Compression Members

Columns II

1. Introduction.

- Main aspects related to the derivation of typical columns buckling lengths for.
- Analysis of imperfections, leading to the derivation of the Ayrton-Perry formula and the European buckling curves.
- Introduce the torsional and flexural-torsional buckling concepts for simple compression members.

2. Buckling Lengths.

For pin-ended columns the buckling length equals the actual length; such columns are, however, relatively rare in practice.

Predicting strength under other than pin-ended conditions can be achieved by using the notion of effective length (L_E).

L_E is the length of a similar pin-ended column (of the same section) that has the same buckling load as the column being considered.

Approximate values for effective length, which can be used in design, are given for a wide range of end-restraint conditions.

For the determination of the elastic Euler critical buckling load

$$N_{cr} = \frac{\pi^2 E I}{L^2} \qquad (1)$$

It is assumed that both ends of the column are pinned (Figure 1)

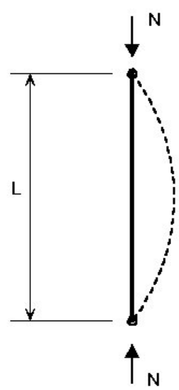


Figure 1 - Buckling of a pin-ended column.

Practical end connections on real columns, however, will often not behave in this manner and this will, therefore, significantly affect the buckling load.

Two aspects of the end condition must be considered:

- Rotation restraints vary between 0 and ∞ (i.e. a hinge without friction or fully restrained);
- Translational restraints (sway or no sway).

The usual design approach consists of reducing the practical case under consideration to an equivalent pin-ended case by means of an effective length factor K.

3. Effective Length of Columns.

The effective length, L_E, of a member hinged at its ends is the distance between the axes of the hinges.

For general end restraints, the effective length L_E, is the length of an end-hinged member which has the same load bearing resistance as the member under consideration.

The application of the above definition is not easy in practice.

The effective length factor, K, is the ratio of the length (L_E) of the equivalent column to the actual length (L); and the length of the equivalent column is the distance between two consecutive points of contra flexure (points of zero moment) in the actual column (Figure 2).

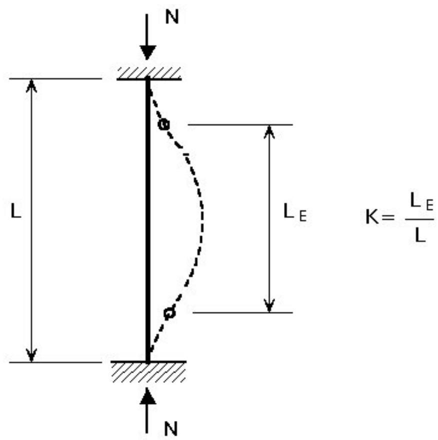


Figure 2 – Equivalent column length.

For the pin-ended column (fundamental case of buckling of a prismatic bar, Figure 1) the effective length factor is equal to 1; the distance between the points of zero moment is equal to the actual column length.

More generally, are for example, the columns of the frame illustrated in Figure 3a.

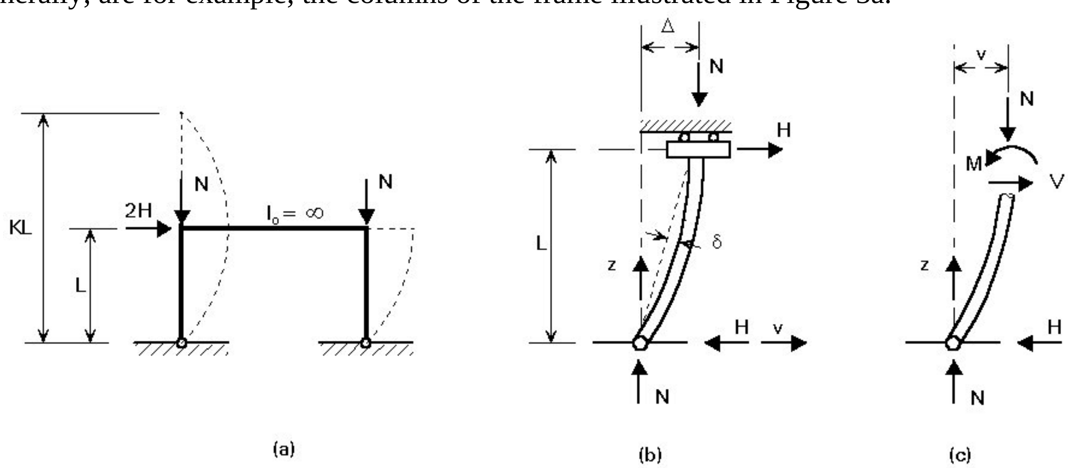


Figure 3 – Determination of the effective length.

If it is assumed that the flexural rigidity of the beam is much higher than that of the columns, no rotation of the upper ends of the columns occurs when the frame moves laterally, Figure 3b.

The bending moment at a point along the column is given by $M = Nv + Hz$ (Figure 3c).

The differential equation becomes:

$$\frac{d^2 v}{dz^2} = -\frac{M}{EI} = \frac{-(Nv + Hz)}{EI} \qquad (2)$$

Using the notation $K^2 = N/EI$:

$$\frac{d^2 y_1}{dx^2} = -\frac{M}{EI} = -\frac{Ny}{EI} \qquad (2b)$$

The solution of Equation (2b) is given by:

$$v = A \cos Kz + B \sin Kz - \frac{Hz}{N} \qquad (3)$$

To find the constants A and B, the boundary conditions are used:

for $z = 0$, $v = 0$ and for $z = L$, $\frac{dv}{dz} = 0$, therefore $A = 0$ and

$$B K \cos K L = 0 \qquad (4)$$

From (4), it follows that either B or $\cos K L = 0$.

If $B = 0$, $v = -Hz/N$ and $d^2v/dz^2 = 0$; in this case, the bending moment M should be zero at any point along the column.

The other possibility is that $\cos KL = 0$ and this condition requires that:

$$K = n\pi/2L \text{ where } n = 1,3,5, ... \qquad (5)$$

To obtain the smallest value of N for which Equation (5) is satisfied, using n=1 gives $KL = \pi/2$ from which $K = \pi/2L$ and $K^2 = N/EI$,

$$N_{cr} = K^2 EI = \pi^2 EI/4L^2 = \pi^2 EI/(2L)^2 \qquad (6)$$

The comparison of Equations (6) and (1) shows that the effective length factor K is equal to 2 and therefore, that the effective length of the column is twice the actual length.

