considering global 2<sup>nd</sup>-order effects

-analysing for vertical loads & sway load effects (magnified) separately

d) Sway and non-sway classification criteria.

Classification  $\rightarrow$  sway or non-sway  $\rightarrow$  ratio of the design value of the total vertical load  $V_{Sd}$  applied to the structure to its elastic critical value  $V_{cr}$  producing sway instability (sway mode failure)

The closer the applied load is to the critical load, the greater is the risk of instability and the greater are the global second-order effects (*P*- $\Delta$  effects)

The classification rule is:

- $V_{Sd} / V_{cr} \leq 0, 1 \rightarrow non-sway.$
- $V_{Sd} / V_{cr} > 0, 1 \rightarrow sway$

This rule can also be expressed:

• 
$$\lambda_{cr} = \frac{V_{cr}}{V_{Sd}} \ge 10 \longrightarrow non-sway$$
  
•  $\lambda_{cr} = \frac{V_{cr}}{V_{Sd}} < 10 \longrightarrow sway$ 

$$\lambda_{cr} = \frac{V_{cr}}{V_{Sd}} < 10 \quad \longrightarrow S$$

#### **3. GENERAL CONSIDERATIONS ABOUT THE MODELLING OF FRAMES.**

Guidelines from Eurocode 3:

- 1.Members and joints should be modelled for global analysis  $\rightarrow$  reflects expected behaviour for the relevant loading
- 2. The basic geometry of a frame should be represented by the centrelines of the members.
- 3.Sufficient to represent the members by linear structural elements located at their centrelines, disregarding the overlapping of the actual widths of the members.
- 4.Account may be taken of the actual width of all or some of the members at the joints

#### **3.1** Choice between a first-order or a 2<sup>nd</sup>-order analysis.

Preliminary design stage  $\rightarrow$  structure  $\rightarrow$  braced or unbraced

This determines how the vertical and horizontal load effects are to be considered in the analysis.

Once established  $\rightarrow$  preliminary design for the members/joints

Preliminary analysis  $\rightarrow$  classify sway/non-sway

For most typical frames  $\rightarrow$  first-order analysis

Whereas a second-order, elastic or plastic analysis can always be used, there are cases where:

- a first-order analysis suffices, without any need to account for second-order effects,
- when  $V_{Sd}/V_{cr} \le 0.25$ , a first-order elastic analysis  $\rightarrow$  sway frames  $\rightarrow$  corrections to account for second-order effects
- Rigid-plastic first-order analysis  $\rightarrow V_{Sd}/V_{cr} \leq 0,20$  and all the internal forces & moments are amplified by  $1/(1-V_{Sd}/V_{cr})$  *i.e.* an Merchant-Rankine Approach

Necessary to evaluate the degree to which the second-order effects may modify the distribution of internal forces (if sway displacements (P- $\Delta$ ) and/or member imperfections/member deformations (P- $\delta$ ) are significant)

Member imperfection/deformation effects  $\rightarrow$  significant for certain types of relatively slender members in a sway frame,  $\rightarrow$  unlikely to be so in a non-sway frame

Effect of sway  $\rightarrow$  critical load parameter  $\lambda_{cr}$ , i.e. the ratio between the total vertical load which would produce in-plane sway instability of the frame and the actual design vertical loads (*Eurocode 3 uses the inverse i.e.*,  $1/\lambda_{cr} = V_{Sd} / V_{cr}$ ,) With columns preliminary sizes  $\rightarrow$  based on  $V_{Sd}/V_{cr}$  for critically loaded columns of the frame

Approximate approach  $\rightarrow$  this parameter for multi-storey buildings

This approach is not suited for typical one-storey pitchedportal industrial frame buildings.

When  $\lambda_{cr} \ge 10$  (or  $V_{Sd} / V_{cr} \le 0,1$ ) the frame is classified as non-sway and a first-order analysis is adequate.

Sway frames  $\rightarrow$  second-order analysis is required.

For many structures  $\rightarrow$  first-order analysis  $\rightarrow$  internal forces and moments  $\rightarrow$  amplified to account for second-order effects

Member deformation effect  $\rightarrow$  relatively slender beamcolumn members of both sway and non-sway frames

 $EC3 \rightarrow examine the importance of local member imperfections for certain types of beam-column in sway frames only$ 

In the absence of a specific requirement about the effect of member deformations due to load, the check for member imperfection significance may also be considered as an evaluation of the significance of the local second-order effects (P- $\delta$ ) on a given member, whatever the source.

Procedure to identify the slender members concerned is similar to frame sway classification.

The difference  $\rightarrow 1/\lambda_{cr} = N_{Sd} / N_{cr}$  for the beam-column member alone

When  $N_{Sd} / N_{cr} \ge 0.25$  for any such member  $\rightarrow$  introduce member imperfections (at least for the members concerned) in the global analysis and to use a 2<sup>nd</sup>-order analysis.

