PLASTIC FRAME IDEALISATION & ANALYSIS

SUMMARY.

- Introduce the plastic frame modelling analysis approaches and basic concepts of analysis.
- Distinctions between elastic and plastic methods of analysis are identified.
- Assumptions and limitations of the various plastic methods of analysis are given
- Plastic analysis results are compared to the predicted and the actual structural behaviours, in particular in terms of the global frame stability.
- Required design efforts associated to each type of plastic analysis is summarised.

OBJECTIVES.

- Understand that the available tools for the plastic analysis of structures have limitations due to the adopted assumptions and simplification.
- Understand the differences between the various methods of elastic and plastic analysis.
- Understand the basis of and limitations of plastic analysis approaches.

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1. METHODS OF GLOBAL PLASTIC FRAME ANALYSIS.

Plastic methods of analysis are permitted only when minimum requirements on:

steel ductility
member cross-section/joint
lateral support at hinges

Guarantee that sections and joints, at least at the locations at which the plastic hinges may form, have sufficient rotation capacity to permit all the plastic hinges to develop

2 ELASTIC-PERFECTLY PLASTIC ANALYSIS (2^{ND}-ORDER).

2.1 Assumptions, limitations, section and joint requirements.

Elastic-perfectly plastic analysis → any section/joint → elastic up to the attainment of the plastic moment resistance, at which point it becomes ideally plastic

Plastic deformations → concentrated at the plastic hinge locations → infinite rotational capacity

Figure 1 → elastic-perfect plastic behaviour of a section/joint

normal force and/or the shear force → sections plastic moment resistance → directly or checked later → design verification stage

Computation of the plastic rotations at the plastic hinges → if required rotation capacity is available

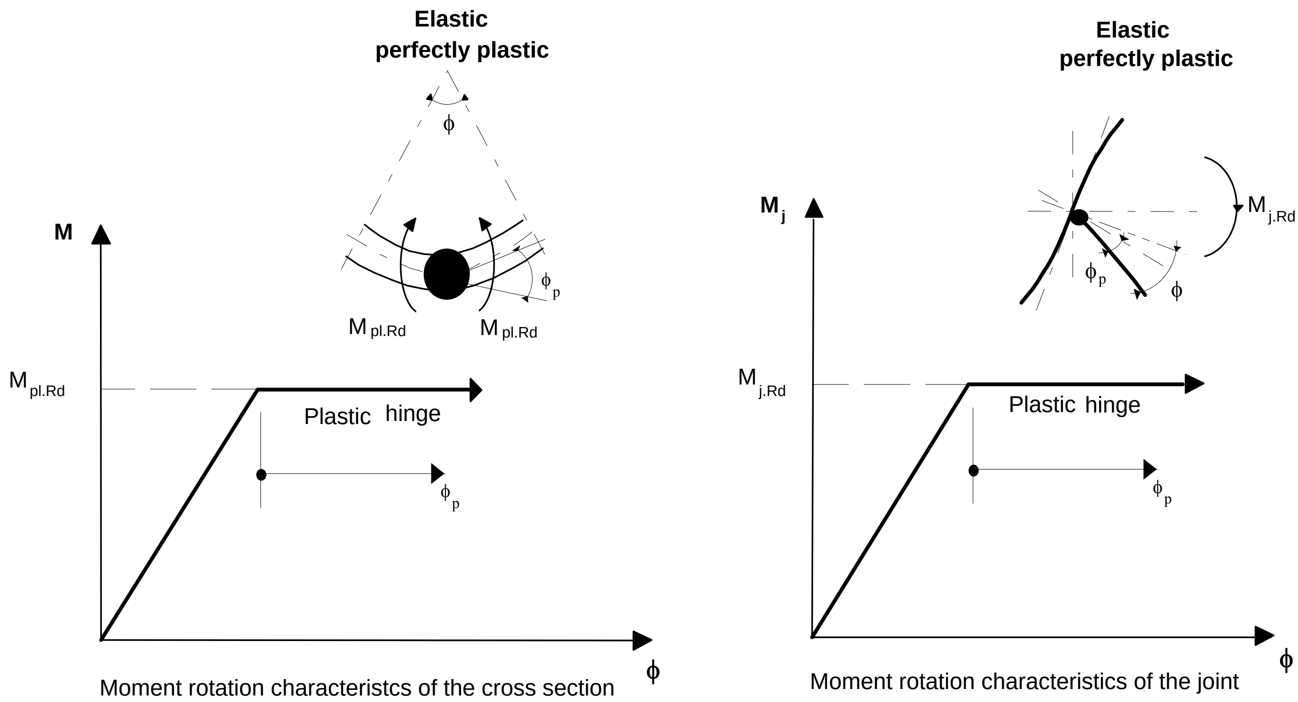


Figure 1 - Behaviour of members and joints.

2.2 Frame analysis and design.

2^{nd}-order elastic-perfect plastic analysis → load by increments

Plastic hinges → formed sequentially / or simultaneously

Starts → elastic second-order analysis displacements (Figure 2, branch 1) → monitoring frame bending moments in the at each load increment

First hinge load → section/joint plastic moment resistance

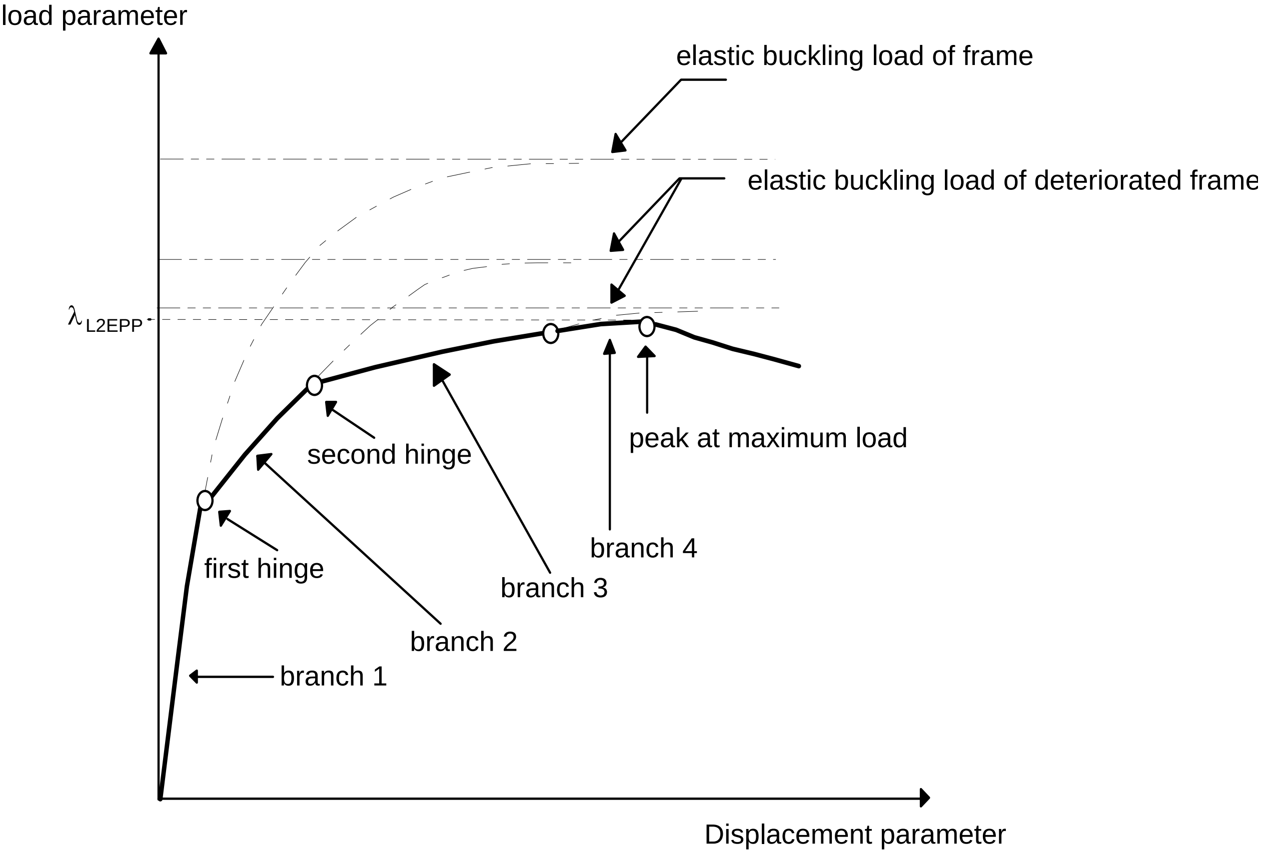


Figure 2 - Load displacement response: second-order elastic-perfectly plastic analysis.