In other words, the critical load for the column of length L, Figure 3, is the same as the critical load of a pin-ended column of length 2L, Figure 3a.

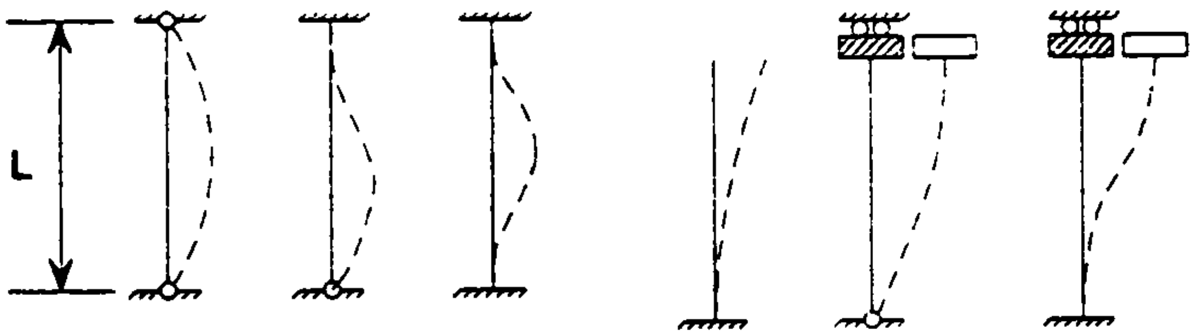
The use of an effective column length is basically a device to relate the behaviour of columns with any form of support to the behaviour of the basic pin-ended case.

The design procedure for columns with particular end conditions is the same as for pin-ended columns but in establishing the design strength from the column design curve, the slenderness (L_E/r_y) would be used instead of L/r_y.

Table 1 gives theoretical K-values for idealized conditions in which the rotational and/or translational restraints at the ends of the column are either fully realized or non-existent.

Table 1 - Effective length factor for centrally loaded columns with various end conditions.

	With lateral restraint (a) (b) (c)			Without lateral restraint (d) (e) (f)		
Ideal buckling conditions	See T1-1			See T1-1		
Theoretical K-values	1,0	0,7	0,5	2,0	2,0	1,0
Recommended K-values when ideal conditions are approximated	1,0	0,8	0,65	2,0	2,0	1,2



T1-1 **(a)** **(b)** **(c)** **(d)** **(e)** **(f)**

Table 1 also recommends K-values that are equal or slightly higher than the equivalent theoretical values derived from elastic stability theory.

When higher values are specified it is usually in recognition of the practical difficulties of providing complete restraint against rotation or translation.

The comparison of cases (b) & (e) shows the influence of the translational restraints on the buckling loads.

Case (e) represents the column of Figure 3a, with lateral displacement, while in case (b) no translation is permitted; the buckling load is multiplied by a factor 8 ($(2,0/0,7)^2$) when translation is prevented.

It is absolutely necessary that the designer knows the difference between sway and non-sway frames.

According to Eurocode 3 [1], a frame may be classified as non-sway if its response to in-plane horizontal loads is sufficiently stiff for it to be acceptably accurate to neglect any additional internal forces or moments arising from horizontal displacement of its nodes.

Any other frame shall be treated as a sway frame and the effects of the horizontal displacements of its nodes taken into account in its design.

A column in a non-sway frame would have no sideways movement at the top relative to the bottom.

The buckling of a non-sway frame would result in a buckled column shape having at least one point of contra flexure between the ends of the member, such as cases (a), (b), and (c) of Table 1 (Figure 4).

The effective length factor K is always less than or equal to 1 ($0,5 \leq K \leq 1$).

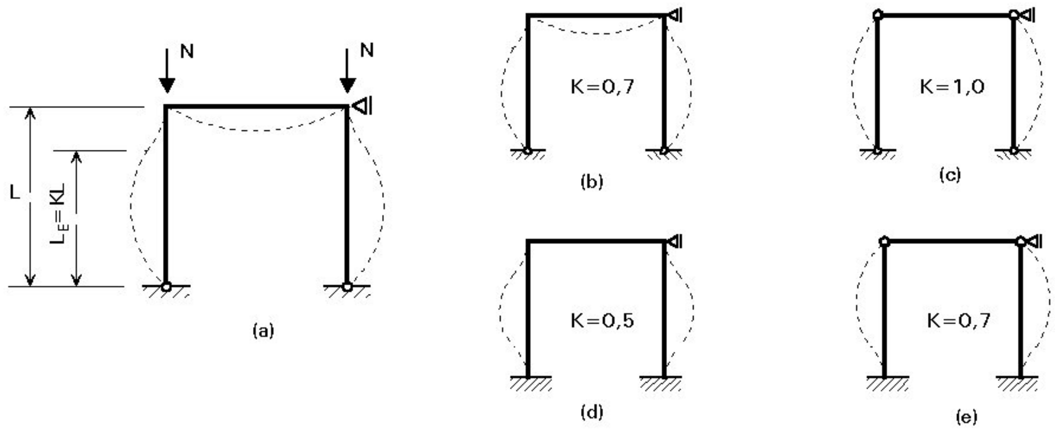


Figure 4 – Buckling of a column in a non-sway frame.

In a sway frame, the top of the column moves relative to the bottom.

Cases (d), (e) and (f) of Table 1 are sidesways buckling cases which are illustrated in Figure 5.