In the calculation of  $N_{cr}$  (Euler load) one uses a buckling length for the member = to the system length.

The imperfection to be used is specific to the used member type and the relevant buckling curve.

#### **3.2** Choice between elastic and plastic methods of analysis.

A plastic method  $\rightarrow$  only appropriate under certain conditions: steel properties, member cross-section classification & joint ductility

The design checks  $\rightarrow$  sophistication of the analysis tool used

In a 2<sup>nd</sup>-order analysis no need to check in-plane stability of frame/members

The choice of global analysis will thus depend not only on EC3, but also on personal choices, available software, etc

Balance between effort devoted to global analysis/remaining ULS checks, Figure 5.





Whatever the design method  $\rightarrow$  identify when 2<sup>nd</sup>-order effects need to be accounted

Typical plane frames  $\rightarrow$  any of the methods of analysis may be used

#### **3.3 Elastic global analysis and design: guidelines.**

In an elastic analysis the frame components choice (sections & joints) is not limited by any ductile behaviour requirements i.e. the method can be used in all cases.

# Figure 6 depicts the different possibilities for elastic global analysis and the relevant checks, Eurocode 3.



Figure 6 - Elastic global analysis and relevant design checks with reference to Eurocode 3

#### **3.3.1 Scope of 1<sup>st</sup>-order elastic analysis.**

First-order elastic analysis, with appropriate account of global frame imperfections  $\rightarrow$  non-sway frames  $\rightarrow 2^{nd}$ -order effects are insignificant

For sway frames under certain conditions.

-Prior to adopting a first-order analysis  $\rightarrow$  evaluate whether this kind of analysis is appropriate for the frame -Once the analysis is carried out and before proceeding with the design checks  $\rightarrow$  examined second-order effects

The frame is analysed for various combination cases.

For a frame subject to sway, but classified as non-sway (when  $V_{Sd}/V_{cr} \le 0,1$ ), **an error of up to 10%** is expected for sway displacements & bending moments when compared to 2<sup>nd</sup>-order analysis results.

## 3.3.2 Frame design following first-order analysis. -Second-order effects.

First-order elastic analysis provides a design safe basis as long as the structure response only slightly deviates from the actual response over a considerable range of loads (i.e. structures with low axial loads).

Structures with high axial loads,  $\lambda_{LI_{,}}$  does not provide a lower bound of the maximum load  $\rightarrow$  it neglects 2<sup>nd</sup>-order effects

First-order elastic analysis  $\rightarrow$  sway frames  $\rightarrow$  appropriate corrections  $\rightarrow$  second-order effects



Fig. 7 - Load displacement response: Range of validity of first order elastic analysis.

#### - Cross-sections.

Resistance (ultimate limit state) check of the cross section members  $\rightarrow$  does not exceed the design resistance (stress).

Elastic analysis  $\rightarrow$  yield stress in the extreme fibres as the member design condition  $\rightarrow$  any class of member cross-section or joint can be adopted.

When class 3 and class 4 sections are used  $\rightarrow$  attainment of the yield stress in the extreme fibres  $\rightarrow$  effective cross section for class 4 sections

 $1^{st}$ -order or  $2^{nd}$ -order elastic analyses  $\rightarrow$  determine the load when the first plastic event takes place

Sections  $\rightarrow$  requirements for ductile behaviour (member section Classes 1 or 2), the section resistance  $\rightarrow$  plastic interaction formula

Furthermore, Eurocode  $3 \rightarrow$  redistribution of the calculated bending moments following a first-order analysis of up to 15% of the peak calculated moment in any member on condition that equilibrium is maintained and that the members concerned are of Class 1 or of Class 2.

#### -Stability.

The design resistance of the most critical section allows one to determine the upper limit, represented by  $\lambda_{L1} \rightarrow$  frame cross-sections  $\rightarrow$  linear elastic

This assumes that the structure/members remain stable.

Investigate instability  $\rightarrow$  in-plane and out-of-plane stability (buckling, lateral-torsional buckling)

Instability reduces the value of  $\lambda_{L1}$ .

For the design to be adequate,  $\lambda_{L1}$  must be at least unity

For Sway Buckling Length Method  $\rightarrow 2^{nd}$ -order sway effects  $\rightarrow$  in-plane stability of the column members  $\rightarrow$  sway mode buckling length

For all the other methods, the in-plane stability of the column members  $\rightarrow$  non-sway buckling length.

No other check of in-plane frame stability in the sway mode is required

#### -Serviceability.

For most frames, a first-order elastic analysis  $\rightarrow$  good tool for predicting the response of the structure/elements at the serviceability limit state (permissible deflections)

At this level of loading, the non-linear effects (2<sup>nd</sup>-order effects) will be small

May also be necessary to check the structure vibrations (office & residential buildings comfort levels)

3.3.3 Scope of second-order elastic analysis.

Second-order elastic analysis, with due allowance being made for global frame imperfections, may be used in all cases.

