Compression Members

Columns I

Summary:

- Structural members subjected to axial compression are known as *columns* or *struts.*
- Stocky columns may not be affected by *overall buckling.*
- Stocky columns may fail by *local buckling* or squashing.
- Columns with high slenderness fail by *elastic buckling.*
- Columns of medium slenderness are sensitive to *imperfections* and fail by *inelastic buckling.*
- Imperfections in real columns reduce their capacity below what is predicted by theory.
- For design, a probabilistic approach using *column curves* is adopted.
- The *design buckling resistance* is found by reducing the *compression resistance of the cross section* by a *reduction factor* for the relevant buckling mode.

Objectives:

- Describe the difference in behaviour of stocky and slender columns.
- Recognise the imperfection sources in real columns and the need for a design probabilistic approach.
- Compare the ECCS column curves.
- Calculate the non-dimensional slenderness of a column.
- Reduction factor evaluation for relevant buckling modes of different cross-sectional shapes columns.

References:

- Structural Stability Research Council, Galambos, T.V., (ed) Guide to Stability Design Criteria for Metal Structure, 4th edition, John Wiley, New York, 1988.
- Eurocode 3: Design of steel structures Part 1.1 General rules and rules for buildings.
- Timoshenko, S.P. and Gere, J.M., Theory of Elastic Stability, McGraw -Hill, New York, 1961.
- Trahair, N.S. and Bradford, M.A., The Behaviour and Design of Steel Structures, E&F Spon, 1994.

Contents:

1. Introduction

The term ‘compression member is generally used to describe structural components only subjected to axial compression loads; this can describe columns (under special loading conditions) but generally refers to compressed pin-ended struts found in trusses, lattice girders or bracing members.

If they are subjected to significant bending moments in addition to the axial loads, they are called beam-columns.

Compression members concerns very few real columns because eccentricities of the axial loads and transverse forces are generally not negligible.

Because most steel compression members are rather slender, buckling can occur; this adds an extra bending moment to the axial load and must be carefully checked.

The lecture briefly describes the different kinds of compression members and explains the behaviour of both stocky and slender columns; the buckling curves used to design slender columns are also given.

2. Main Kinds Of Compression Members.

2.1 Simple Members with Uniform Cross-Section

For optimum performance compression members need to have a high radius of gyration, *i*, in the direction where buckling can occur; circular hollow sections should, therefore, be most suitable in this respect as they maximize this parameter in all directions.

The connections to these sections are, however, expensive and difficult to design.

It is also possible to use square or rectangular hollow sections whose geometrical properties are good (the square hollow sections being the better); the connections are easier to design than those of the previous shape, but again rather expensive.

Hot-rolled sections are, in fact, the most common cross-sections used for compression members. Most of them have large flanges designed to be suitable for compression loads.

Their general square shape gives a relatively high transverse radius of gyration i_z and the thickness of their flanges avoids the effect of local buckling.

The open shape, produced by traditional rolling techniques, facilitates beam-to-column and other connections.

Welded box or welded H-sections are suitable if care is taken to avoid local flange buckling.

They can be designed for the required load and are easy to connect to other members; it is also possible to reinforce these shapes with welded cover plates, Figure 1.

It should be noted that:

- Type of connection is important in the design of simple compression members because it defines the effective length to be taken into account in the evaluation of buckling.
- Circular sections do not represent the optimum solution if the effective length is not the same in the two principal directions; in this case, non symmetrical shapes are preferable.

- Members are frequently subjected to bending moments in addition to axial load; in these conditions I-sections can be preferable to H-sections.

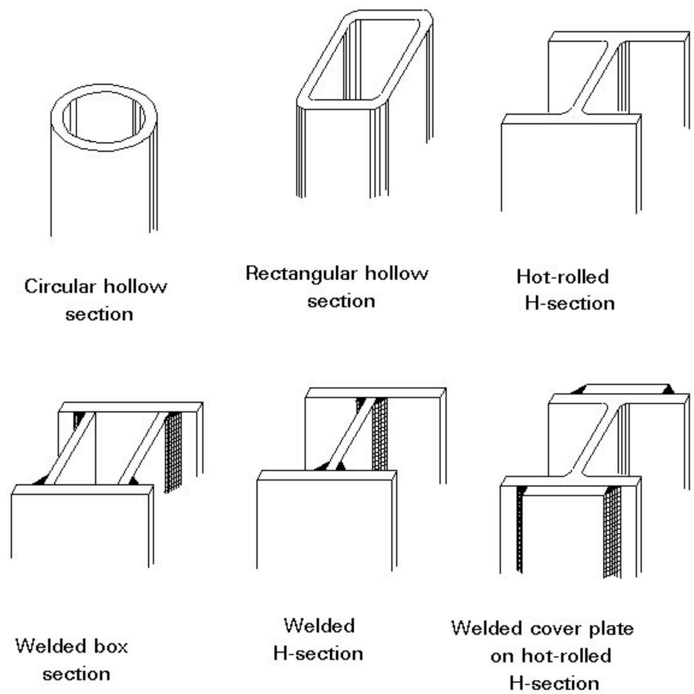


Figure 1 – Simple compression members.