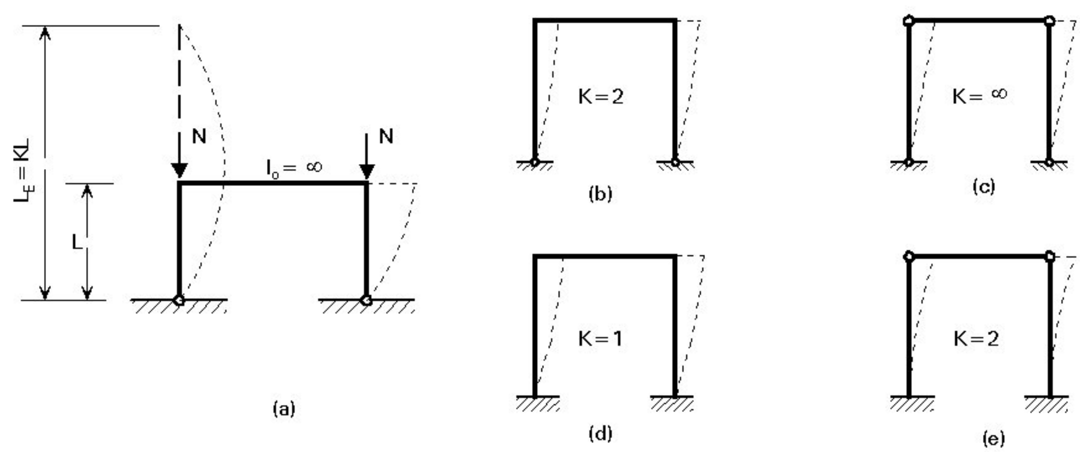


Figure 5 Buckling of a column in a sway frame.

The effective length factor K is always greater than or equal to 1 and is unlimited ($1 \leq K \leq \infty$).

The above considerations concerning single storey frames can be generalized, so as to extend to frames of more than one storey.

The fully rigid end-restraints (Figures 4b, 4d, 5b and 5d) can rarely be achieved in practice and partial end-restraints are much more common.

In the case of partial end-restraint the effective length factor K can be determined either by a generalized second order rotation method or by using stability functions [2].

The solution to the problem is expressed in the form:

$$K = f(\eta_t, \eta_b) \qquad (7)$$

where η_t and η_b, are elastic restraint coefficients at the top and bottom of the column considered.

Simplified approaches are available for evaluating the effective length factor K, [3-7].

In Eurocode 3 [1], the approach suggested by Wood [4] has been adopted and two cases are considered: non-sway and sway frames.

In some cases, a compressed bar can be supported elastically at intermediate points along the length.

Figure 6 shows, the compressed chord of a truss girder; the intermediate supports of the chord are represented by the framed cross girder.

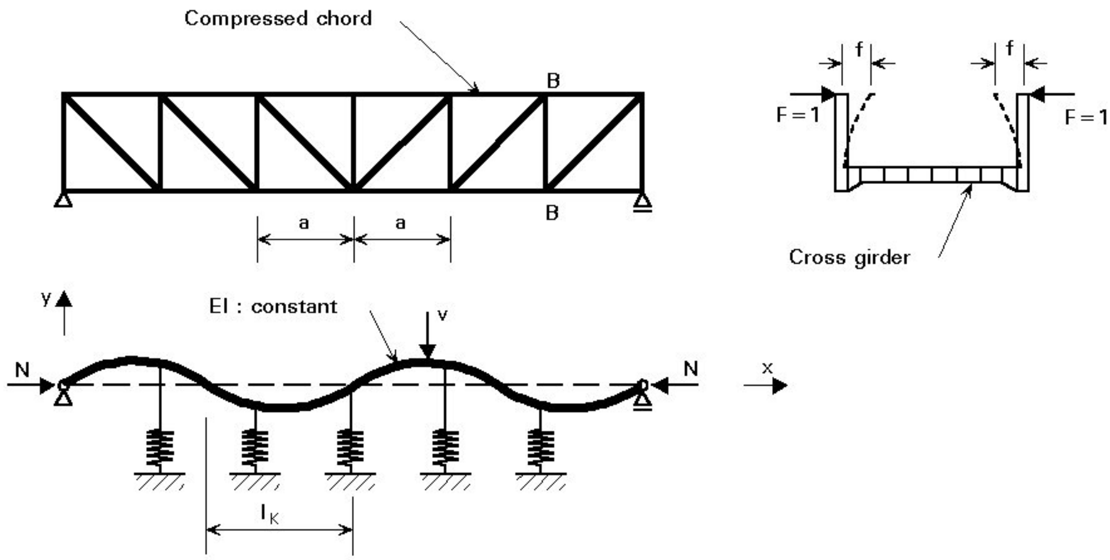


Figure 6- Buckling of a bar with elastic supports.

In such case the effective length is greater than the distance "a" between the cross girders [8] and is:

$$l_k = \pi \sqrt[4]{\frac{1}{4} EI f a} \qquad (8)$$

where f = $1/K_r$, the displacement of a spring (intermediate support) due to a unit force.

3.1 Columns of non-sway frames.

Wood [4], considers a sub-element of a non-sway frame as illustrated in Figure 7b (part AB of the frame represented in Figure 7a.).

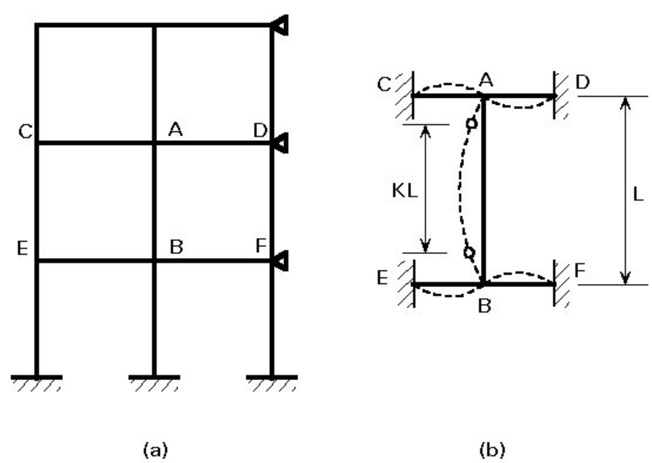


Figure 7 - Example of substitute frame.

The two elastic restraint coefficients η_t and η_b (which are closely analogous to the cross-distribution coefficients at the upper and lower ends of the column) are calculated using the following formulae:

$$\eta_t = K_C/(K_C + \Sigma K_{b,t}) \qquad (9)$$

$$\eta_b = K_C/(K_C + \Sigma K_{b,b}) \qquad (10)$$

where: K_C is the column stiffness I/L

ΣK_b is the effective beam stiffness sum at a joint and the suffices b and t indicate the column bottom or top end.

When the beams are not subject to axial forces, their effective stiffness can be determined by reference to Table 2, provided that they remain elastic under the design moments.