Next analysis → further incremental loads → frame behaves differently → introduction of a pinned joint at the first plastic hinge (branch 2)

Joint introduced at the plastic hinge → acts as a pin only for the subsequent incremental increases in the loading → transferring the same moment = plastic moment resistance

Next plastic hinge formed → load increase → repeat process

Figure 2 solid curve → 2^{n}d-order elastic-perfectly plastic analysis results

Branch 1 → fully elastic → curve → asymptotic to elastic buckling load → only if → infinite elastic behaviour

First hinge → formed → frame behaves under further load increments as if one hinge exists in it (branch 2) → until the formation of the next hinge

Unlimited elastic behaviour → assumed after the first hinge → branch 2 → asymptotic to the "deteriorated" buckling load → frame with a pin introduced at the first hinge location

Process is repeated → new hinges being formed → till the structure becomes unstable (mechanism or frame instability)

2nd-order elastic-plastic analysis maximum load → this load level → reference load multiplier λ_{L2EPP} → Figure 2

No additional design checks of the resistance of sections and joints are required if the influence of the normal force and/or the shear force is accounted for

As the rotations at the plastic hinges have been calculated, → required rotation capacity is available

2nd-order theory → in-plane frame stability → covered by structural analysis

3 ELASTO-PLASTIC ANALYSIS (2ND ORDER THEORY)

3.1 Assumptions, limitations, section/joint requirements

2nd-order elasto-plastic analysis → better estimation of structural response → (relative to a 1st-order or 2nd-order elastic-perfectly plastic analysis)

Yielding of members and joints → progressive process → elastic to plastic transition is gradual

Once yielding commences → moment in the member cross section increases → plastic zone extends partially along the member / depth of the cross-section → plastic zone theory

Figure 3 → moment rotation characteristics of members → are usually adopted in this analysis

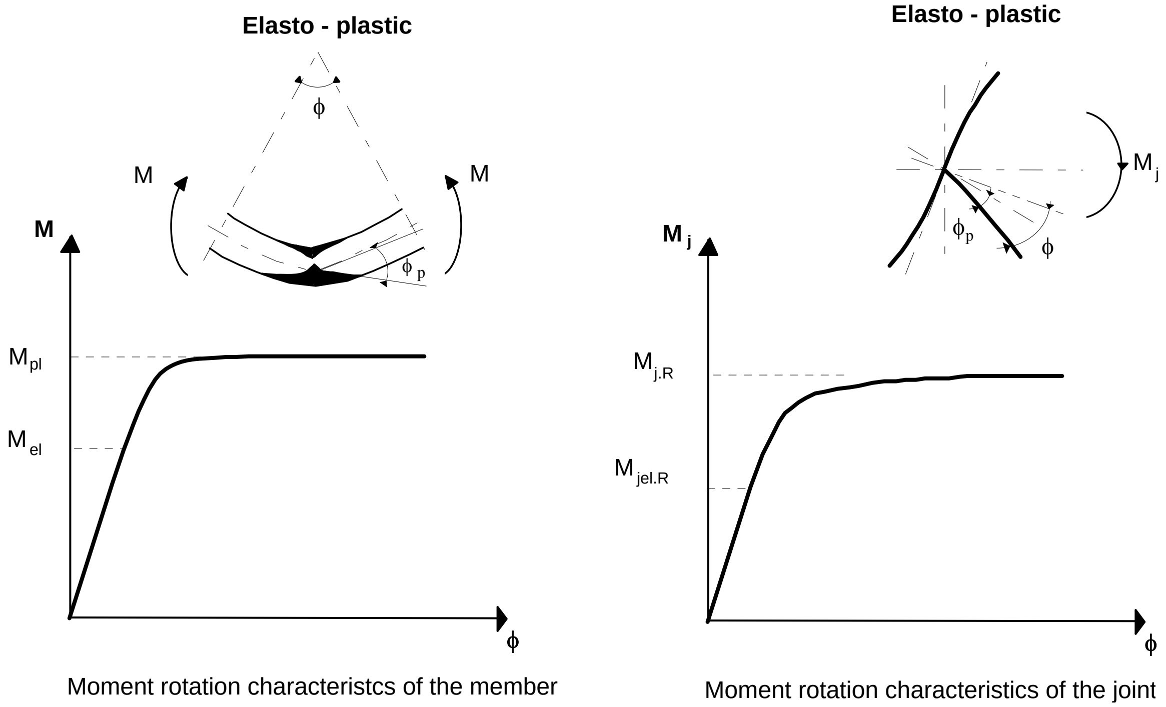


Figure 3 - Moment rotation characteristics of member/joint

Model have not included the beneficial effects of:
- material strain hardening
- membrane action

Ductility requirements + procedure for analysis/checks → = 2nd-order elastic-perfectly plastic analysis

Elasto-plastic method → complexity, → not used for practical design purposes → research applications

4 RIGID-PLASTIC ANALYSIS (FIRST-ORDER THEORY).

4.1 Assumptions, limitations, section and joint requirements

Contrary to the elastic-plastic analysis → elastic deformations (members, joints and foundations) → small compared to the plastic deformations → ignored in the rigid-plastic analysis

Elastic-perfectly plastic analysis → plastic deformations → concentrated in sections where plastic hinges are likely to occur → These sections → infinite rotational capacity

Figure 4 → idealised rigid-plastic response

Design moment resistance + structural configuration + loading → parameters that affect rigid-plastic analysis

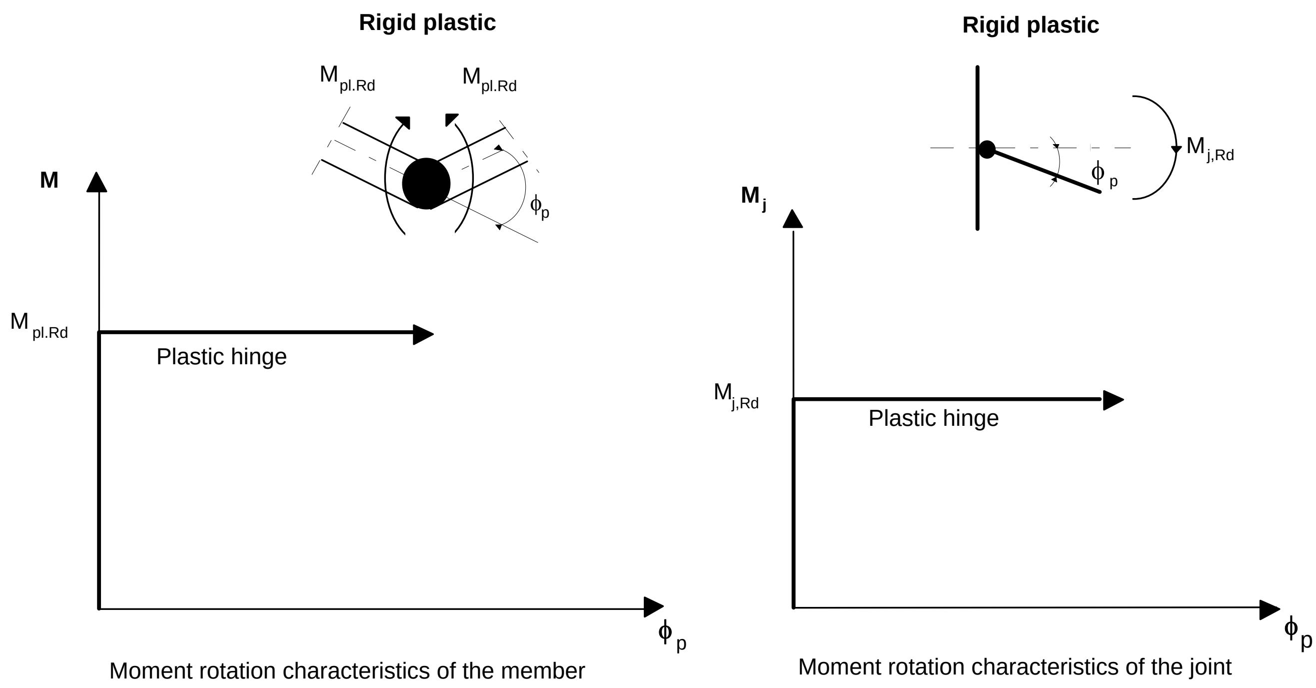


Figure 4 - Moment rotation characteristics of member/joint

Members ductility requirements → =elastic-perfectly plastic

Rigid-plast. methods → not usually suited → 2nd-ord analysis

4.2 Frame analysis

structure maximum load → collapse → realistic plastic mechanism has been created → analysis → identifying the critical mechanism

Collapse load → fundamental theorems of plastic design

Equilibrium (Statical) Method

1. Assume moments → equilibrium → applied forces
2. Satisfy that $M_d \leq M_{pl}$
3. Check to see if a mechanism exists

If a mechanism does not exit → additional load must be applied → evaluated load is a lower bound to collapse load

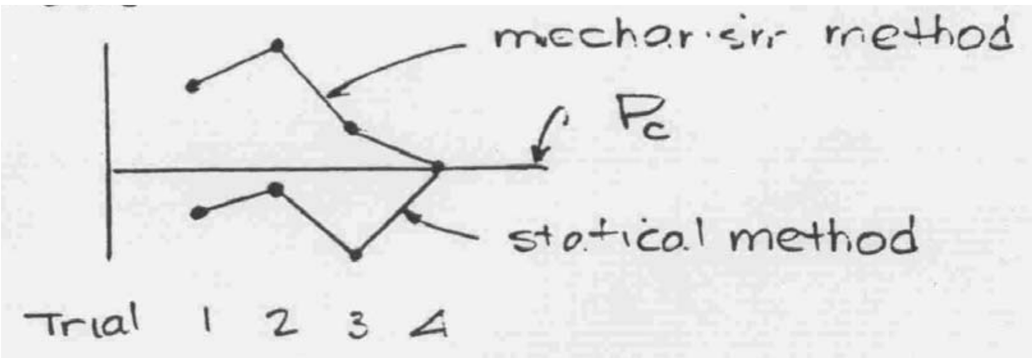
Mechanism (Kinematic) Method

1. Assume a mechanism.
2. Satisfy equilibrium equations.
3. Check that $M_d \leq M_{pl}$

If M_d is greater than M_{pl} → mechanism exists → remove loads → evaluated load is an upper bound to collapse load.