2.2 Simple Members with Non-Uniform Cross-Sections

Members with changes of cross-section within their length, are called non-uniform members; tapered and stepped members are considered in this category.

In tapered members (Figure 2) the cross-section geometry changes continuously along the length; these can be either open or box shapes formed by welding together elements, including tapered webs, or flanges, or both.

A classical example is that of a hot-rolled H or I-section whose web is cut along its diagonal; a tapered member is obtained by reversing and welding the two halves together.

Stepped columns (Figures 3) vary the cross-section in steps.

A typical example of their use is in industrial buildings with overhead travelling cranes; the reduced cross section is adequate to support the roof structure but must be increased at crane level to cater for the additional loads.

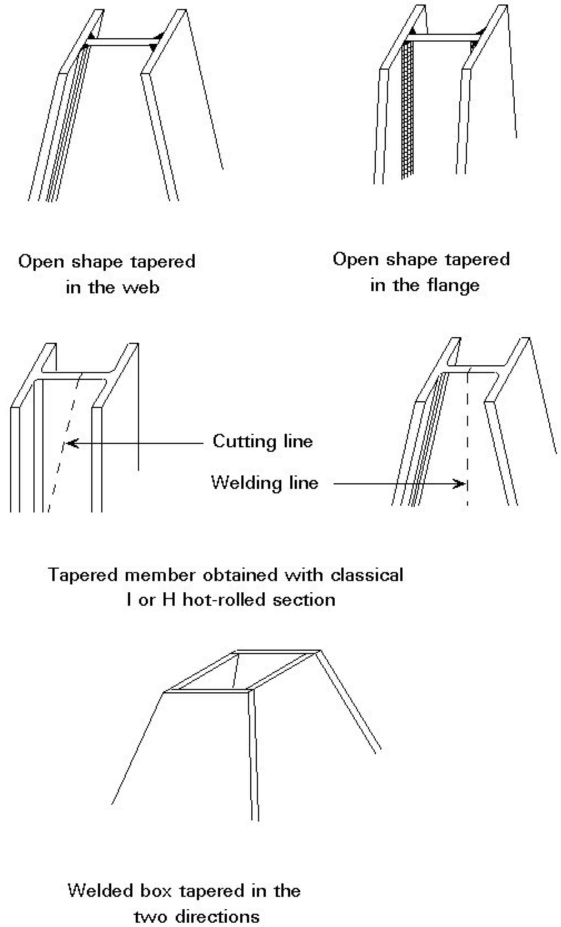


Figure 2 – Tapered members.

Stepped columns can also be used in multi-storey buildings to resist the loads at the lower levels columns.

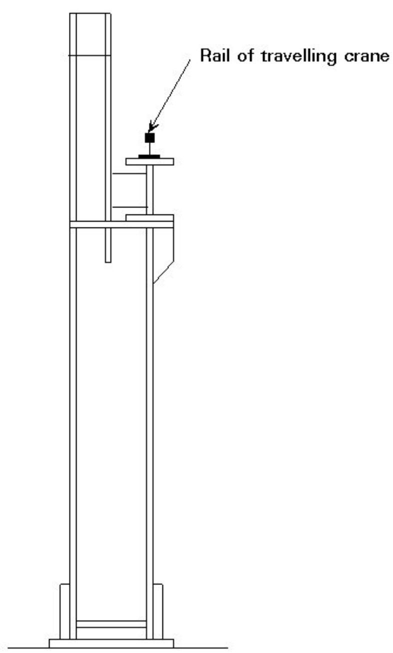


Figure 3 – Stepped column.

2.3 Built-up Columns

Built-up columns are fabricated from various different elements; they consist of two or more main components, connected together at intervals to form a single compound member (Figure 4).

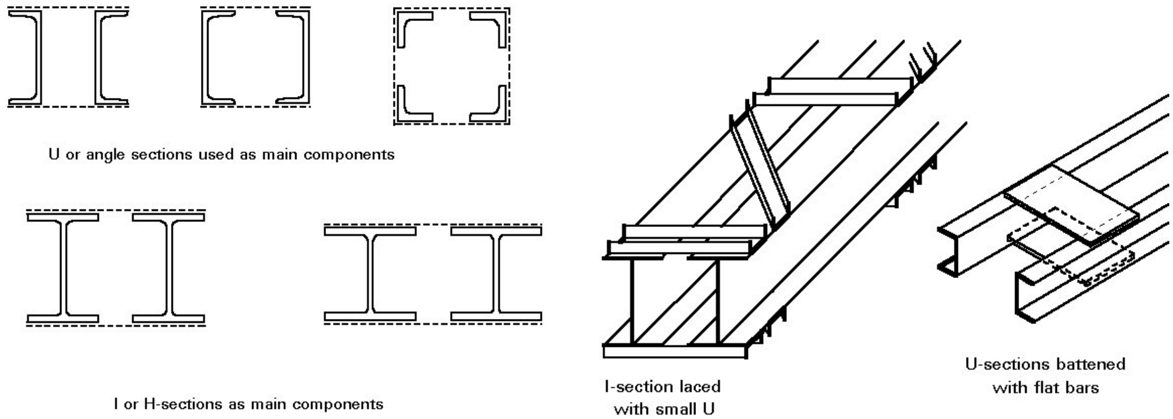


Figure 4 - Built-up columns.