Table 2 - Effective stiffness of a beam

Conditions of rotational restraint at far end of beam	Effective beam stiffness (provided beam remains elastic)
Fixed at far end	1,0 I/L
Pinned at far end	0,75 I/L
Rotation as at near end (double curvature)	1,5 I/L
Rotation equal and opposite to that at near end (single curvature)	0,5 I/L
General case. Rotation Θ_A at near end and Θ_B at far end	$(1 + 0,5\ \Theta_B/\Theta_A)$ I/L

Where, for the same load case, the design moment in any of the beams exceeds the elastic moment, the beam should be assumed to be pinned at the point or points concerned.

Where a beam has semi-rigid connections its effective stiffness should be reduced accordingly.

When the beams are subject to axial forces, their effective stiffness should be adjusted accordingly.

As a simple alternative, the increased stiffness due to axial tension can be neglected and the effects of axial compression can be allowed for by using the conservative approximations given in Table 3.

Table 3 - Approximate formulae for reduced stiffness due to axial compression

Far end condition	Effective beam stiffness
Fixed	1,0 I/L $(1 - 0,4\ N/N_{cr})$
Pinned	0,75 I/L $(1 - 1,0\ N/N_{cr})$
Double curvature	1,5 I/L $(1 - 0,2\ N/N_{cr})$
Single curvature	0,5 I/L $(1 - 1,0\ N/N_{cr})$
Where $N_{cr} = \pi^2 EI/L^2$	

Considering the sub-element illustrated in Figure 7b, and the distribution coefficients given above yields results that can be graphically illustrated [4] by the curves of Figure 8.

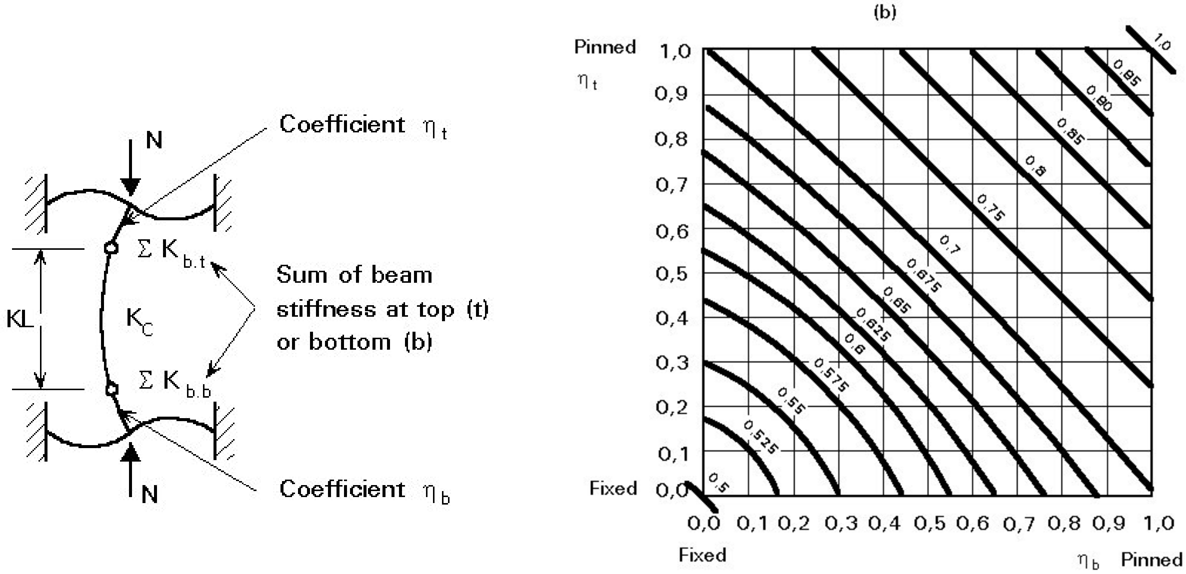


Figure 8 - Effective length factor for a column in a non-sway frame.

These can also be represented by the following expression:

$$K = \frac{1 + 0,145\,(\eta_b + \eta_t) - 0,265\,\eta_b \cdot \eta_t}{2 - 0,364\,(\eta_b + \eta_t) - 0,247\,\eta_b \cdot \eta_t} \qquad (11)$$

The model can be adapted for the design of continuous columns, by assuming that each length of column is loaded to the same value of the ratio (N/N_{cr}).

When (N/N_{cr}) varies, this leads to a conservative value of K for the most critical length of column.

For each length of a continuous column this assumption can be introduced using the model shown in Figure 9 and obtaining the distribution coefficients η_t and η_b as follows:

$$\eta_t = \frac{K_c + K_t}{K_c + K_t + \sum K_{b.t}} \qquad (12)$$

$$\eta_b = \frac{K_c + K_b}{K_c + K_b + \sum K_{b.b}} \qquad (13)$$

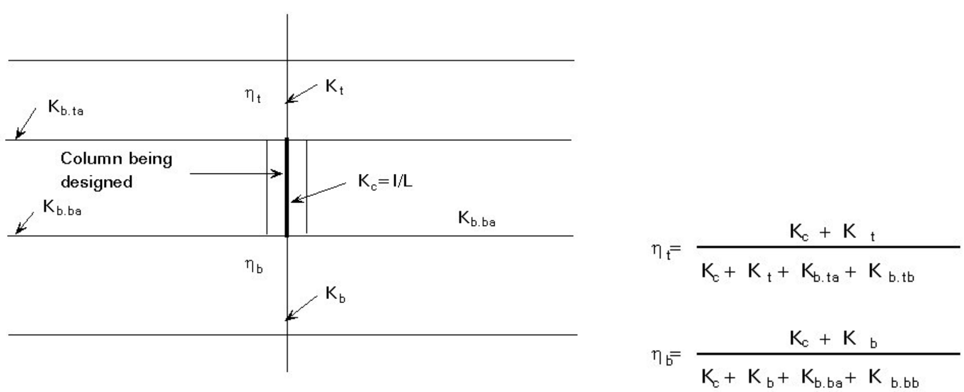


Figure 9 – Elastic restraint coefficients for continuous columns.

3.2. Columns of sway frames.

In unbraced frames (and some braced frames) sway is permitted; the effective length factor, K, is therefore greater than unity and can tend towards infinity if the horizontal beams are very flexible.

K can be calculated using the same approach as that adopted for frames in which sway is prevented; it should, however, be pointed out that the results for sway frames must be considered as even more approximate than those given for non-sway frames.

Wood's method can be considered as acceptable when sway is permitted only if the frames are regular, i.e. heights, moments of inertia and axial forces in the columns do not differ considerably.