UNIQUENESS THEOREM

Collapse Load is the unique load that satisfies both methods



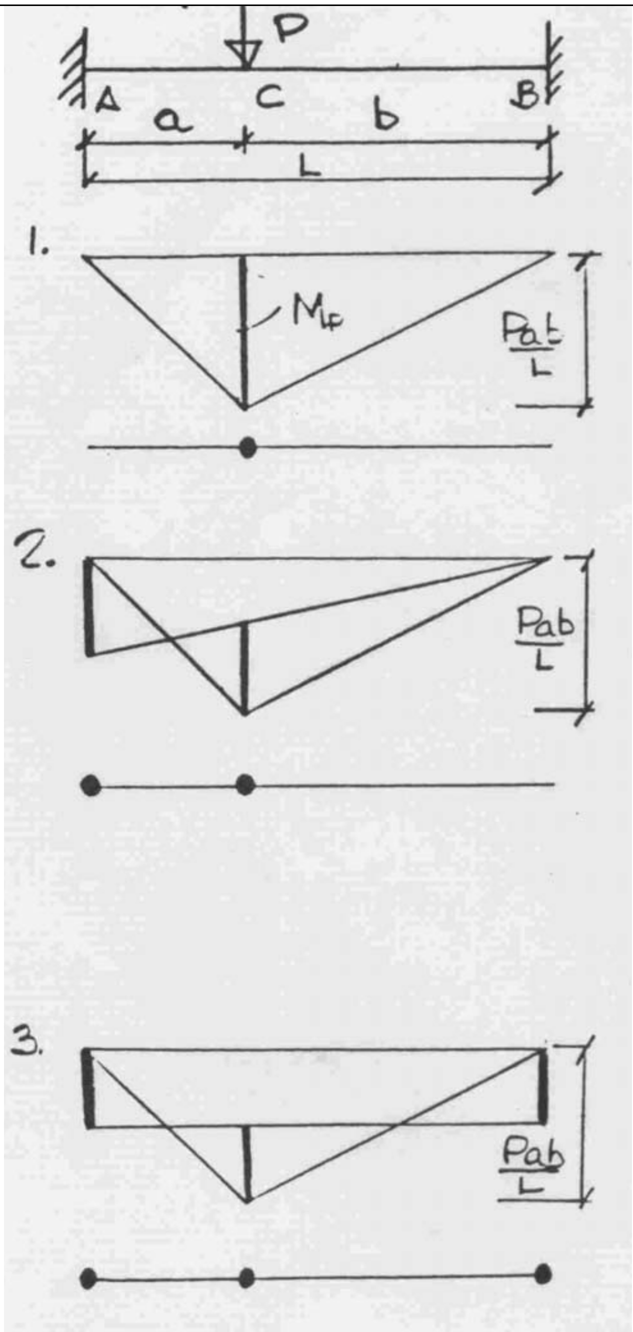
LOWER BOUND THEOREM

An estimate of the load capacity of a structure, based on some assumed distribution of internal forces and external reactions, will be a lower bound estimate, provided;

1- All the internal and external forces are in equilibrium.
2- Internal forces nowhere exceed the relevant force capacity
3- The behaviour is ductile, i.e. any sections at any point, when loaded to its force capacity can maintain that force during any subsequent deformation.

Equilibrium Method

	Trial 1 Draw static moment

	diagram of height Pab/L Make $M_c = M_{pl}$ $\therefore$ P = M_{pl} L/ab but no mechanism Trial 2 M_s = Pab/L Make $M_a = M_c = M_{pl}$ $\therefore$ [b/L] M_{pl} + M_{pl} =Pab/L M_{pl} [b+L]/L = Pab/L P = M_{pl} [b+L]/ab but no mechanism Trial 3 M_s = Pab/L Make $M_a = M_c = M_b = M_{pl}$ $\therefore$ 2 M_{pl} =Pab/L P = 2 M_{pl} L/ab and a mechanism exist.

Mechanism Method

1. Assume a mechanism.
2. Satisfy statical equilibrium by virtual work.
3. If $M_d > M_{pl}$ and $P > P_c$ an upper bound load was found

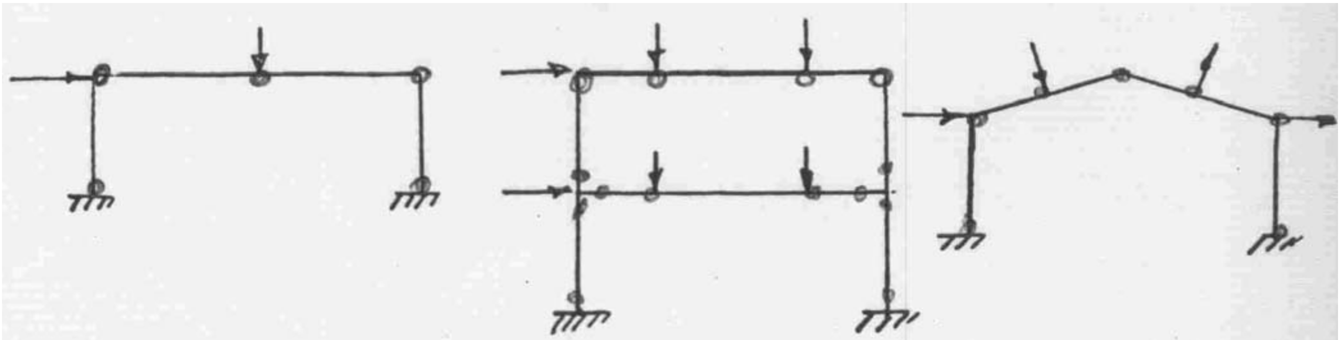
Procedure:

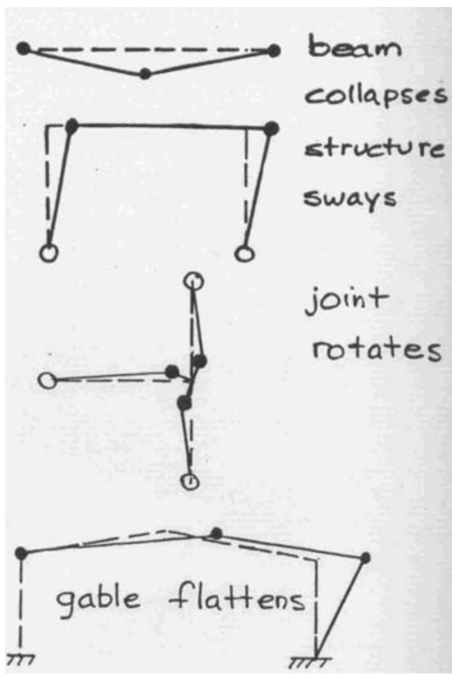
1. Determine points of possible plastic hinges.

2. Select a mechanism.
3. Solve equilibrium equations by virtual work.
4. Check $M_d \le M_{pl}$; if $M_d > M_{pl}$ upper bound load was found
   if $M_d = M_{pl}$ the correct solution is found

Types of Frame Mechanisms

Beam mechanisms are a subset of frame mechanisms.



1-Beam mechanism. 2-Sway mechanism. 3-Joint mechanism. 4-Gable mechanism.	

According to the uniqueness theorem, for a given structure and loading, any arbitrarily assumed plastic collapse mechanism occurs at a value of the load multiplier greater or equal than collapse load multiplier

Examining the various possible mechanisms → identifies the collapse mechanism for which the value of the load multiplier is least and which is both statically and plastically admissible

collapse load for each mechanism → Virtual Work → external work = internal work forming the mechanism

Example 1, elementary mechanisms 1 & 2 and the combined mechanism 3 for a simple portal frame

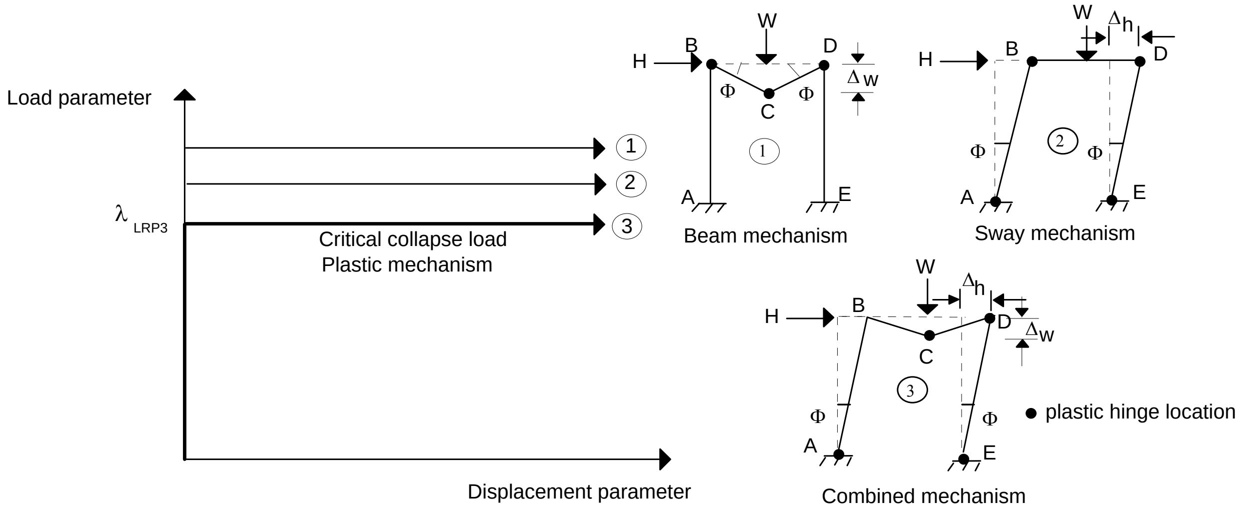


Figure 5 - Load displacement - Rigid plastic analysis

In order to establish the analysis/design equations for the simple frame in Figure 5, it is assumed that:

- Ratio of the design vertical load W_{Sd} acting at mid-span of the beam, to the design horizontal load H_{sd} acting at the eaves, α, is known from load combination case evaluation.
- Columns AB and DE, of height h, have the same cross-section design resistances.
- The joints at A and at E have the same design resistances.
- The joints at B and at D have the same design resistances.