Channel sections and angles are often used as main components but I or H-sections can also be used.

They are laced or battened together with simple elements (bars or angles or smaller channel sections) and it is possible to find columns where both methods are combined (Figure 5).

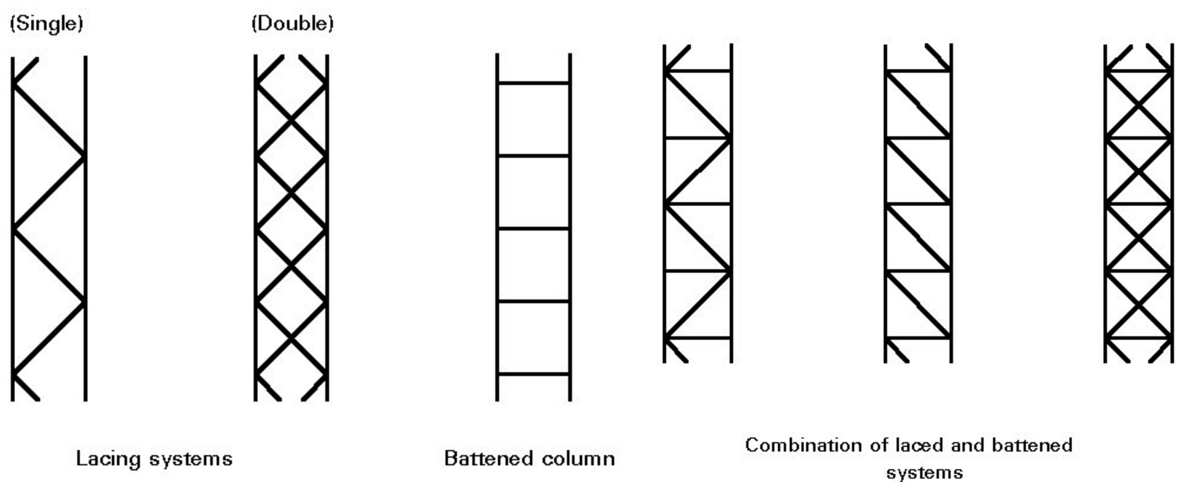


Figure 5 – Laced and battened columns.

Columns with perforated plates (Figure 6) are also considered in this category.

The advantage of this system is that it gives relatively light members because most of the steel is placed quite far from the centre of gravity of the cross-section have a relatively high radius of gyration.

The most important disadvantage comes from the high fabrication costs involved.

These members are generally designed for large structures where the compression members are long and subjected to heavy loads.

It is to be noted that buckling of each individual element must be checked very carefully.

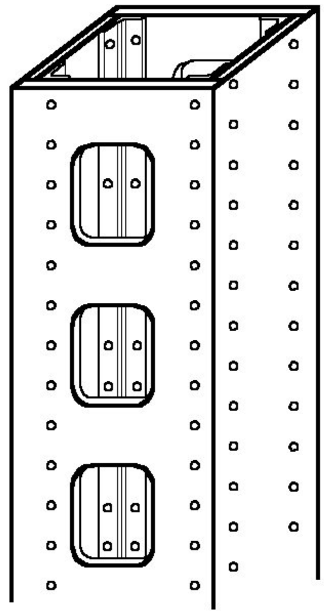
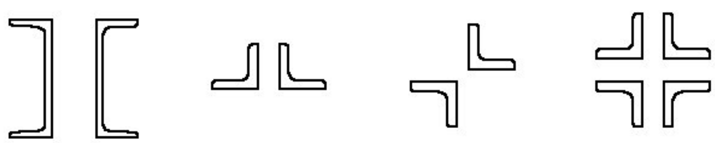
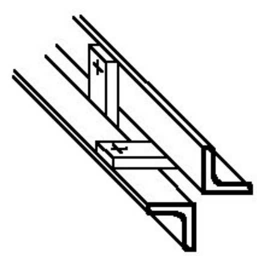


Figure 6 – Perforated plate column.

Figure 7 shows some closely spaced built-up members and gives details of star-battened angle members.



Closely spaced built-up members



Detail of star-battened member

Figure 7 – Built-up members.

These are not so efficient as the previous ones because of their smaller radius of gyration; however, the ease with which these can be connected to other members may make their use desirable.

They behave in compression in a similar way to those described above.