The effective length factor of a column in a sway frame may be obtained from Figure 10 or Eq. (14):

$$K = \sqrt{\frac{1 - 0{,}2(\eta_t + \eta_b) - 0{,}12\,\eta_t\,\eta_b}{1 - 0{,}8(\eta_t + \eta_b) + 0{,}6\,\eta_t\,\eta_b}} \qquad (14)$$

The elastic restraint coefficients η_t and η_b are calculated as for the case of non-sway frames.

Introducing the effective length concept in the elastic design of sway columns requires that second order or load destabilizing effects (due to the displacement of the top of the columns) are approximately taken into account by an effective length factor K, greater than 1.

The advantage of this approach is its simplicity but it should be recognised that it is limited and may, in some cases, be inaccurate.

A design considering the whole structure, based on approximate methods for elastic critical load analysis, is recognized as more reliable.

These methods consider the effects of horizontal sway forces on the structure that subject the column to bending moments as well as axial loads.

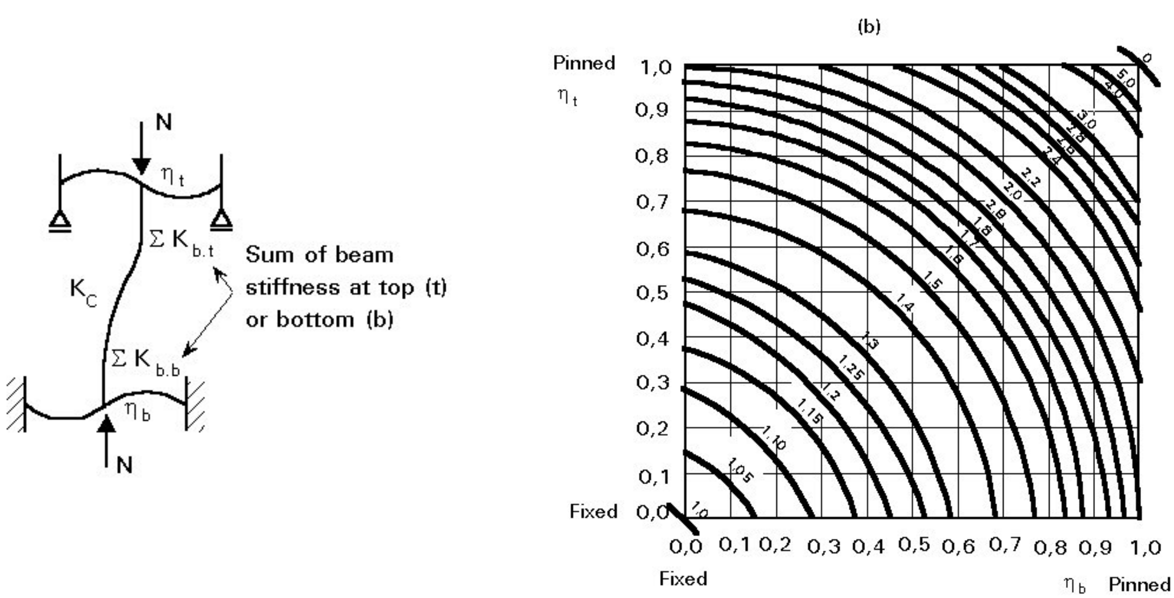


Figure 10 - Effective length factor for a column in a sway frame.

4. Analytical formulation of the European buckling curves.

The behaviour of real steel structures is always different from that predicted theoretically; the main reasons for this discrepancy are:

- geometrical imperfections, due to defects causing lack of straightness, unparallel flanges, asymmetry of cross-section etc;
- material imperfections, due to residual stresses (caused by the rolling or fabrication process) or material inelasticity;
- deviation of applied load from idealised position due to imperfect connections, erection tolerances or lack of verticality of the member.

In reality, all the imperfections act together simultaneously and their effect depends on their individual intensity and on the slenderness of the column.

4.1 Initial deflection.

Assuming that the initial deflection of a pin-ended column of length l, has a half sine-wave form with magnitude e_o (Figure 11), the initial deformation along the column can be written as:

$$y_o = e_o \sin \frac{\pi x}{\ell} \qquad (15)$$

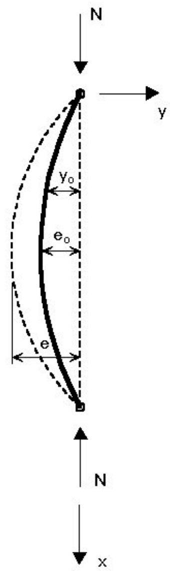


Figure 11 – Pin-ended column with initial curvature

The differential equation for the deformation of such a pin-ended column loaded by an axial force N is:

$$\frac{d^2 y}{d^2 x} + \frac{N(y + y_o)}{E I} = 0 \qquad (16)$$

Combining this, the expression for y_o, and taking into account the boundary conditions, the solution is:

$$y = \frac{e_0}{\left(N_{cr}/N - 1 \right)} \sin \frac{\pi x}{l} \qquad (17)$$

The maximum total deflection, e, of the column is then:

$$e = e_o + \frac{e_0}{N_{cr}/N - 1} = \frac{e_0}{1 - N/N_{cr}} \qquad (18)$$

and the ratio $1/(1 - N/N_{cr})$ is generally called the "amplification factor".

Taking into account the maximum bending moment, Ne, due to buckling, the equilibrium of the column requires that:

$$\frac{N}{A} + \frac{Ne}{W} = f_y \qquad (19)$$

N is the maximum axial load, limited by buckling, and σ_b the maximum normal stress ($\sigma_b = N/A$), then:

$$\frac{N}{A} + \frac{N}{A}\frac{eA}{W} = \sigma_b + \sigma_b \frac{eA}{W} = f_y \qquad (20)$$

or, introducing σ_{cr}, the Euler critical stress ($\sigma_{cr} = \pi^2 E/\lambda^2$) and including the value of e:

$$\sigma_b + \sigma_b \frac{e_0}{1 - \sigma_b/\sigma_{cr}} \frac{A}{W} = f_y \qquad (21)$$

which can be written as:

$$(\sigma_{cr} - \sigma_b)(f_y - \sigma_b) = \sigma_b \sigma_{cr} e_o A/W \qquad (22)$$

This equation is the basic form of the Ayrton-Perry formula.