- The design moment at A and E, denoted $M_{pl,Rd,1}$, will be the smaller of the design resistances for column section/joint
- The design moment at B and D, denoted $M_{pl,Rd,2}$, will be smaller of the resistances for the column section, for the beam section and for the joint.
- The design moment at C, denoted $M_{pl,Rd,3}$, is related to a beam cross-section that has a length of L.
- The positive and negative design moments at any section or joint are the same.

The equations corresponding to each mechanism obtained from application of the Virtual Work Principle

- ***Mechanism 1***: $W_{Rd,1}\Delta_{w1} = 2M_{pl,Rd,2}\phi_1 + 2M_{pl,Rd,3}\phi_1$

Since $\Delta_{w1} = (\phi_1)L/2$, we obtain $W_{Rd,1} = \frac{4(M_{pl,Rd,2} + M_{pl,Rd,3})}{L}$

so that $\lambda_{LRP1} = W_{Rd,1}/W_{Sd}$, which should have a value at least unity

- ***Mechanism 2***: $H_{Rd,2}\Delta_{h2} = 2M_{pl,Rd,1}\phi_2 + 2M_{pl,Rd,2}\phi_2$

Since $\Delta_{h2} = (\phi_2)h$, we obtain $H_{Rd,2} = \frac{2(M_{pl,Rd,1} + M_{pl,Rd,2})}{h}$

and $\lambda_{LRP2} = H_{Rd,2}/H_{Sd} = W_{Rd,2}/W_{Sd}$, should have a value at least unity

- ***Mechanism 3***: $H_{Rd,3}\Delta_{h3} + W_{Rd,3}\Delta_{w3} = 2M_{pl,Rd,1}\phi_3 + 2M_{pl,Rd,2}\phi_3 + 2M_{pl,Rd,3}\phi_3$

Since $\Delta_{h3} = (\phi_3)h$ and $\Delta_{w3} = (\phi_3)L/2$, we obtain:

$$H_{Rd,3} + W_{Rd,3}(L/2h) = H_{Rd,3}[1 + \alpha(L/2h)] = \frac{2(M_{pl,Rd,1} + M_{pl,Rd,2} + M_{pl,Rd,3})}{h}$$

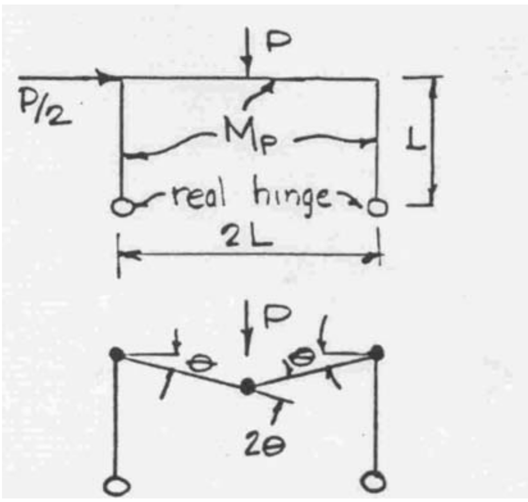
$\lambda_{LRP3} = {}^{H_{Rd,3}}\!/\!_{H_{Sd}} = {}^{W_{Rd,3}}\!/\!_{W_{Sd}}$, should have a value of at least unity

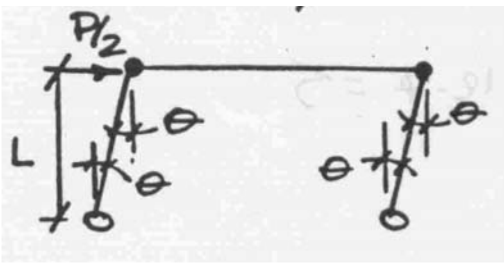
Any load-displacement response → horizontal line → ordinate → associated collapse load multiplier value

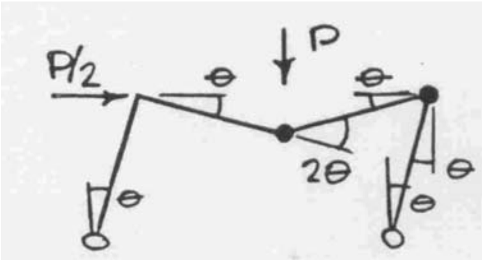
The lowest curve shall be retained, mechanism 3 in this case

Therefore, the collapse load given by the rigid-plastic analysis → load multiplier λ_{LRP3} shown in Figure 5.

Example 2

	Unknowns = 4 Equations = E =3 Degrees ind. = X = 1 Hinges to form mechanism = N =2 1. Beam Mechanism $P_1\ \theta\ L = W_e = W_i = M_{pl}\ (\theta + 2\theta + \theta)$ $P_1 = 4\ M_{pl}\ /L$

	2. Sidesway mechanism $(P_2\ /2\ \theta)\ L = M_{pl}\ (\theta + \theta)$ $P_2 = 4\ M_{pl}\ /L$

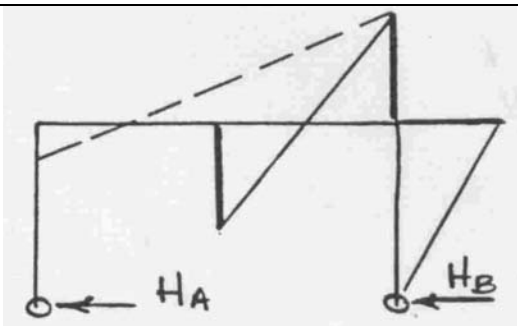
	3. Combined mechanism – beam and sidesway In combining mechanism try; (i) eliminate hinges, (ii) activate loads. (iii) combine those with lowest Pc: $(P_3/2\theta)L + P_3\ \theta\ L=M_{pl}(2\theta+2\theta)$ $P_3 = P_{cr} = 8/3\ M_{pl}\ /L$

If this is the critical load the moment diagram can be drawn with $M_d \leq M_{pl}$

Calculate Ms that is equal to the statical moment for the beam

$$Ms = \text{“PL”}/4 = (P\ 2L)/4 = 8/3\ (M_{pl}/L)\ (L/2) ==4/3\ M_{pl}$$

Now the moment diagram can be drawn in terms of M_{pl} on the tension side of the members

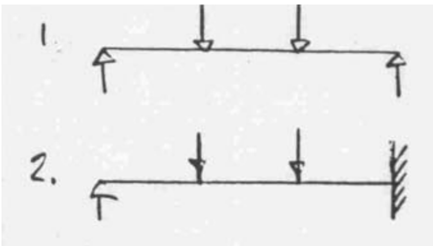
	First establish moments M_{pl} at plastic hinges $Hb = M_{pl}/L =3/8\ PL/L = 3/8\ P$ $\therefore\ Ha = 4/8\ P – 3/8\ P =P/8$

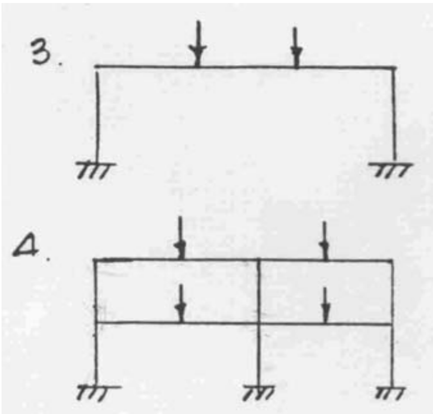
NUMBER OF MECHANISMS

Number of independent mechanisms, beam, sway, joint, gable:

$$n = N - X$$

where N is the number of possible plastic hinges
X is the number of redundancies

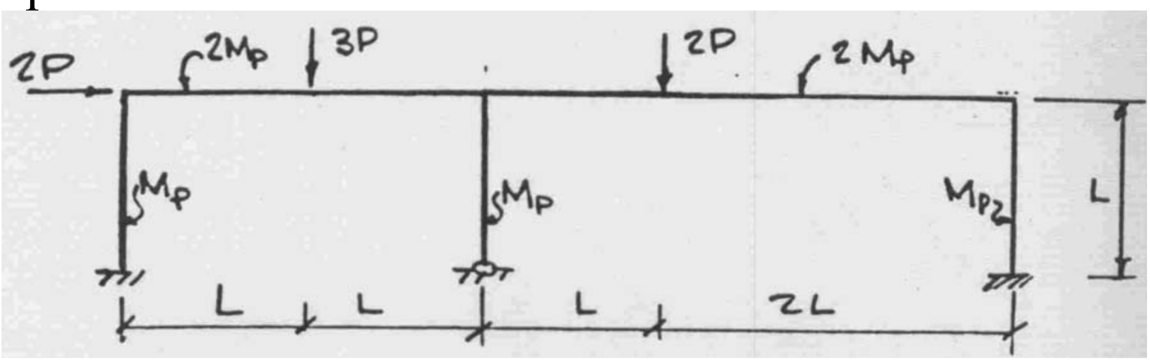
	$n = N - X$ $N = 2$ $X = 0$ $n = 2$ (2 beam) $n = N - X$ $N = 3$ $X = 1$ $n = 2$ (2 beam)