Built-up columns may have uniform or non-uniform cross-sections; it is possible, for example, to find stepped or tapered built-up columns (Figure 8).

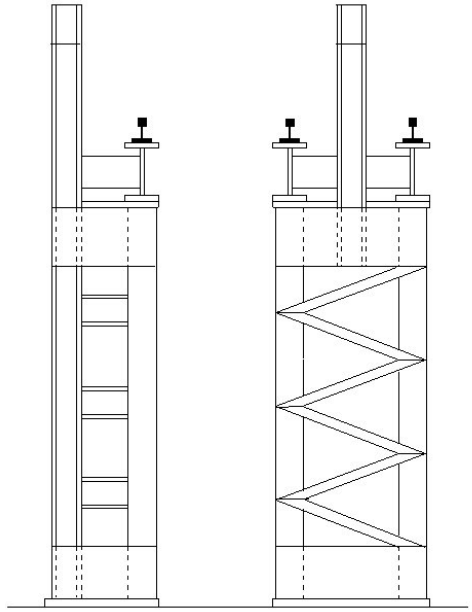


Figure 8 – Stepped built-up columns.

3. Stub columns

Stocky columns have a very low slenderness such that they are unaffected by the member overall buckling.

In such cases the compressive capacity of the member is dictated by the compressive resistance of the cross-section, which is a function of the section classification.

It is to be noted that residual stresses and geometric imperfections are practically without influence on the ultimate strength of this kind of column and that most experimental stub columns fail above the yield stress because of strain-hardening.

Class 1, 2, 3 cross-sections are all unaffected by local buckling and hence the design compression resistance is taken as the design plastic resistance,

$$N_{c.Rd} = N_{p\ell.Rd} = Af_y / \gamma_{M0} \quad (1)$$

Eurocode 3
5.4.4(1) a) or eq. 6.9 & 6.10

For Class 4 cross-sections, local buckling in one or more elements of the cross-section prevents the attainment of the squash load and thus the design compression resistance is limited to the local buckling resistance,

$$N_{c.Rd} = N_{o.Rd} = A_{eff} f_y / \gamma_{M0} \quad (2)$$

Eurocode 3
5.4.4(1) b) or eq. 6.9 & 6.11

Where A_{eff} is the area of the effective cross-section.

4. Slender steel columns

Depending on their slenderness, columns exhibit two different types of behaviour: those with high slenderness present a quasi elastic buckling behaviour whereas those of medium slenderness are very sensitive to the effects of imperfections.

If ℓ_{cr} is the critical length, the Euler critical load N_{cr} is equal to:

$$N_{cr} = \frac{\pi^2 EI}{\ell_{cr}^{\ 2}} \tag{3}$$

and it is possible to define the Euler critical stress σ_{cr} as:

$$\sigma_{cr} = \frac{N_{cr}}{A} = \frac{\pi^2 EI}{\ell_{cr}^{\ 2} A} \tag{4}$$

By introducing the radius of gyration, i = $\sqrt{I / A}$, and the slenderness, $\lambda = \ell_{cr} / i$, for the relevant buckling mode, Equation (4) becomes:

$$\sigma_{cr} = \frac{\pi^2 E}{\lambda^2} \tag{5}$$

Plotting the curve σ_{cr} as a function of λ on a graph (Figure 9), with the horizontal line representing perfect plasticity, $\sigma = f_y$, shown, it is interesting to note the idealised zones representing failure by buckling, failure by yielding and safety.

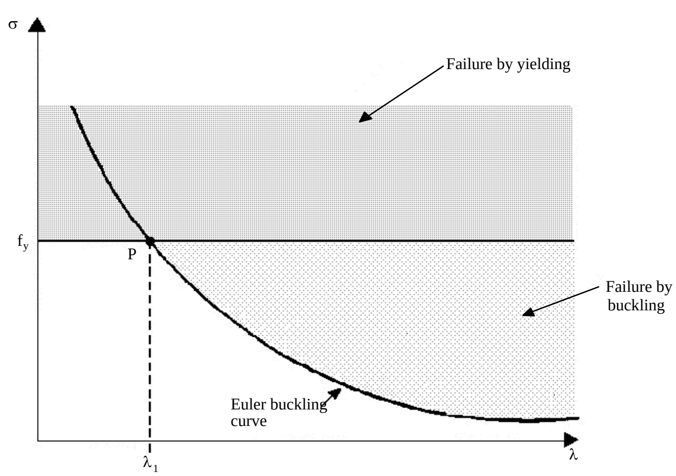


Figure 9 - Euler buckling curve and modes of failure.

The intersection point P, of the two curves represents the maximum theoretical value of slenderness of a column compressed to the yield strength.

This limiting slenderness when σ_{cr} is equal to the yield strength of the steel is given by:

$$\lambda_1 = \pi[E / f_y]^{0,5} = 93{,}9\varepsilon \tag{6}$$

5.5.1.2 (1)

where:

$$\varepsilon = [235 / f_y]^{0,5} \quad (7)$$

therefore λ_1 is equal to 93,9 for steel grade S275 and to 76,4 for steel grade S355.