4.2 Eccentricity of the applied load.

If the axial compression load is applied with an eccentricity e_c on an initially straight pin-ended column (Figure 12), a bending moment ($N e_c$) is introduced which increases the buckling effect.

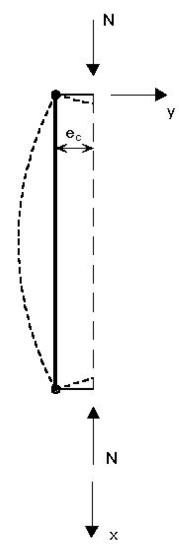


Figure 12 – Pin-ended column with load eccentricity.

This effect obviously increases along with axial load.

It is possible to show that the total maximum deflection e of the column is equal to:

$$e = e_c - e_c/\{\cos[l/2\,(N/EI)^{1/2}]\} \qquad (23)$$

and the "amplification factor" to: $1/\cos\,[\pi/2\,(N/N_{cr})^{1/2}]$

Now, if the combined effect of the initial deflection and of the eccentricity of loading is considered, the stress is approximately equal to:

$$\sigma_b + \sigma_b \frac{e_0 + e_c + 0{,}23 e_c \,\sigma_b/\sigma_{cr}}{1 - \sigma_b/\sigma_{cr}} \frac{A}{W} = f_y \qquad (24)$$

This relationship is correct within a few percent for all values of σ_b from 0 to σ_{cr}.

4.3 Ayrton-Perry formula.

The classical form of the Ayrton-Perry formula is:

$$(\sigma_{cr} - \sigma_b)\,(f_y - \sigma_b) = \eta\,\sigma_{cr}\,\sigma_b \qquad (25)$$

This is the form of Equation (22) if $\eta = (e_o\,A) / W$

The coefficient η represents the initial out-of-straightness imperfection of the column but it can also include other defects i.e. residual stresses in which case it is called the "generalized imperfection factor".

It is possible to write the Ayrton-Perry formula under another form:

$$(\sigma_{cr} / f_y - \overline{N})\,(1 - \overline{N}) = \eta\,\overline{N}\,\sigma_{cr} / f_y \qquad (26)$$

where: $\overline{N} = \sigma_b / f_y$

If $\overline{\lambda}^2 = f_y / \sigma_{cr}$ then, dividing by σ_{cr} / f_y, gives:

$$(1 - \overline{N}\,\overline{\lambda}^2)\,(1 - \overline{N}) = \eta\,\overline{N} \qquad (27)$$

or:

$$\overline{\lambda}^2\,\overline{N}^2 - \overline{N}\,(\overline{\lambda}^2 + \eta + 1) + 1 = 0 \qquad (28)$$

This form leads to the European formulation [1].

4.4 Generalized imperfection factor.

The generalized imperfection factor takes into account all the relevant defects in a real column when considering buckling: geometric imperfections, eccentricity of applied loads and residual stresses; inelastic properties are not considered because they only influence stub columns.

The generalized imperfection factor can be expressed through the coefficient η representing the effect of deflections:

$$\eta = \frac{\ell A}{\gamma W} \qquad (29)$$

where $\gamma = \lambda / \varepsilon_o$, represents the equivalent geometrical imperfection (which is the ratio of the length over the equivalent initial curvature of the column).

Then using $L = \lambda . i$, $W = I / v$ and $i^2 = I / A$, η can be written as:

$$\eta = \lambda / \gamma \,(i/v) \qquad (30)$$

where (i/v) is the relative diameter of the inertia ellipse in the axis where buckling occurs.

As $\lambda = \bar{\lambda}\ \eta\ (E/f_y)^{1/2}$, introducing the plateau $\bar{N} = 1$ when $\bar{\lambda} \leq \bar{\lambda}_o$, the previous relationship is:

$$\eta = \frac{90{,}15\left(\bar{\lambda} - \bar{\lambda}_0\right)}{\gamma\left(i/v\right)} \qquad (31)$$

4.5 European formulation.

Using η expressed as:

$$\eta = \alpha(\bar{\lambda} - \bar{\lambda}_o) \qquad (32)$$

the smallest solution of the Equation (28) is:

$$\bar{N} = \{1 + \alpha\,(\bar{\lambda} - \bar{\lambda}_o) + \bar{\lambda}^2 - [1 + \alpha\,(\bar{\lambda} - \bar{\lambda}_o) + \bar{\lambda}^2]^2 - 4\bar{\lambda}^2\}^{1/2} / 2\bar{\lambda}^2 \qquad (33)$$

Multiplying by the conjugated term and choosing $\bar{\lambda}_o = 0{,}2$, this leads to the European formulation:

$$\chi = 1 / \{\phi + [\phi^2 - \bar{\lambda}^2]\}^{1/2} \leq 1 \qquad (34)$$

where:

$$\phi = 0{,}5\,[1 + \alpha\,(\bar{\lambda} - 0{,}2) + \bar{\lambda}^2] \qquad (35)$$

χ is the reduction factor considered in Eurocode 3 [1].

It can be written as $\alpha = \alpha_1 + \alpha_2$, where α_1 and α_2 represents the mechanical and geometrical imperfections.

Considering only the geometrical imperfections, the European buckling curves were established with an initial curvature equal to L/1000

$$\alpha_2 = 90{,}15/[1000\,(i/v)].$$

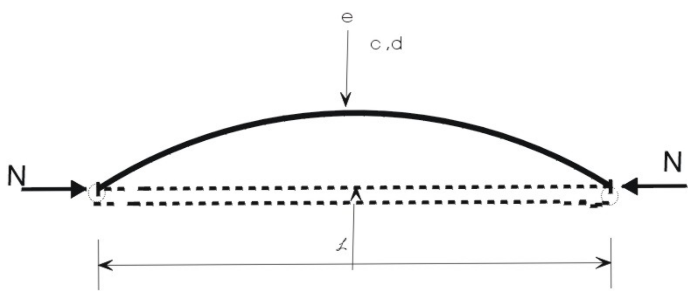
Considering now the equivalent initial deflection: $e_o = L/\gamma$, linked to the generalized imperfection factor η (Equation (29)) and using Equation (32), gives:

$$e_o = \alpha(\bar{\lambda} - 0{,}2)\, W / A \qquad (36)$$

which represents the equivalent initial bow imperfection of a pin-ended column including the initial crookedness and the residual stresses; this has to be taken into account in a second order analysis.