It is always required for sway frames and when member imperfections must be included

 $Ec3 \rightarrow$  number of procedures to provide such an analysis

Some of these methods (the indirect methods) are not strict second- order analysis methods  $\rightarrow$  simple but acceptable means of obtaining safe results under certain conditions

For instance, neither the "equivalent lateral load method" nor any of the indirect methods are suitable for accounting local  $2^{nd}$ -order effects (P- $\delta$ ) due to either member imperfections or deformations.

#### -General method.

General method  $\rightarrow$  direct method for second-order analysis

2<sup>nd</sup>-order effects due to global frame imperfections, sway displacements and (usually) in-plane local member deformations are taken into account directly when using the general method of second order analysis

2<sup>nd</sup>-order effects due to local member imperfections can also be accounted for when using this analysis.

#### -Equivalent lateral load method.

Iterative procedure, using the results of a first-order analysis at each iterative step

Can be considered to be a direct method for second-order analysis  $\rightarrow$  only suitable for accounting sway effects (P- $\Delta$ )

Except  $\rightarrow$  some relatively slender members, which is rare in typical sway frames  $\rightarrow$  gives quite satisfactory results

#### -Amplified Sway Moment Method.

Can be adopted if the  $2^{nd}$  order bending moment distribution  $\rightarrow$  approximately affinity to the  $1^{st}$  order bending moment (low to moderate sway structures).

This guaranteed by limiting to cases:  $\lambda_{cr} > 4 (V_{sd}/V_{cr} < 0,25)$ 

The sway moments arise  $\rightarrow$  applied horizontal loads and also through the effects of frame or the vertical loading asymmetry.

Non-sway internal forces and moments due to vertical loads only  $\rightarrow$  first order analysis with sway prevented

Internal sway forces and moments  $\rightarrow$  amplifying those given by a separate analysis for the horizontal forces including those liberated at the floor levels

Design moments/forces caused by sway are obtained in steps:

- 1.Analyse the frame for vertical loading only, with floor levels laterally restrained (sway prevented)
- 2.Determine the horizontal reactions at floor level restraints
- 3.Analyse the frame with the floor level horizontal restraints removed, under the horizontal forces including a system of lateral forces equal but opposite to the horizontal reactions found in Step 2
- 4.Moments for joint, member cross-section and out-of-plane beam stability design are the moments Step 1 + moments Step 3 which have been amplified by an appropriate factor. (Shear forces and axial loads are also amplified)
- 5. Magnifying factor is:

$$\left[\frac{1}{1-\frac{V_{sd}}{V_{cr}}}\right] \text{ (the method is valid for } \frac{V_{sd}}{V_{cr}} \le 0,25\text{ )}$$

Amplified sway moments & forces are added to the non-sway values  $\rightarrow$  design values

Moments which arise due to sway only are amplified by the given amplification factor while retaining the non-sway moments at their original values

For the in-plane buckling check  $\rightarrow$  non-sway buckling length + amplified moments and forces

Out-of-plane stability must also be checked.

#### -Sway-Mode Buckling Length Method.

 $2^{nd}$ -order sway effects  $\rightarrow$  using a first-order elastic analysis  $\rightarrow$  unknown sensitivity structures

Internal forces  $\rightarrow$  first-order analysis

Sway moments in beams/joints  $\rightarrow$  amplified by a nominal factor of 1.2 and added to the remainder of the moments (those not due to sway)

Amplified forces  $\rightarrow$  design checks of joints and member cross-sections and member in-plane and out-of-plane stability

For in-plane buckling  $\rightarrow$  in-plane the sway-mode buckling length must be used

#### **3.3.4 Design following a second-order elastic analysis.**

#### -Cross-section.

Same design checks for the first-order analysis

Critically loaded section/joint  $\rightarrow$  upper limit  $\rightarrow$  load multiplier  $\lambda_{L2} \rightarrow$ , elastic analysis is valid

Design is adequate  $\rightarrow \lambda_{L2} \ge 1$ 



Figure 8 - Load displacement response: Range of validity second order elastic analysis.

- Stability.

#### a) General second-order analysis

General second-order elastic analysis  $\rightarrow$  overall in-plane sway-mode frame stability is covered

When a general method is used it includes both frame and member imperfections  $\rightarrow$  critical load is obtained, neither inplane frame nor member stability need to be checked.

General method of second-order analysis is rarely brought to the point where the elastic critical load is obtained.

Furthermore, neither member imperfections nor out-of-plane behaviour are usually considered

Checks against out-of-plane instability of the members/frame are required but in-plane frame stability check is not required.