	$n = N - X$ $N = 6$ $X = 3$ $n = 3$ (2 beam, 1 sway) $n = N - X$ $N = 22$ $X = 12$ $n = 10$ (4beam, 4sway, 2joint)

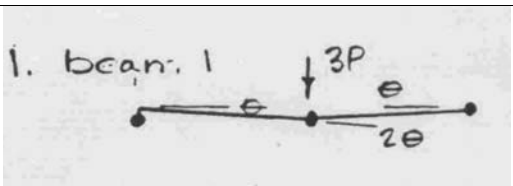
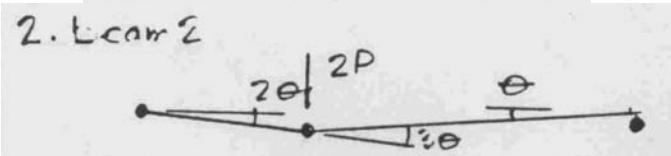
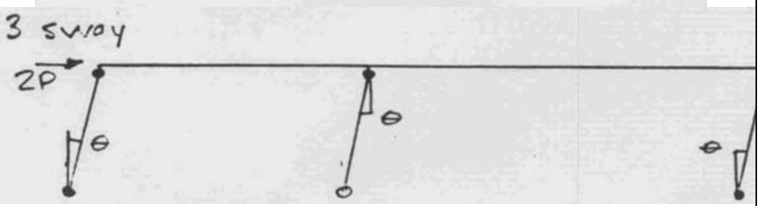
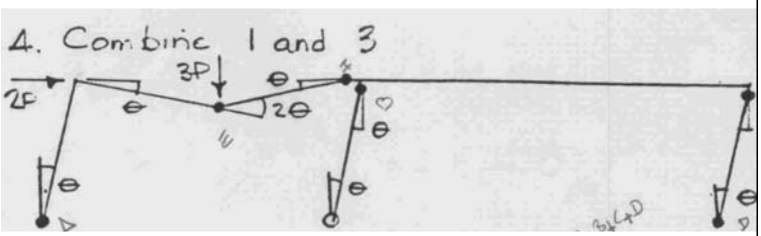
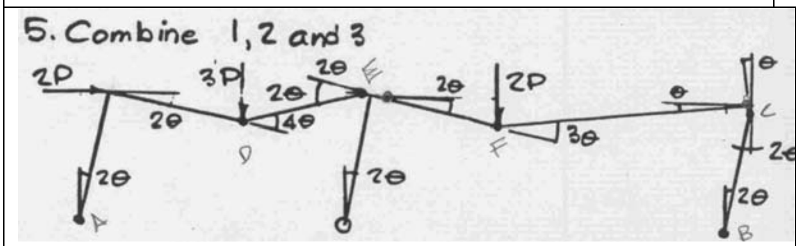
The number of combined mechanisms is:

$$N_c = 2^n - 1$$

Example 2



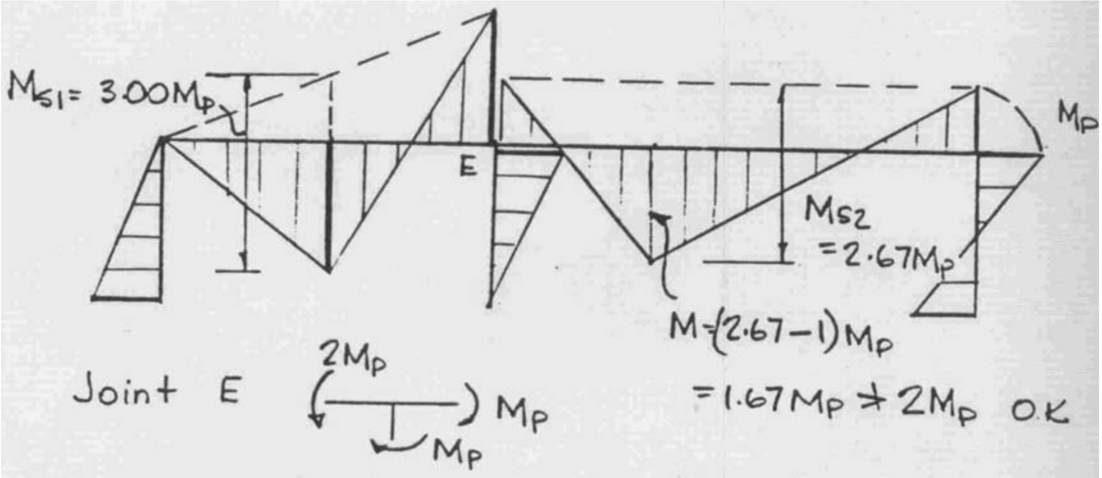
$n = N - X = 9 - 5 = 4$ (2 beam + 1 sway =1 joint)

	$3P\theta L = 2M_{pl}(2\theta+\theta)+M_{pl}(\theta)$ $\therefore\ P = 2.33M_{pl}/L$
	$2P2\theta L = 2M_{pl}(5\theta)+M_{pl}\theta$ $\therefore\ P = 2.75M_{pl}/L$
	$2\ P\theta L = 5\ M_{pl}\theta$ $\therefore\ P = 2.5M_{pl}/L$
	$2\ P\theta L + 3P\theta L = 5P\theta L =$ $M_{pl}(4\theta)+2M_{pl}(3\theta)=10M_{pl}\theta$ $\therefore\ P = 2.00M_{pl}/L$
	$2\ P2\theta L + 3P2\theta L +$ $2P2\theta L = 14P\theta L =$ $M_{pl}\theta\ [2+2+3]+$ $2M_{pl}\theta[4+4+3] = 29M_{pl}\theta$ $\therefore\ P = 2.07M_{pl}/L$

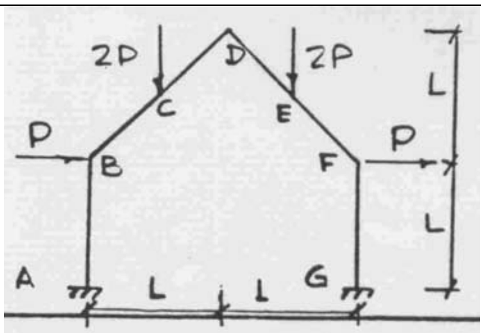
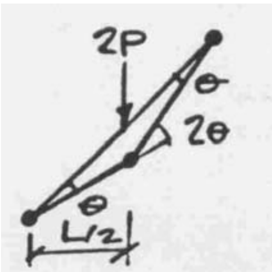
Check 4 with $P = 2\ M_{pl}/L \quad \therefore\ M_{pl} = 0.5PL$

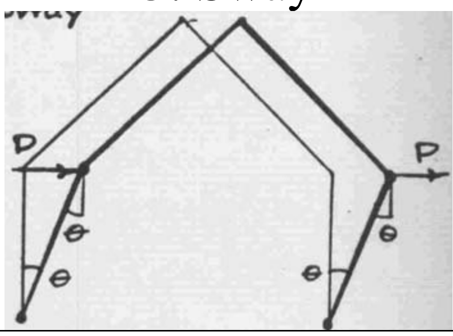
$M_{s1} = 3P\ 2L/4 = 150PL = 3M_{pl}$

M_{s2} = "Pab/L" $= 2PL\ 2L/3L = 4PL/3 = 2.67M_{pl}$

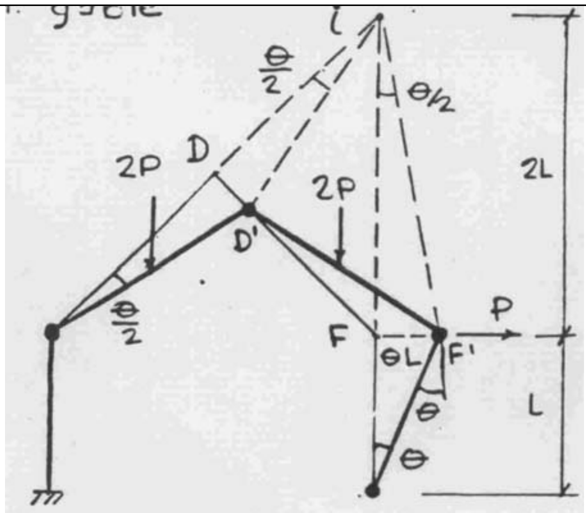


Gable Frame

 1,2 Beam mechanisms 	N= 7 X = 3 n=4 (2beam, 1 sway, 1 gable) M_{pl} throughout $2P\theta L/2 = 4M_{pl}\theta$ $\therefore\ P = 4M_{pl}/L$