The design values relative to each European buckling curve are given in Table 4.

Table 4 - Design values of equivalent initial bow imperfection $e_{o,d}$ (from Figure 5.5.1, Eurocode 3).



Cross-section		Method of global analysis	
Method used to verify resistance	Section type and axis	Elastic,or Rigid - Plastic, or Elastic - Perfectly Plastic	Elasto-Plastic (plastic zone method)
Elastic [5.4.8.2]	Any	$\alpha(\bar{\lambda} - 0,2)k_\gamma W_{el}/A$	-
Linear [5.4.8.1(12)]	Any	$\alpha(\bar{\lambda} - 0,2)k_\gamma W_{pl}/A$	-
Plastic [5.4.8.1(1) to (11)]	I-section yy-axis	$1,33\alpha(\bar{\lambda} - 0,2)k_\gamma W_{pl}/A$	$\alpha(\bar{\lambda} -0,2)k_\gamma W_{pl}/A$
	I-section zz-axis	$2,0\ k_\gamma\ e_{eff}/\varepsilon$	$k_\gamma\ e_{eff}/\varepsilon$
	Rectangular hollow section	$1,33\alpha(\bar{\lambda} - 0,2)k_\gamma W_{pl}/A$	$\alpha(\bar{\lambda} - 0,2)k_\gamma W_{pl}/A$
	Circular hollow section	$1,5\ k_\gamma\ e_{eff}/\varepsilon$	$k_\gamma\ e_{eff}/\varepsilon$

$k_\gamma = (1 - k_\delta) + 2\ k_\delta \bar{\lambda}$ but $\geq 1,0$

Buckling curve	α	e_{eff}	k_δ			
			γ_{M1} =1,05	γ_{M1} =1,10	γ_{M1} =1,15	γ_{M1} =1,20
a	0,21	l/600	0,12	0,23	0,33	0,42
b	0,34	l/380	0,08	0,15	0,22	0,28
c	0,49	l/270	0,06	0,11	0,16	0,20
d	0,76	l/180	0,04	0,08	0,11	0,14

Non-uniform members: Use value of W_{el}/A or W_{pl}/A at centre of buckling length l

5. TORSIONAL AND FLEXURAL-TORSIONAL BUCKLING.

For hot-rolled steel members, with the type of cross-sections commonly used for compression members, the relevant buckling mode is generally flexural buckling;

However, in some cases, torsional or flexural-torsional modes may govern and these must be investigated for all sections with small torsional resistance.

5.1 Cross-section subjected to torsional or flexural-torsional buckling.

Concentrically loaded columns can buckle by flexure about one of the principal axes (classical buckling), twisting about the shear centre (torsional bucking) or a combination of both flexural and twisting (flexural-torsional buckling).

Torsional buckling can only occur if the shear centre and centroid coincide and the cross-section can rotate; this leads to a twisting of the member. Z-sections and I-sections with broad flanges can be subject to torsional buckling; pylons, fabricated from angle sections, must also be checked for this instability.

Symmetrical sections with axial load not in the plane of symmetry, and non-symmetrical sections such as C-sections, hats, equal-leg angles, T-sections and singly symmetrical I-sections, i.e. sections where the shear centre and the centroid do not coincide, must be checked for flexural-torsional buckling.

Figure 13 gives examples of sections that must be checked for torsional or flexural-torsional buckling.

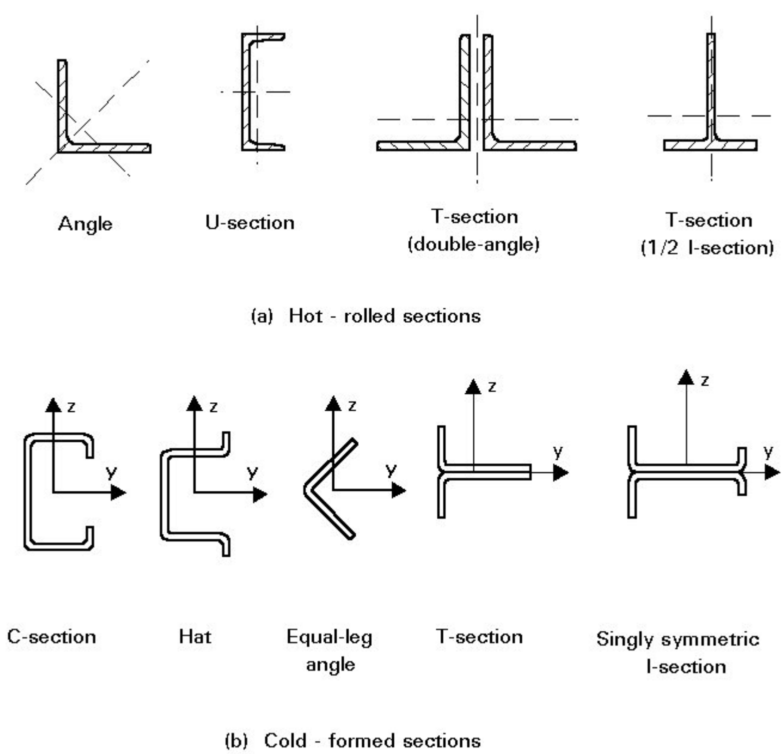


Figure 13 – Typical cross- sections requiring checks for torsional or flexural – torsional buckling.

5.2 Torsional buckling.

The critical stress depends on the boundary conditions and it is very important to evaluate precisely the possibilities of rotation at the ends.

The critical stress depends on the torsional stiffness of the member and on the resistance to warping deformations provided by the member itself and by the restraints at its ends.

The differential equation for torsional buckling is:

$$GI_p \frac{d^4\theta}{dx^4} - EI_w \frac{d^2\theta}{dx^2} = -Nr_0^2 \frac{d^2\theta}{dx^2} \qquad (37)$$

and the critical load for pure torsional buckling, $N_{cr\theta}$, is:

$$N_{cr\theta} = \frac{1}{r_0^2}\left[G\,I_D + \frac{\Pi^2\,E\,I_w}{\ell_{cr}^2}\right] \qquad (38)$$

where r_o is the polar radius of gyration, G the shear modulus of elasticity, N the axial load, θ the twist angle, I_D the torsion constant, and I_w the warping constant.