It is advised to carry out in-plane stability checks  $\rightarrow$  relatively slender members  $\rightarrow$  member imperfections were not considered

Non-sway in-plane buckling lengths  $\rightarrow$  for sway or non-sway

#### b) Alternative indirect methods

Amplified Sway Moment or Sway Mode Buckling Length Methods  $\rightarrow$  out-of plane and in-plane frame and member stability must be checked

Amplified Sway Moment Method  $\rightarrow$  in-plane non-sway mode buckling length is used + *amplified* moments

Sway-Mode Buckling Length Method  $\rightarrow$  in-plane sway-mode buckling length

Suggested moment including the *non- amplified* sway moments  $\rightarrow$  for this buckling check although not stated in Ec3

Amplified moments  $\rightarrow$  sections design checks + lateraltorsional buckling

 $Ec3 \rightarrow$  these checks guarantee the overall sway stability of the frame when any of the direct or indirect methods are used

# **3.3.5** Assessment of the elastic critical load for a frame in the sway mode.

-Approximate procedure.

Frames in building structures, i.e. beams connecting each column at each storey level, the sway mode elastic critical buckling load can be calculated by:

- 1<sup>st</sup>-order elastic analysis for the specific load combination. The horizontal displacement of each storey due to the loads (both horizontal & vertical) is determined.
- Elastic critical load of the frame (sway mode) under the specific load combination case may be estimated from:

$$\frac{V_{Sd}}{V_{cr}} = max \left[\frac{\delta}{h}\frac{V}{H}\right]_{i}$$

- where i designates the  $i^{th}$  storey
  - $v_{sd}$  design value of the total vertical foundation reaction
  - $v_{cr}$  is the elastic critical load of the sway mode frame,
  - δ horizontal displacement at the top of the  $i^{th}$  storey relative to the bottom of the  $i^{th}$  storey,
  - h is the  $i^{th}$  storey height,
  - H total horizontal reaction at the bottom of  $i^{th}$  storey,

v is the total vertical reaction at the bottom of  $i^{th}$  storey.

#### -Grinter Frame Procedure.

Replace the actual frame by the equivalent Grinter frame

For multi-bay frames  $\rightarrow$  first find an intermediate single-bay substitute frame with rigid joints before determining the Grinter frame equivalent

A multi-story multi-bay frame with rigid or semi-rigid joints is first replaced by an equivalent substitute single bay frame having rigid joints and of columns and beams with equivalent stiffnesses

This equivalent  $\rightarrow$  lateral displacements of each storey are the same as for the original frame  $\rightarrow$  elastic critical load for both structures should be similar



Figure 9 - (a) actual frame (semi-rigid joints), (b) substitute frame (rigid joints), (c) Grinter frame.

Assumed columns  $\rightarrow$  elastically and are continuous  $\rightarrow$  stiffness of the column at each storey is:

$$K_{c} = \frac{1}{2} \sum_{j} K_{c,j}$$

where  $\kappa_{c,j}$  is the column j stiffness coefficient,  $\frac{I_{c,j}}{I_{c,j}}$ .

Equivalent stiffness coefficient of the beam with linear restraints at each storey is:

$$K_{b} = \sum_{i} K_{b,equi,i}$$

$$K_{b,equi,i} = \frac{I_{b,equi,i}}{L_{b,i}} \text{ in which } I_{b,equi,i} = \left[\frac{1}{1+3\alpha_{i}}\right] I_{b,i} \& \alpha_{i} = \frac{2EI_{b,i}}{S_{j,ini,i}L_{b,i}} (\alpha_{i} = 0 \text{ for a rigid joint})$$

$$\frac{EI_{b,i}}{L_{b,i}} \text{ is the flexural stiffness of the considered beam i,}$$

 $s_{j,ini,i}$  is the initial joint stiffness at the end of the considered beam in the actual structure.

For a beam in which the joint stiffnesses are not the same at each end  $\rightarrow$  uses lowest joint stiffness (conservative) or  $\rightarrow$  appropriate single value for individual equivalent beam stiffness

Since the lateral displacements at each storey of the real, the substitute and the equivalent Grinter frames are similar, the values of the elastic critical loads for all three frames can be expected to be similar

Stiffness of the members in the Grinter frame are :

$$K_b^* = 3\sum_i K_{b,equi,i}$$
 and  $K_c^* = \sum_j K_{c,j}$ 

Elastic critical load of the actual frame with semi rigid joints can be computed by referring to the associated Grinter frame

1-The critical load of each column V<sub>cr</sub><sup>\*</sup> → buckling length in the sway mode considering the end restraints
2-Each column of the Grinter frame → V<sub>cr</sub><sup>\*</sup>. The lowest of all the, v<sub>cr,min</sub><sup>\*</sup>, is selected as being a safe lower bound for the elastic critical load of the Grinter/actual frame

## **5.IMPERFECTIONS.**

Appropriate account for the effects of practical imperfections in the global analysis, in the analysis of bracing systems and in member design.