3. Sway 	$2P\theta L = 4M_{pl}\theta$ $\therefore\ P = 2M_{pl}/L$

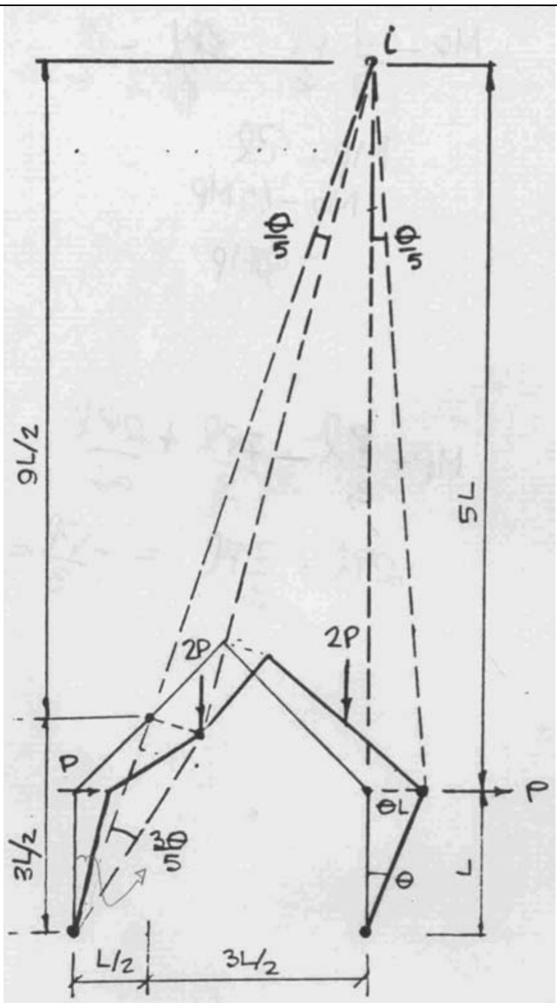
4. Gable	i is the instantaneous centre of rotation for DEF as part of BCD, D must move D to D’



as part of FG, F must move f to F'this locates I on BD projected and GF projected

$P\theta L + 2P\ \theta/2\ L/2 +$
$2P\ \theta/2\ L/2 = 2P\theta L =$
$M_{pl}\theta\ [1+1/2+1/2+1/2+1+1/2]$
$= 4M_{pl}\theta$

$\therefore\ P = 2.00M_{pl}/L$



Try a combined gable plus beam eliminating plastic hinges at B and D (where they are opposite in the gable and beam)

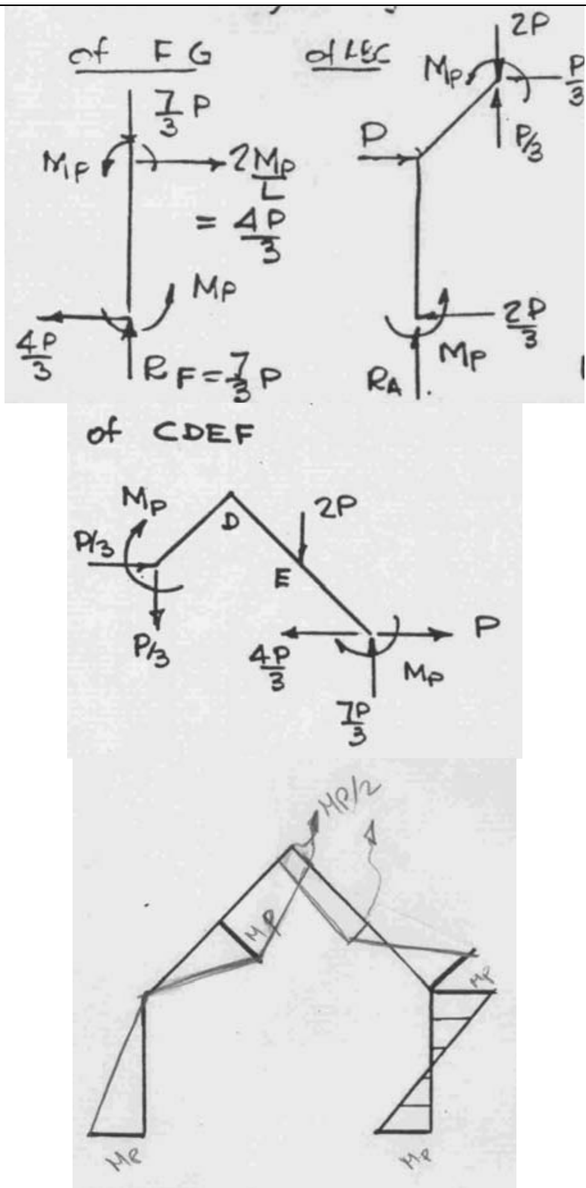
$P\ 3/5\ \theta L + P\theta L + 2P\ 3/5\ \theta\ L/2 +$
$2P\ \theta/5\ L/2 = 12/5 P\theta L =$
$M_{pl}\theta\ [3/5+4/5+6/5+5/5] =$
$18/5\ M_{pl}\theta$

$\therefore\ P = 1.50M_{pl}/L\ \ \therefore$
$M_{pl} = 2/3\ PL$
$M_{s1}=M_{s2}=\text{“}Pl/4\text{”}=2PL/4= 0.75M_{pl}$

Free-Body Diagram

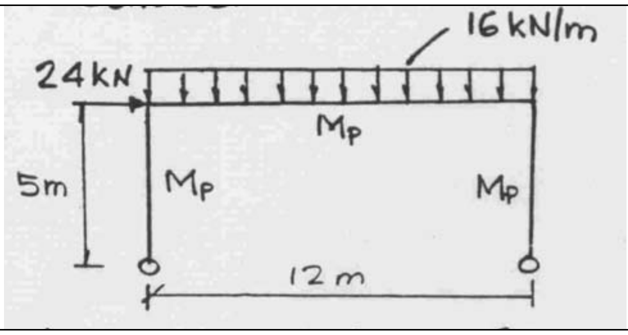
$M_c =0= PL/2- 2/3\ P\ 3/2\ L -$
$R_aL/2 +2\ M_{pl} =0$

$\therefore\ R_a= 4M_{pl}/L - P = 5/3\ P$

	$M_b = = M_{pl} - 2/3\ PL$ $M_d = = -M_{pl} – P/3\ L/2 + P/3L/2 = -M_{pl} – PL/3 = -0.5M_{pl}$ $M_E = = -M_{pl} – P/3\ L = -0.5M_{pl}$ The complete moment diagram

DISTRIBUTED LOADS

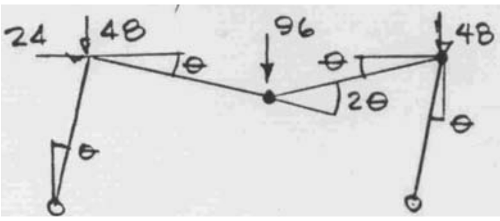
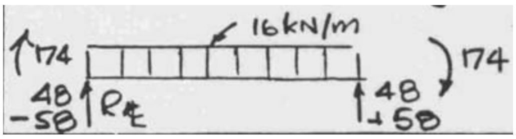
For beams with distributed loads the location of the hinge is not known in advance. Consider:

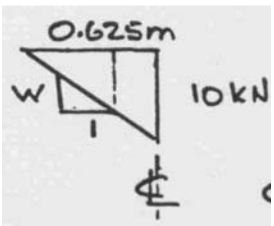
	$n = N – X \quad N = 3$ $X = 1$ $n = 2$ (1 beam, 1 sway)

Beam mechanism $2M_{pl} = wL^2/8 = 16\ 12^2/8;\ M_{pl} = 144$ KNm

Sway mechanism $5\ 24\theta = 2M_{pl}\,\theta$; $M_{pl} = 60$ kNm

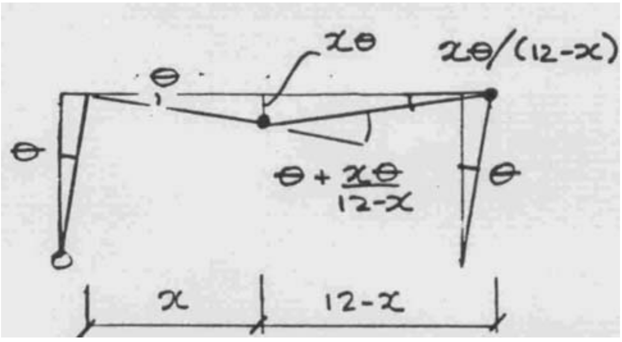
A. For the combine mechanism assume one-half of uniformly distributed load acts at centre and ¼ of U.D. L. act at each column because on average virtual displacement is half of the maximum.

 	½ of U.D.L.=1.2 16 12=96kN $4M_{pl}\,\theta = 24\ \ 5\,\theta + 96\ \ 6\,\theta$; M_{pl} =174 KNm examine right half of beam (174+174)/6 = 58 kN R_a = -10 kN ; R_b = 106 kN

	But this means shear is not zero at the beam midpoint but at a point 10/16 = 0.635 m to the left. The area of the shear diagram is 3.12kNm ∴ M_{max} where V=0 is 174+3 1=177 kNm.

If force hinge at incorrect location an upper bound to the collapse load is obtained (i.e. underestimate M_{pl} required.