To check a compression member with torsional buckling, a new reference slenderness $\bar{\lambda}$ must be evaluated:

$$\bar{\lambda} = \sqrt{\frac{f_y}{\sigma_{cr\theta}}} \qquad (39)$$

where $\sigma_{cr\theta}$ is the elastic critical stress for torsional buckling obtained with the critical load $N_{cr\theta}$ (Eq. (38)).

Generally flexural buckling occurs at a lower critical stress than torsional buckling.

Figure 14 illustrates this phenomenon for the case of a cruciform strut.

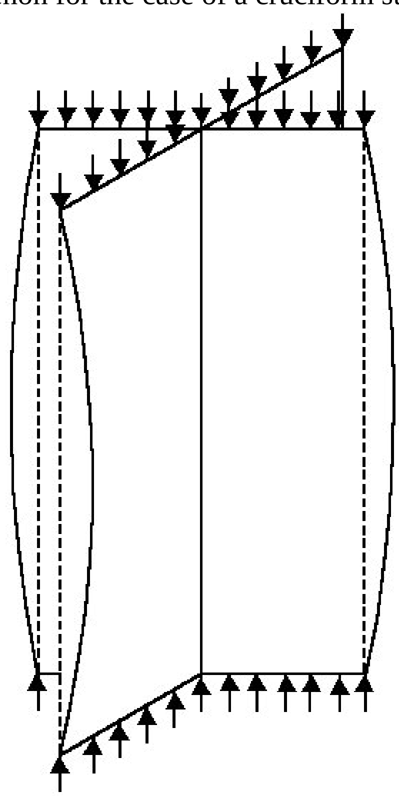


Figure 14 - Torsional buckling of a cruciform strut.

3.3 Flexural-torsional Buckling

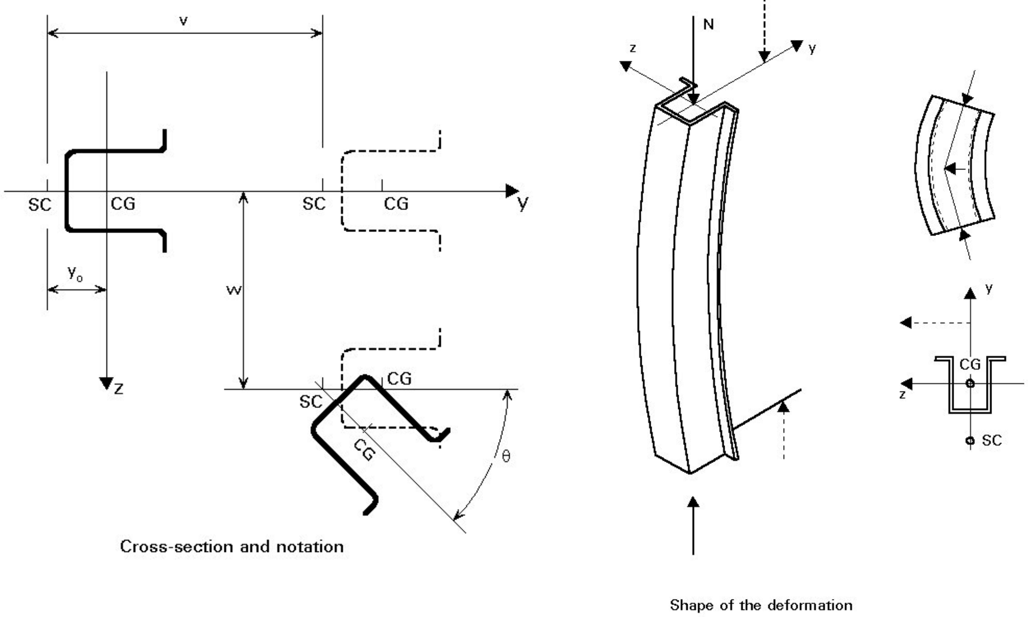
This is the combination of flexural and torsional buckling. The three basic equilibrium equations are:

$$EI_y \frac{d^2 w}{dx^2} = -N(w + y_0\theta) \tag{40}$$

$$EI_z \frac{d^2 v}{dx^2} = -N(v + z_0\theta) \tag{41}$$

$$EI_w \frac{d^4\theta}{dx^4} - (GI_p - r_0^2 N)\frac{d^2\theta}{dx^2} - Ny_0 \frac{d^2 w}{dx^2} + Nz_0 \frac{d^2 v}{dx^2} = 0 \tag{42}$$

where, y_o and z_o are the coordinates of the shear centre and v and w are the deflections, Figure 15.



Figures 15 – Flexural buckling and Flexural torsional buckling of a hat section strut.

The critical load for pure torsional buckling is obtained from the lowest root of the following equation:

$$r_o^2 (N_{cr} - N_{crz}) (N_{cr} - N_{cry}) (N_{cr} - N_{cr\theta}) - \ldots\ N_{cr}^2 z_o^2 (N_{cr} - N_{cry}) - N_{cr}^2 y_o^2 (N_{cr} - N_{crz}) = 0 \tag{43}$$

where, N_{cry} and N_{crz} are respectively the critical loads for pure flexural buckling about the axes y and z, and $N_{cr\theta}$ is defined by Equation (38).

Cross-sections with one (or two) axis of symmetry give y_o (or z_o) = 0 leading to a simplification of the previous equation; for example, a section with two axes of symmetry gives:

$$(N_{cr} - N_{crz}) (N_{cr} - N_{cry}) (N_{cr} - N_{cr\theta}) = 0 \tag{44}$$

and the members buckle at the lowest of the critical loads without interaction of modes.

To check a compression member with flexural-torsional buckling, a new reference slenderness $\bar{\lambda}$ must be evaluated in a similar way as for torsional buckling (Equation (39)).

In this case $\sigma_{cr\theta}$ is the elastic stress for flexural buckling obtained with the critical load relative to flexural-torsional buckling.

6. CONCLUDING SUMMARY.

- Effective lengths enable column design curves, for pin-ended columns, to be used for the design of practical columns which have a wide range of end-restraint conditions.
- Simplified methods are available for evaluating the effective length of a compressed bar.
- For sway columns, effective lengths are greater than the actual lengths.
- For non-sway columns, effective lengths are less than the actual lengths.
- The Ayrton-Perry formula describes the behaviour of real columns. It is the basis of the European buckling curves.
- European buckling curves are explained; these include a generalized imperfection factor.
- Torsional buckling and flexural-torsional buckling are introduced.

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