Practical imperfections, which include residual stresses, are geometrical imperfections such as lack of verticality, lack of straightness, lack of fit and the unavoidable eccentricities present in practical joints.

Eurocode 3 requires that two kinds of imperfection be included in the global analysis of all frames:

- initial sway imperfections (frame imperfections),
- member imperfections, where necessary.

Member imperfections may neglected  $\rightarrow$  non-sway frames

For sway frames with slender columns the analysis may have to incorporate member imperfections

#### 4.1 Frame imperfections.

Global frame imperfections  $\rightarrow$  global analysis  $\rightarrow$  equivalent geometric imperfection, i.e. an initial sway (see figure 10(a)).

Resulting forces and moments  $\rightarrow$  member design



Figure 10 - (a) Global frame imperfections (b) Local member imperfections.

Frame imperfections  $\rightarrow$  load case to be used in conjunction with all the critical load combinations acting on the frame

Initial sway imperfections  $\rightarrow$  all horizontal directions, but one direction at a time

Particular attention  $\rightarrow$  anti-symmetric sways on two opposite faces  $\rightarrow$  introduce torsional effects.

Global imperfections can also be accounted  $\rightarrow$  introducing equivalent lateral loads at the floor levels  $\rightarrow$  two possible ways for introducing frame imperfections  $\rightarrow$  4.2 & 4.3

#### 4.2 Global geometric imperfection for frames.

Frame imperfections are quantified  $\rightarrow$  initial sway rotation of the frame relative to the foundation of the columns (figure 11).



Sway imperfection  $\Phi$ 

Figure 11 - Global frame imperfections.

The initial sway imperfections are determined directly from:

$$\Phi =_{k_c k_s} \Phi_{0}$$

where:  $\Phi_0 = 1/200 k_c = (0.5 + \frac{1}{n_c})^{0.5}$  but  $k_c \le 1$ ;  $k_s = (0.2 + \frac{1}{n_c})^{0.5}$  but  $k_s \le 1$ 

n<sub>c</sub> is the number of full height columns per plane n<sub>s</sub> is the number of storeys

#### 4.3 Closed system of equivalent horizontal forces.

Alternative method  $\rightarrow$  use a closed system of equivalent horizontal forces

Procedure  $\rightarrow$  same as that for obtaining the equivalent lateral load to account for the  $P-\Delta$  sway effect due to the loading

Equivalent horizontal forces at each roof and floor leve  $\rightarrow$  multiplying the proportion of the vertical load applied at the level by the initial sway imperfection

May be applied in any horizontal direction, but only in one sway direction at time

The equivalent horizontal forces  $\rightarrow$  vertical reactions x initial sway imperfections are applied at the supports (in the opposite direction to those applied at the other levels)

Equivalent horizontal forces on the entire frame form a closed system  $\rightarrow$  net equivalent horizontal force  $\rightarrow$  entire structure=0

#### 4.4 Imperfections for analysis of bracing systems.

Bracing systems  $\rightarrow$  required to provide lateral stability within the length of beams or compression members must also be analysed with allowance being made for an equivalent geometric imperfection of the members to be restrained

Initial bow imperfections or an equivalent stabilising force

When the restrained members are spliced  $\rightarrow$  bracing system shall be capable of resisting a local force applied to it at that point by each beam or compression member which is spliced

#### 4.5 Local member imperfections and member deflections.

Local member imperfection to be used (bowing)  $\rightarrow$  Fig. 10(b)

Effect  $\rightarrow$  same as that due to the actual deflection of the member itself due to bending and axial load, i.e. a *P*- $\delta$  effect

Member imperfections effects  $\rightarrow$  neglected when considering frame global analysis, except  $\rightarrow$  specific slender members

Those cases for which it can be neglected  $\rightarrow$  effect is presumed to be included in the appropriate buckling formula.

Effect must be considered  $\rightarrow$  members in sway frames subjected to axial compression  $\rightarrow$  moment-resisting connections and in which:

 $\overline{\lambda} > 0.5 [Af_y/N_{sd}]^{0.5}$  (alternatively  $N_{sd}/N_{cr} > 0.25$  Or  $\lambda_{cr} = N_{cr}/N_{sd} < 4$ )

where :  $N_{Sd}$  is the design value of the compressive force

- $N_{cr}$  is Euler member buckling load using the buckling length = the system length
- $\bar{\lambda} = [Af_y/N_{cr}]^{0.5}$  (class 1,2 or 3 sections) is the in-plane nondimensional slenderness.

Incorporation of the initial local member imperfection in the global analysis  $\rightarrow$  modification of the member forces and moments along the entire length of the member.