B. Using the uniformly distribute load the location of the plastic hinge in the beam can be calculated



$$W_i = M_{pl}\left(\vartheta + \frac{x\vartheta}{12-x}\right)2 = 2M_{pl}\vartheta\left(\frac{12}{12-x}\right)$$

W_e=24 5θ+16xθ/2 12 = 120θ+96xθ

120 + 96x = 24M_{pl} / (12 - x)

(5 + 4 x)(12 – x) = M_{pl}

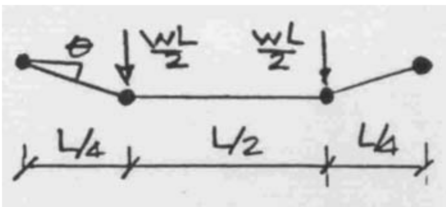
at x dM_{pl}/dx=0=(5+4x)(-1) + (12 –x)4

8x =43 ∴ x = 5.37 m and

M_{pl}=265 6.63=175.6kNm

C. A reasonable answer is obtained if the uniformly distribute load is replaced by two concentrated loads equal to wl/2 at the beam ¼ and ¾ points.

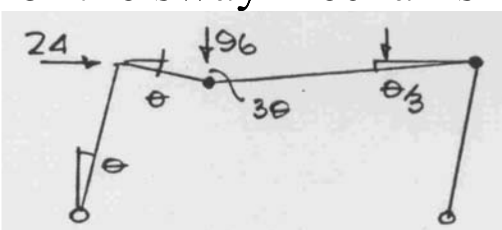
For the beam mechanism



WL/2 θL/4 2 = M_{pl} 4 θ

M_{pl} = $wl^2/16$

For the sway mechanism



24 5 θ + 96 3 θ + 96 θ = 25 M_{pl} (4θ/3) 2

M_{pl}=[120+384] 3/8=189 kNm

This is conservative by 9%

For multi-storey and/or multi-bay frames, for which particular care has to be taken to identify hinges that form and later unload the use of a computer programme is usually required.

Most typical frame structures → considering complete collapse mechanisms (mechanisms 2 and 3 are examples) and partial collapse mechanisms (mechanism 1 is an example)

Complete collapse mechanisms → entire frame → statically determinate at collapse

Single-storey pitched-roof portal frames → analysed using the approach given above → partly graphical "trial and error" method is often preferred → pinned bases are normally adopted → plastic hinges in the joints → avoided → haunches at the beam (rafter) ends

5. PLASTIC GLOBAL ANALYSIS AND DESIGN CHECKS.

Rigid-plastic analysis → direct information → design frame resistance

Adequate design → critical mechanism ≥ 1

Allowance for in-plane stability and 2^{nd}-order effects → reduction of load multiplier

Additional design checks → sections/joints → influence of the normal forces and/or the shear forces → design moment resistances → not negligible

Plastic hinges rotations → supposed infinite + no evaluation for them is made → sections sufficiently ductile must be used

This analysis → not provide structural deflections due to loads → complemented by an elastic analysis → serviceability loading conditions

Little difference between the other design tasks (stability for instance) → compared to those of a linear elastic analysis

5.1 Criteria to be respected for plastic analysis.

Plastic methods of analysis → following main restrictions:

1.Steel requirements :

- specified minimum tensile strength f_u to the specified minimum yield strength f_y ratio satisfies :

$$\frac{f_u}{f_y} \rangle = 1{,}2$$

- Failure elongation at on a gauge length of $5{,}65\sqrt{A_0} \geq 15\%$ (A_0 original cross section area)
- ultimate strain e_u → ultimate strength $f_u \geq 20$ x e_y yield strain → yield strength f_y.

2.Lateral restraint → at all plastic hinge locations at which plastic hinge rotation may occur under any load case.

Restraint → within a distance along the member from the theoretical plastic hinge location ≤ 1/2 depth of the member

3. Member section classifications → particular where plastic hinges occur → class 1 requirements.

Section classes 2 and 3 → may also be allowed → where hinges do not occur.

Class 2 sections → used at a hinge location → only when a large rotation capacity is not needed

4. Where cross-sections of the members vary along their length → restrictions are placed on the distances from a hinge at which → reductions of web thicknesses + changes in the web/compression flange class can be affected

Restrictions → guarantee that sections/joints → at least at locations where plastic hinges may form → sufficient rotation capacity → permit all the plastic hinges to develop

5.2. Application of plastic analysis.

loads → increase in a proportional and monotonic → collapse load multiplier →produce collapse by a plastic mechanism ≥ 1

Figure 6 → Eurocode 3 choices for a plastic global analysis + relevant checks

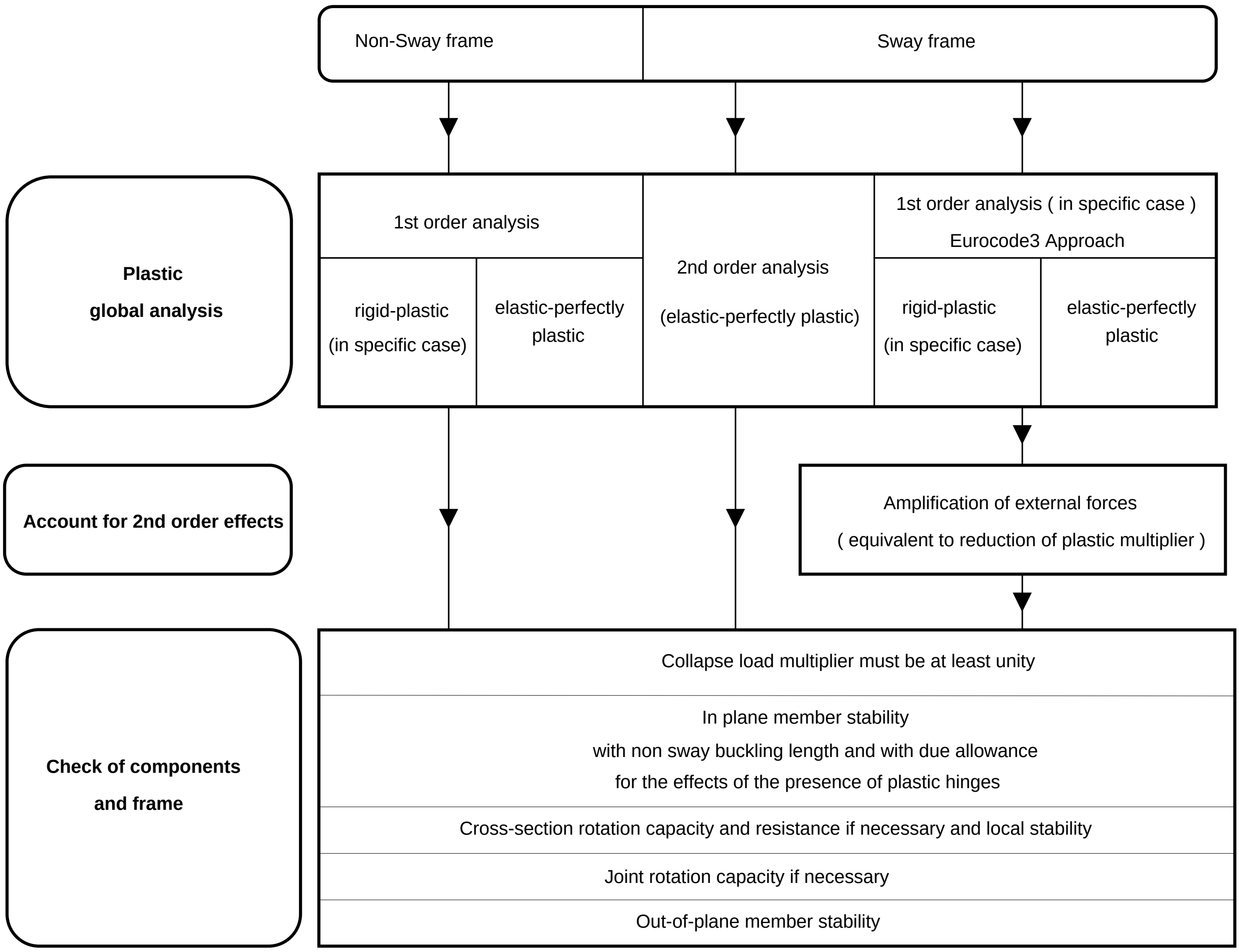


Fig. 6 – Ec3 Plastic global analysis & design checks

5.3 First-order plastic analysis and design.

1^{st}-order analysis (rigid-plastic or elastic-perfectly plastic) → non-sway frames → while for sway frames → limited to specific cases → single-storey pitched-portal frames

When using 1^{st}-order plastic analysis → especially rigid-plastic method → allow for frame imperfections → by "equivalent horizontal force" method

1^{st}-order plastic method → does not make allowance → any member buckling phenomena (in or out-of-plane) → checks carried out → allowance for presence of plastic hinges

First-order rigid-plastic method can be used → in-plane buckling lengths → non-sway mode → member design → allowance for effects of plastic hinges.