Figure 9 may be redrawn in a non-dimensional form, figure 10, by dividing the Euler critical stress by the yield strength (σ_{cr} / f_y) and the slenderness by the limiting slenderness (λ / λ_1).

The same plot applies to struts of different slenderness and material strengths.

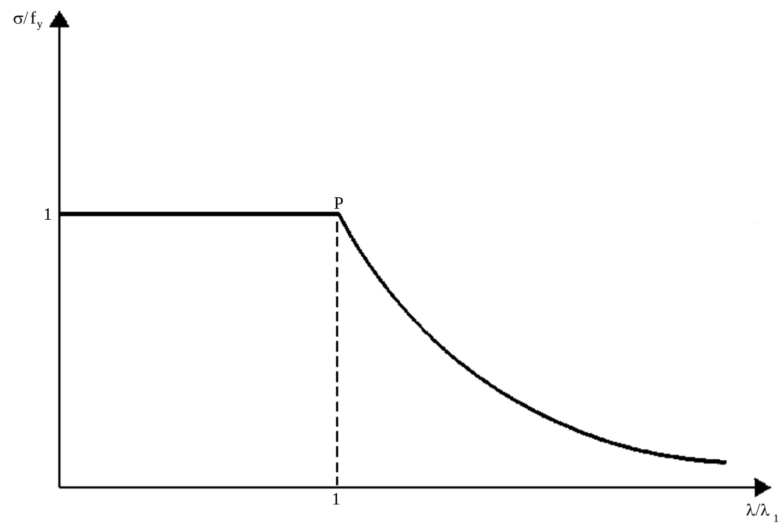


Figure 10 - Non-dimensional buckling curve.

The real behaviour of steel columns is rather different from the idealised behaviour described above.

Columns generally fail by inelastic buckling before reaching the Euler buckling load due to various imperfections in the "real" element: initial out-of-straightness, residual stresses, eccentricity of axial applied loads and strain-hardening.

Experimental studies of real columns give results as shown in Figure 11.

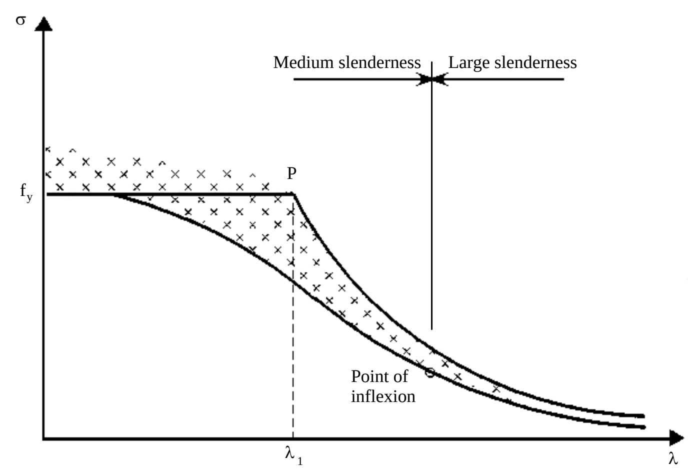


Figure 11 - Real column test results and buckling curves.

Compared to the theoretical curves, the real behaviour shows greater differences in the range of medium slenderness than in the range of large slenderness.

In the zone of the medium values of λ (representing most practical columns), the effect of structural imperfections is significant and must be carefully considered.

The greatest reduction in the theoretical value is in the region of limiting slenderness λ_1.

A lower bound curve is obtained from a statistical analysis of test results and represents a safe load limit.

A column can be considered slender if its slenderness is larger than that corresponding to the point of inflexion of the lower bound curve, shown in Figure 11.

The ultimate failure load for such slender columns is close to the Euler critical load (N_{cr}) and is thus independent of the yield stress.

Columns of medium slenderness are those whose behaviour deviates most from Euler's theory.

When buckling occurs, some fibres have already reached the yield strength and the ultimate load is not simply a function of slenderness; the more numerous the imperfections, the larger the difference between the actual and theoretical behaviour.

Out-of-straightness and residual stresses plays a fundamental role on the behaviour of these columns.

Residual stresses can be distributed in various ways across the section as shown in Figure 12.

Residual stresses combined with axial stresses cause yielding to occur in the cross-section and the effective area able to resist the axial load is, therefore, reduced.