Local 2<sup>nd</sup>-order effect due to member deflections (also a P- $\delta$  effect)  $\rightarrow$  further aggravate these changes

Prudent to use a general 2<sup>nd</sup>-order analysis for very slender members in sway & non-sway frames.

Care should be taken  $\rightarrow$  direction of the initial bow imperfection may have on the resulting values of forces and moments in the member

# **5. GLOBAL FRAME ANALYSIS METHODS.**

#### 5.1 General.

Actual load-deformation  $\rightarrow$  use of sophisticated s methods

Models  $\rightarrow$  from simple elastic analysis or the rigid-plastic analysis to complex, elasto-plastic analysis, which can provide a close representation of the real behaviour of the structure

Only one loading combination is discussed, but all loading combinations must be analysed

#### **5.2 Second-order effects**

Deflections due to the external loads modify the structural response and the distribution of the internal forces

For frames, the most significant modifications to the linear response  $\rightarrow$  sway & axial loads

Fixed bar cantilever subject to combined axial and transverse loads applied at the free end, shown in Figure 12, is taken

Cantilever is representative of part of the height of a column, i.e. from its base up to the point of inflexion near mid-height, in a frame structure subjected to sway displacement Lateral displacement of the point of inflexion will be close to half of the relative sway between the floor above and the floor below the column



 $\Delta$  is the sway relative to the column base of the infexion point

Figure 12 - First and second order moments in a beam-column.

In the presence of the axial load, the lateral (sway) displacement at the top of the member and the curvature of the member itself,  $2^{nd}$ -order effects in form of secondary moments are induced.

The consequences will be that the actual deformations of the column under a given loading are greater than predicted by a first order analysis.

Global second-order moment, the *P*- $\Delta$  effect  $\rightarrow$  relative lateral (sway) displacement ( $\Delta$ ) between the member top & bottom

Local second-order moment  $\rightarrow P-\delta$  effect  $\rightarrow$  axially loaded member due to the deflections ( $\delta$ ) relative to the chord line connecting the member ends For frames, whilst the  $P-\delta$  effect still arises when sway deformation is prevented, both the  $P-\delta$  effect and the  $P-\Delta$  effect arise when sway can occur. In the case of sway of typical frames, the  $P-\Delta$  effect is usually much more significant than the  $P-\delta$  effect

Always necessary to evaluate whether sway effects in a frame are significant or not  $\rightarrow P-\delta$  effect needs only be considered for particularly slender members

Both effects occur independent of whether the axial load is one of compression or is one of tension

In terms of member stability and of overall frame stability, secondary effects due to tension forces  $\rightarrow$  beneficial while compression forces  $\rightarrow$  adverse

#### **5.3 Slope-deflection method**

Elastic frame analysis  $\rightarrow$  examining the slope-deflection equations for a simple beam or beam-column member

#### **5.3.1 First-order elastic analysis.**

The basic slope-deflection equations express the moment at the end of a member as the superposition of the end moments due to external loads on the member with the ends assumed fixed and of the moments caused by the actual end displacements and rotations The slope-deflection method is an application of the more general displacement method of first-order elastic analysis to plane frames in which the effects of axial and shear strain energy can be disregarded compared to bending strain energy

Figure 13 shows the sign convention  $\rightarrow$  (*displacement*  $\Delta$  *is much smaller than the member length* L)



Figure 13 - Deformation of beam-column.

Basic slope-deflection equations for the member:

$$\overline{M}_{AB} = \left[\frac{EI}{L} (4\theta_{A} + 2\theta_{B} - 6\psi_{AB}) + M_{AB}\right]$$

$$\overline{M}_{BA} = \left[\frac{EI}{L} (2\theta_{A} + 4\theta_{B} - 6\psi_{AB}) + M_{BA}\right]$$
(1)

Where:  $\overline{M}_{AB}$  and  $\overline{M}_{BA}$  are the moments at the joint nodes A & B,

 $M_{\scriptscriptstyle AB}$  and  $M_{\scriptscriptstyle BA}$  a fixed-end moments for member lateral loads alone,

 $\Psi_{AB} = \frac{\Delta}{L}$  is the slope of the chord line AB caused by sway. Corresponding shear forces at the member ends are obtained from the following relationships:

$$\overline{V}_{AB} = \frac{\left(\overline{M}_{AB} + \overline{M}_{BA}\right)}{L} + V_{AB}$$

$$\overline{V}_{BA} = -\frac{\left(\overline{M}_{AB} + \overline{M}_{BA}\right)}{L} + V_{BA}$$
(2)

 $v_{AB}$  and  $v_{BA}$ , end shears  $\rightarrow$  simply supported beam, span L

Slope-deflection equations, shear equations and usual relations between axial load and axial deformation  $\rightarrow$  for the derivation of the 1<sup>st</sup>-order stiffness matrix for each member

When sway is prevented the contribution of the sway term  $(6_{\Psi_{AB}})$  can be omitted