No further checks of the in-plane frame stability for sway buckling is required

1^{st}-order rigid-plastic analysis should not be used for unbraced frames with more than two storeys → see the exceptions under 2^{nd}-order elastic-plastic analysis

When plastic hinges occur → columns must be checked for in-plane buckling → buckling length = system length

These columns → adequate rotation capacity → in-plane slendernesses satisfing (EC3 §5.2.7(3)):

- Braced frames: $\bar{\lambda} \leq 0{,}4\left[\mathrm{Af_y}/\mathrm{N_{Sd}}\right]^{0{,}5}$,
- or alternatively $\dfrac{1}{\lambda_{cr}} = \dfrac{N_{Sd}}{N_{cr}} \leq 0{,}16$
- Unbraced frames: $\bar{\lambda} \leq 0{,}32\left[\mathrm{Af_y}/\mathrm{N_{Sd}}\right]^{0{,}5}$,
- or alternatively $\dfrac{1}{\lambda_{cr}} = \dfrac{N_{Sd}}{N_{cr}} \leq 0{,}10$

N_{cr} is the column member in-plane Euler buckling load

When the rotations at the plastic hinges have been calculated (elastic-plastic analysis), a check to ensure that the required rotation capacity is available can be carried out

rigid-plastic analysis →information not available → class 1 sections and ductile joints if necessary (when plastic hinges are located there) → must be used at plastic hinge locations

1st order plastic analysis methods → direct information → design frame resistance

Checks for cross-sections/joints resistance → required → influence of axial and/or shear forces → when these have not been included in the analysis method

Rigid-plastic method → does not provide any information → deflections/rotations → complemented by an elastic analysis → serviceability loading conditions

All other design checks → = 1st-order elastic analysis

5.4 Second-order plastic analysis and design

2nd-order plastic analysis → with allowance for global frame imperfections → may be used in all cases for which a plastic analysis is allowed →in particular → must be used for sway frames → where plastic design is chosen

Alternative to general 2nd-order elastic-plastic analysis → 1st-order rigid-plastic method → is allowed for certain types of frames → appropriate amplification of moments/forces

5.4.1 General method.

General method usually used → 2^{nd}-order elastic-perfectly plastic analysis method → used for all sway/non-sway frames

Elasto-plastic method → mostly used for research

Plastic global analysis restrictions on member classification, joint ductility and material properties apply.

2^{nd}-order effects → global frame imperfections + sway displacements are considered when performing global analysis

2^{nd}-order effects → local member imperfections, when required + in-plane member deflections are usually considered

Axial and/or shear forces influence → sections/joints plastic moment resistance → may also be allowed for in the formulation of the design resistances used in the analysis.

2^{nd} order elastic-perfectly plastic analysis has the advantages (over a 1^{st} order rigid-plastic):

- Frame collapse (plastic mechanism/instability) is identified
- All plastic hinges are identified, including any that may form but then unload (not appear in the frame collapse mechanism) but which need restraint as do all plastic hinges
- Hinges forming beyond the ultimate design loads can be identified
- Internal forces & moments, including 2^{nd} order effects, at stages up to collapse can be calculated

No additional design checks for the cross-sections are required → axial/shear forces influence is considered in the analysis.

As the rotation of the plastic hinges have been calculated, this permits checking → required rotation capacity

In most case of when elastic-perfectly plastic analysis is used in calculating frames, only in-plane behaviour of members is considered → separate out-of-plane stability checks are needed

No further checks of the in-plane frame stability for sway buckling are required →been covered by structural analysis

All other checks → as for 1st order elastic analysis case

5.4.2 Simplified second-order plastic analysis.

When plastic analysis is used, allowance shall be made for 2nd-order effects → sway mode

Rigid-plastic analysis → not normally be used for 2nd-order analysis → 2--order elastic-plastic analysis → usually required for sway frames

Ec3 Alternative to a 2nd-order elastic-plastic analysis → use of rigid-plastic first-order analysis (Ec3 § 5.2.6.3) → particular types of sway frames

Indirect methods with 1st-order elastic analysis → 2nd-order sway effects are accounted for indirectly → magnifying moments (and associated forces) → in this case → all internal moments/associated forces are magnified → not just those due to sway alone as it is the done in the elastic analysis case

The limitation on its use excludes the use of slender members for which member imperfections would have to be accounted

King → this method is derived → Merchant-Rankine criterion

Magnification factor = 1st-order elastic analysis:

$$\frac{1}{1 - V_{Sd}/V_{cr}}\ .$$

Method is limited to:

$$V_{Sd}/V_{cr} \leq 0{,}20$$

- excludes the use of slender members

also limited to structures that:

1. Frames one or two storeys high in which either:
   - no plastic hinge locations occur in the columns, or
   - Columns have in-plane slendernesses → buckling length = system length, satisfying conditions for plastic hinges columns designed with a 1^{st} order rigid-plastic analysis

2. Frames with fixed bases, in which the sway failure mode involves plastic hinges in the columns at the fixed bases only. The design is based on an incomplete mechanism → columns are designed to remain elastic at the calculated hinge moment and to meet the in-plane slenderness condition for columns with hinges

1st-order rigid-plastic method → allowed for specific cases of sway frames only → (one or two storey frames but also very specially designed multi-storey frames)

Internal forces/moments → ultimate design load under consideration → amplified to generate a consistent set of internal forces/moments → allowance for 2nd-order effects

Alternative → reanalyse the structure for loads increased by the magnification factor

Cross-section safety checks + joint resistance are required to account for the influence of axial and/or shear forces on the resistance moment

In-plane and out-of-plane member stability checks → using non-sway buckling length → allowance being for the presence of plastic hinges →According to Ec3 → these checks guarantee the overall in-plane & out-of-plane frame stability

All other design checks → = 1st-order rigid-plastic analysis

5.4.3 Merchant-Rankine approach.

Merchant-Rankine approach is not cited explicitly in Ec3 → criterion limits of application → used in sway frame classification

Amplified moment method applied to frames analysed by first-order rigid-plastic analysis → based upon it

It can be used for sway frames → included in national codes

The following limits on its use have been proposed:

$$4 \leq \frac{\lambda_{cr}}{\lambda_p} \leq 10$$

where: λ_{cr} is the linear elastic critical load multiplier

λ_p 1st-order collapse (plastic mechanism) load multiplier

Safety check of the entire frame → ensuring that the collapse load multiplier λ_f → calculated from the Merchant-Rankine:

$$\frac{1}{\lambda_f} \leq 1,0$$

Collapse load multiplier λ_f → Merchant-Rankine formula (modified version of the original Rankine formula):

$$\frac{1}{\lambda_f} = \frac{1}{\lambda_{cr}} + \frac{0,9}{\lambda_p}$$

This criterion is very simple to apply for checking frames

A safe and consistent set of internal forces & moments, needed for the design checks, can be generated by a 1st order elastic-perfectly plastic analysis

The limits on its use will exclude slender columns so that no account need be taken for the second-order effects due to member imperfections or member deflections

Cross-section/joint resistance safety checks are required → influence of axial/shear forces

When the frame is checked using the Merchant Rankine criterion, the out-of-plane member stability needs to be checked.

local buckling resistances may have to be checked for some members

All other design checks → = 1st-order rigid-plastic analysis

5.4.4 Origins of the Merchant-Rankine method.

Load multiplier ratio limiting values for the Merchant-Rankine approach → found in an inverted form in Ec3 → used for other purposes

A sway frame is defined as when:

$$V_{Sd}/V_{cr} > 0{,}1 \quad \text{or} \quad V_c/V_{Sd} < 10$$

Limit for the application of the Amplified Sway Moment Method is given as:

$$V_{Sd} \,/\, V_{cr} \leq 0{,}25 \;\; or \;\; V_{cr} \,/\, V_{Sd} \geq 4$$

These limits → proposed by Wood and Merchant → good "engineers guess" → validity range of the empirical design formula → its validity domain → beyond the limits given above

The original Rankine formula, which is empirical in nature, is:

$$\frac{1}{\lambda_f} = \frac{1}{\lambda_{cr}} + \frac{1}{\lambda_p}$$

This formula provides a safer lower bound than the Perry-Robertson formula for column buckling while the Merchant-Rankine variant fits better with test results since it allows a "squash buckling" range

Merchant → same formula → finding sway frame resistance

The formulae are drawn in Figure 7

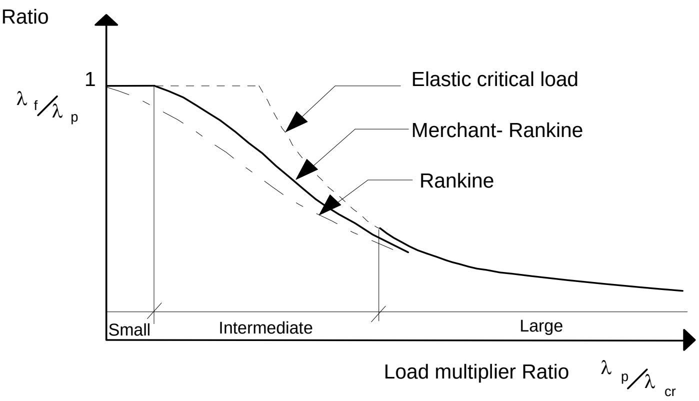


Figure 7 - Rankine and Merchant-Rankine formulae.

6. CONCLUDING SUMMARY.

- Distinctions between elastic/plastic analysis methods are identified

- Assumptions and limitations of the various plastic methods of analysis are given

- Results of each analysis is described so as to permit a comparison of the predicted and the actual structural behaviours, in particular the evaluation of frame stability

- Design effort required subsequent to using each type of plastic analysis is summarised so as to give an understanding of the essential implications of the use of the method