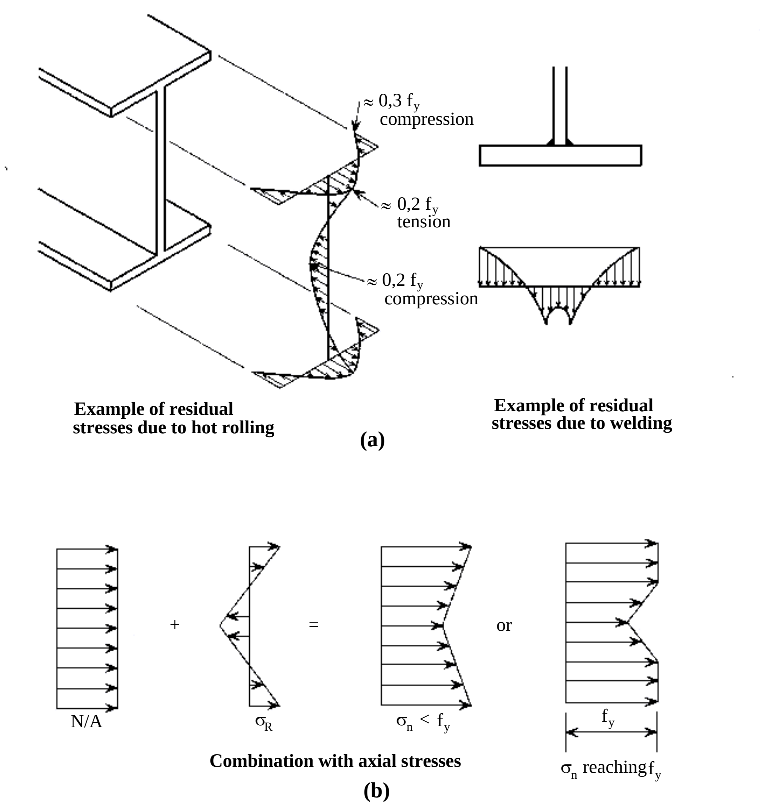


Figure 12 - Residual stress pattern.

An initial out-of-straightness e_O, produces a bending moment giving a maximum bending stress σ_B (Figure 13a), which when added to the residual stress, σ_R gives the stress distribution, Figure 13b.

If σ_{max} is greater than the yield stress the final distribution will be partly plastic and sections of the member will have yielded in compression, as shown in Figure 135c.

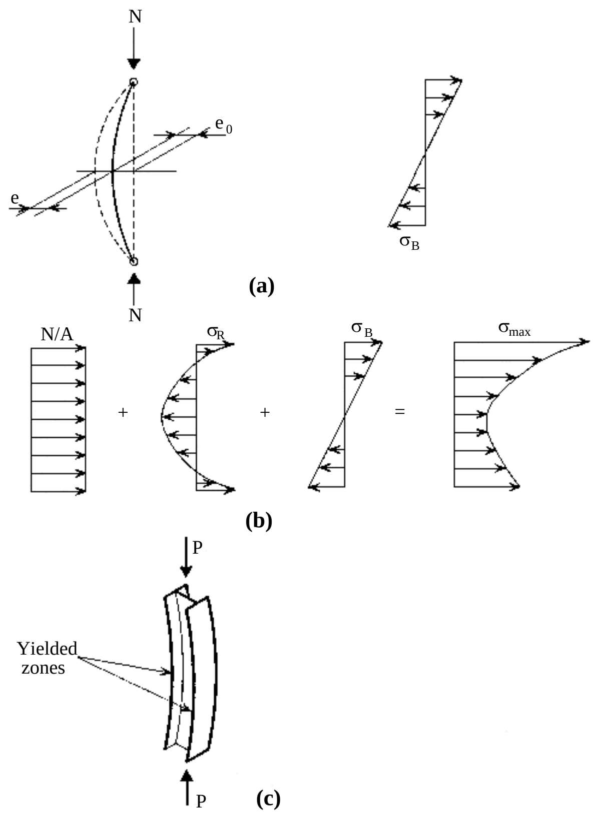


Figure 13 - Partially yielded compression member

5. Non-dimensional slenderness $\bar{\lambda}$

EC3 defines the non-dimensional slenderness $\bar{\lambda}$ as the following

$$\bar{\lambda} = \left[\beta_A \frac{Af_y}{N_{cr}} \right]^{0,5} \qquad (8)$$

Eurocode 3 5.5.1.2.(1) or eq. 6.49

which may be written (with eq. 5 and 6) and used as

$$\bar{\lambda} = \left(\frac{\lambda}{\lambda_1} \right) [\beta_A]^{0,5} \qquad (9)$$

where $\beta_A = 1$ for class 1,2,3 cross-sections and $\beta_A = A_{eff} / A$ for class 4.

6. ECCS buckling curves

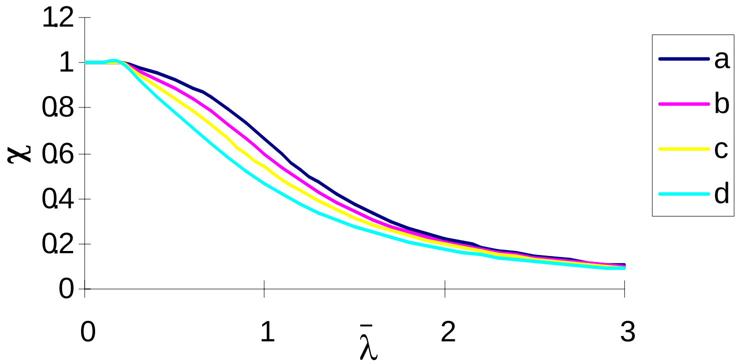
The ECCS buckling curves are based on the results of more than 1000 tests on various types of members (I H T [⊥ □ O), with different values of slenderness (between 55 and 160).