It is usual to analyse the sway frames structure first for all design loads acting but with sway prevented

Sway effects  $\rightarrow$  separate analysis and the results from both analyses are superimposed

For the separate sway analysis, equations which express the equilibrium between the external horizontal forces acting at each storey (i.e. the sum of the column shears) and the corresponding moments in the columns in the storey are used

Horizontal forces acting on the structure  $\rightarrow$  first analysis as the values for the horizontal reactions (needed to prevent sway) at each floor level, but applied in the opposite direction

First-order sway equilibrium equation for each floor  $\rightarrow$  summation being over all the columns of the given storey *i*,:

$$\sum_{j} \left( \overline{M}_{AB,sway} + \overline{M}_{BA,sway} \right)_{j} = h_{i} [H_{i}]$$
(3)

where  $H_i$  is the total external shear force acting on the floor *i* of height  $h_i$  and A & B denote the typical column *j* two ends.

Initial fixed-end moments due to sway in each column for this analysis, in which further sway at every floor is prevented:

$$M_{AB} = M_{BA} = \frac{6EI\Delta_i}{h_i^2}$$
(4)

where  $\Delta_i$  is the sway of the floor supported by the columns of storey *i* relative to the floor below and which is the unknown to be solved

The approach involves carrying out a separate analysis for the sway of one floor at a time

Final sways solved using the series of sway equations, there being as many equations  $\rightarrow$  unknown floor sways

#### Second-order analysis.

When the P- $\Delta$  and the P- $\delta$  effects are ignored, each structural element is characterised by a linear stiffness matrix for a first-order elastic analysis.

This approach is only acceptable only when the column members have relatively low axial loads.

Importance of the P- $\delta$  axial load effect  $\rightarrow$  ratio of the member's axial load N<sub>Ax</sub> to N<sub>E</sub> the Euler buckling load

For a member of length L and second moment of area I, the Euler load is given by  $N_E = \frac{\pi^2 EI}{I^2}$ 

Account for the P- $\delta \rightarrow$  modifying the terms of the linear stiffness matrix so that they include terms (stability functions) which are functions of the ratio  $\varepsilon = \pi \sqrt{\frac{N_{Ax}}{N_{E}}}$ 

Axial load effectively modifies the stiffness of the member

Following are modified member slope-deflection equations:

$$\overline{M}_{AB} = \left[\frac{EI}{L}\left(s\theta_{A} + sc\theta_{B} - s(1+c)\psi_{AB}\right) + mM_{AB}\right]$$

$$\overline{M}_{BA} = \left[\frac{EI}{L}\left(sc\theta_{A} + s\theta_{B} - s(1+c)\psi_{AB}\right) + mM_{BA}\right]$$
(5)

The formulae for the terms s, c and m have values of 4, 0,5 and 1 respectively when  $\varepsilon = 0$  so the equations are then the same as for the first-order analysis.

The other terms in the equations are as defined for the firstorder analysis (see figure 13).

The parameter m indicates that the fixed-end moments are slightly different than for the member without axial load and it can be expressed in terms of the parameters s and c.

For instance, for a uniformly distributed load  $\rightarrow 6/[s(1 + c)]$ 

Its effect is to increase the fixed end moments when the axial load is compressive and to reduce them when it is tensile Compared to those for the first-order analysis, the equations for the end shear forces are also modified as they now include a new term to account for the effect of sway.

This new term represents the  $P-\Delta$  effect  $\rightarrow$  axial load N is + for a compressive axial load, they become as follows:

$$\overline{V}_{AB} = \frac{\left(\overline{M}_{AB} + \overline{M}_{BA}\right)}{L} + N\psi_{AB} + V_{AB}$$

$$\overline{V}_{BA} = -\frac{\left(\overline{M}_{AB} + \overline{M}_{BA}\right)}{L} - N\psi_{AB} + V_{BA}$$
(6)

Equations 5) and 6) indicate the most important modifications to the member shear and bending stiffness terms while a more in depth treatment shows that sway introduces other higherorder terms into the shear and axial load stiffness terms.

Resulting modified stiffness matrix for the structure as a whole is non-linear since the stiffness terms are now functions of the actual sway displacements as well as the member axial loads

Equilibrium of the structure  $\rightarrow$  deformed shape

 $2^{nd}$ -order analysis  $\rightarrow$  increasing all loads incrementally and convergence by an iteration procedure

Typical frames with sway where very slender columns are not used, since the  $P-\delta$  effect is negligible compared to the  $P-\Delta$  effect  $\rightarrow$  direct use of the non-linear theory is not required

Simpler approaches, based on iteration of a first-order analysis for instance, can be safely used

Caution should be exercised when very slender members or curved members are used since the moments occurring along their entire lengths are modified

Simpler methods of  $2^{nd}$ -order analysis are not usually suited for such special cases and the more exact second-order analysis may be needed

Member is best modelled by a number of elements  $\rightarrow$  member imperfections

Moments/forces at the joints thus created along the member length as well as at its ends can be obtained from the analysis

#### 5.3.3 Equivalent lateral load approach $\rightarrow 2^{nd}$ -order analysis

From figure 12, it is observed that the column base end moment has increased from the 1<sup>st</sup>-order: [Hh]  $\rightarrow$  [Hh+P $\Delta$ ]

The effective horizontal load (shear force) was increased by  $[P\Delta/h]$  due to sway

The approach involves using the initial values of the axial loads and floor sways given by a first-order linear-elastic analysis of the entire structure→both vertical/horizontal loads

From these axial loads and sways one determines the extra "equivalent" horizontal force of  $[P \Delta_f / h_s] \rightarrow$  at the top of each column of each storey in the direction of the sway, figure 14

In this expression,  $h_s$  is the storey height of a given column

and  $\Delta_{\rm f}$  is the relative sway between the column top and bottom

As equal but opposite horizontal forces are applied at the bottom of each column, there is no increase in the resultant horizontal force applied to the structure as a whole

All building storeys are assumed to sway in the same direction

Storey shear at the top of columns is opposite in direction to that at the bottom of columns

Total additional horizontal force at any given floor level will be given by the sum for all the columns of the storey below minus the sum for all the columns of the storey above (i.e. closed system of forces)

A new first-order analysis is carried out so as to incorporate the effects of the "equivalent" horizontal loads applied at each floor (including those at the foundation level)



Figure 14 - Equivalent lateral force procedure.

The procedure is repeated until the values for the floor sways converge to an accuracy acceptable level

If it does not converge within a few iterations  $\rightarrow$  structure is unstable

After convergence the resulting internal forces/moments in every member now include the  $P-\Delta$  effects

The initial sway displacements are denoted by  $\Delta_i$ , where *i* denotes the storey level

Total additional storey shear for any storey level is:

$$V_i' = \frac{\sum P_i}{h_i} (\Delta_i - \Delta_{i-1})$$
(7)

where:  $v_i$  additional shear in storey *i* due the sway forces;

 $\sum P_i$  sum of the column axial loads for storey level *i*;

 $h_i$  is the height of storey *i* which is between floors levels *i*-1 and *i*;

Δ<sub>i</sub>, Δ<sub>i-1</sub> total sway displacements of floor levels *i* and *i-1* respectively, relative to the foundation level (level 0) where zero lateral movement is assumed.

The total "equivalent" sway force acting at any given floor  $\rightarrow$  difference between the "additional" storey shear from the columns of the storey below and above, i.e.:

 $H'_{i} = V'_{i} - V'_{i+1} \tag{8}$ 

The structure is analysed again using the  $1^{st}$ -order theory, either for the sway forces H'acting alone and then combining the results with those from the  $1^{st}$  analysis, or including them in the lateral forces when all loads are acting

When the  $\Delta_i$  values at the end of a cycle are close (say within 5%) to those given by the previous cycle  $\rightarrow$  converged sufficiently

#### **5.3.4 Modified slope-deflection method.**

Sum of all the storey columns axial loads = total vertical load applied the storey

When using the slope-deflection method, a simple way of including  $P_{-\Delta}$  effects is to use a modified sway equation expressing the lateral equilibrium of a storey i.e.:

$$\sum_{j} \left( \overline{M}_{AB,sway} + \overline{M}_{BA,sway} \right) - \Delta_{i} \sum_{j} P_{j} = h_{i} [H_{i}]$$
(9)

where  $\sum_{j} P_{j}$  is the known sum of the axial loads in the columns *j* of the storey *i* and  $\Delta_{i}$  the storey sway *i* 

This approach leads to a sway direct solution, including the P- $\Delta$  effects, without iterations

# 6.CONCLUDING SUMMARY.

- 7.
- Approaches used for the modelling of frames for analysis, the basic concepts of analysis and the different methods of global frame analysis in use.
- Modelling of typical frames was described with the various aspects to be considered such as: resistance to horizontal forces and accounting for imperfections.
- Classification of frames as braced/unbraced and sway/nonsway, the assessment of the elastic critical load, and what frame and member imperfections are and when to account for them in the global analysis,
- Different sources of non-linear structural behaviour are identified
- 2<sup>nd</sup>-order effects are explained as well as its methods and limitations.
- Assumptions and limitations of the various elastic methods of analysis are given in particular as concerns the evaluation of frame stability.

- The idea that the choice of the analytical tool is partly guided by design to be undertaken and partly by the user as a function of software availability is introduced. The more sophisticated the analysis tool is, the less will be the resulting design tasks after the analysis.
- Different methods, "direct" and "indirect", for second-order analysis was outlined and the consequence that each has for the design of the frame and its components was explained