A probabilistic approach, using the experimental strength, associated with a theoretical analysis, allows curves to be drawn describing column strength as a function of the reference slenderness.

A half sine-wave geometric imperfection of magnitude equal to 1/1000 of the length of the column and the effect of residual stresses relative to each kind of cross-section are accounted for.

The ECCS buckling curves (a_0, a,b,c or d) are shown in Figure 14.

These give the value for the reduction factor χ of the resistance of the column as a function of the reference slenderness for different cross-sections (for different values of the imperfection factor α).



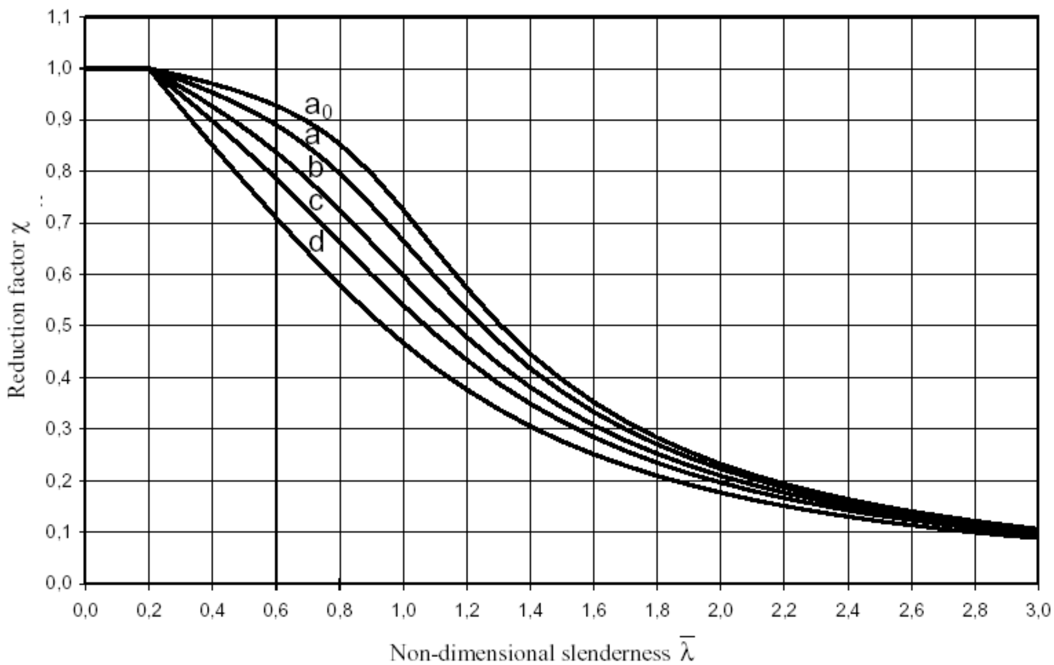


Figure 14 - European buckling curves

EC3 expresses the ECCS curves by the mathematical expression for χ:

$$\chi = \frac{1}{\phi + [\phi^2 - \bar{\lambda}^2]^{0,5}} \leq 1 \qquad (10)$$

where:

$$\phi = 0,5[1 + \alpha(\bar{\lambda} - 0,2) + \bar{\lambda}^2] \qquad (11)$$

Eurocode 3
5.5.1.2.(1)
or eq. 6.49

Table 5.5.2 in EC3 gives values of the reduction factor χ as a function of the reference slenderness $\bar{\lambda}$.

The imperfection factor α depends on the shape of the column cross-section considered, the direction in which buckling can occur (y axis or z axis) and the fabrication process used on the compression member (hot-rolled, welded or cold-formed).

Values for α, which increase with the imperfections, are given in Table 1.

Buckling curve	a	b	c	d
Imperfection factor α	0,21	0,34	0,49	0,76

Table 6.1: Imperfection factors for buckling curves

Buckling curve	a_0	a	b	c	d
Imperfection factor α	0,13	0,21	0,34	0,49	0,76

Table 1 - Imperfection factors.

Table 2 helps with the selection of the appropriate buckling curve as a function of the type of cross-section, of its dimensional limits and of the axis about which buckling can occur.

It is important to note that the buckling curves are established for a pin-ended, end loaded member; it is necessary carefully to evaluate the buckling lengths if the boundary conditions are different.

7. Design steps for compression members

To design a simple compression member it is first necessary to evaluate its two effective lengths, in relation to the two principal axes, bearing in mind the expected connections at its end.

The verification procedure should then proceed as follows:

- geometric characteristics of the shape, and its yield strength give the reference slenderness $\bar{\lambda}$.
- χ is calculated, taking into account the forming process and the shape thickness, using one of the buckling curves and $\bar{\lambda}$.

The design buckling resistance of a compression member is then taken as:

$$N_{b.Rd} = \chi \beta_A \frac{A f_y}{\gamma_{M1}} \tag{12}$$

Eurocode 3
eq. 6.47& 6.48

where $\beta_A = 1$ for class 1,2,3 cross-sections and $\beta_A = A_{eff} / A$ for class 4.

If this is higher than the design axial load, the column is acceptable; if not, another larger cross-section must be chosen and checked.

Table 6.2: Selection of buckling curve for a cross-section

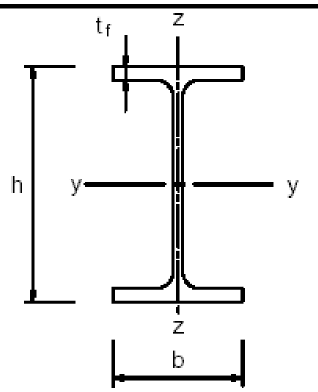
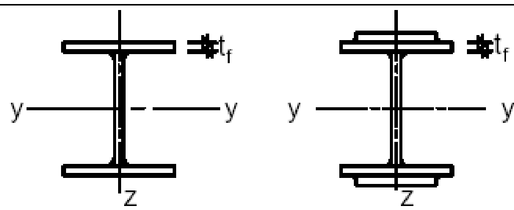
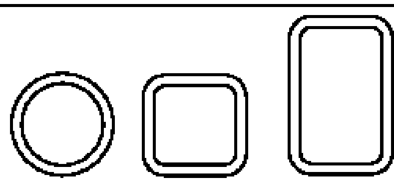
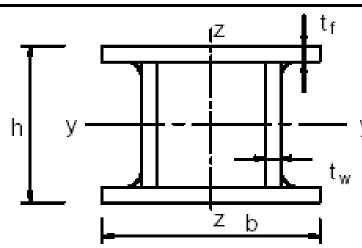
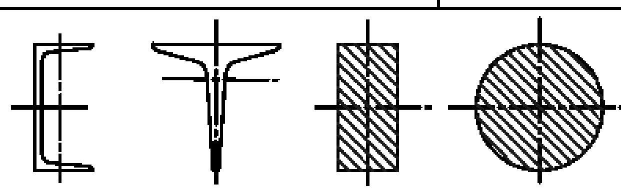
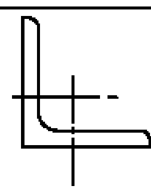
Cross section		Limits		Buckling about axis	Buckling curve S 235 S 275 S 355 S 420	Buckling curve S 460
Rolled sections		h/b > 1,2	$t_f \le 40$ mm	y – y z – z	a b	a_0 a_0
			40 mm < $t_f \le 100$	y – y z – z	b c	a a
		h/b ≤ 1,2	$t_f \le 100$ mm	y – y z – z	b c	a a
			$t_f > 100$ mm	y – y z – z	d d	c c
Welded I-sections		$t_f \le 40$ mm		y – y z – z	b c	b c
		$t_f > 40$ mm		y – y z – z	c d	c d
Hollow sections		hot finished		any	a	a_0
		cold formed		any	c	c
Welded box sections		generally (except as below)		any	b	b
		thick welds: $a > 0{,}5t_f$ $b/t_f < 30$ $h/t_w < 30$		any	c	c
U-, T- and solid sections				any	c	c
L-sections				any	b	b

Table 2 - Selection of appropriate buckling curve for a cross section

8. Concluding summary

- Many different kinds of cross-sections are used as compression members; these include simple members, built-up, tapered and stepped columns.

- A stub column (with $\bar{\lambda} \leq 0{,}2$) can achieve the full plastic resistance of the cross-section and buckling does not need to be checked, although the local buckling may reduce the capacity of class 4 sections.

- If $\bar{\lambda} > 0{,}2$, reduction of the load resistance must be considered because of buckling

- Columns with medium slenderness fail by inelastic buckling and slender columns by elastic buckling.

- European buckling curves give the reduction factor for the relevant buckling mode depending on the shape of the cross-section, the forming-process, the reference slenderness and the axis about which buckling can occur.

- They take into account experimental and theoretical approaches and give reliable results.

- The design buckling resistance is calculated as a "knocked down" design compression resistance of the cross-section, using the reduction factor for buckling, χ .

ADDITIONAL READING

1. Dowling P.J., Knowles, P. and Owens G.W., "Structural Steel Design", The Steel Construction Institute, Butterworths, 1988.
2. European Convention for Constructional Steelwork, "Manual on Stability of Steel Structures", June 1976.
3. Structural Stability Research Council, "Guide to Stability Design Criteria for Metal Structures", Edited by B.G. Johnson, John Wiley & Sons, 1976.
4. MacGinley T.J. and Ang T.C., "Structural Steelwork: Design to Limit State Theory", Butterworths, 1987.