TORSIONAL SECTION PROPERTIES OF STEEL SHAPES

Canadian Institute of Steel Construction, 2002

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Introduction

Structural engineers occasionally need to determine the section properties of steel shapes not found in the current edition of the Handbook of Steel Construction (CISC 2000). The following pages provide the formulas for calculating the torsional section properties of structural steel shapes.

The section properties considered are the St. Venant torsional constant, J, the warping torsional constant, C_w, the shear centre location, y_O , and the monosymmetry constant, β_x . Although not a torsional property, the shear constant, C_{RT} , is also included for hollow structural sections (HSS), as it is not easily found in the literature.

Some of the formulas given herein are less complex than those used in developing the Handbook and the Structural Section Tables (CISC 1997). Effects such as flange-to-web fillet radii, fillet welds, and sloped (tapered) flanges have not been taken into account. Likewise, some of the formulas for monosymmetric shapes are approximations which are only valid within a certain range of parameters. If needed, more accurate expressions can be found in the references cited in the text.

Simple example calculations are provided for each type of cross section to illustrate the formulas. A complete description of torsional theory or a detailed derivation of the formulas for torsional section properties is beyond the scope of this discussion; only the final expressions are given. The references can be consulted for further information.

Although no effort has been spared in an attempt to ensure that all data contained herein is factual and that the numerical values are accurate to a degree consistent with current structural design practice, the Canadian Institute of Steel Construction does not assume responsibility for errors or oversights resulting from the use of the information contained herein. Anyone making use of this information assumes all liability arising from such use. All suggestions for improvement will receive full consideration.

St. Venant Torsional Constant

The St. Venant torsional constant, *J*, measures the resistance of a structural member to *pure* or *uniform* torsion. It is used in calculating the buckling moment resistance of laterally unsupported beams and torsional-flexural buckling of compression members in accordance with CSA Standard S16.1-94 (CSA 1994).

For open cross sections, the general formula is given by Galambos (1968):

$$J = \sum \left(\frac{b' t^3}{3} \right) \qquad [1]$$

where *b′* are the plate lengths between points of intersection on their axes, and *t* are the plate thicknesses. Summation includes all component plates. It is noted that the tabulated values in the Handbook of Steel Construction (CISC 2000) are based on net plate lengths instead of lengths between intersection points, a mostly conservative approach.

The expressions for *J* given herein do not take into account the flange-to-web fillets. Formulas which account for this effect are given by El Darwish and Johnston (1965).

For thin-walled closed sections, the general formula is given by Salmon and Johnson (1980):

$$J = \frac{4\,A_O^2}{\int_S ds / t} \qquad [2]$$

where A_O is the enclosed area by the walls, *t* is the wall thickness, *ds* is a length element along the perimeter. Integration is performed over the entire perimeter *S*.

Warping Torsional Constant

The warping torsional constant, C_w, measures the resistance of a structural member to *nonuniform* or *warping* torsion. It is used in calculating the buckling moment resistance of laterally unsupported beams and torsional-flexural buckling of compression members in accordance with CSA Standard S16.1-94 (CSA 1994).

For open sections, a general calculation method is given by Galambos (1968). For sections in which all component plates meet at a single point, such as angles and T-sections, a calculation method is given by Bleich (1952). For hollow structural sections (HSS), warping deformations are small, and the warping torsional constant is generally taken as zero.

Shear Centre

The shear centre, or torsion centre, is the point in the plane of the cross section about which twisting takes place. The shear centre location is required for calculating the warping torsional constant and the monosymmetry constant. It is also required to determine the stabilizing or destabilizing effect of gravity loading applied below or above the shear centre, respectively (SSRC 1998). The coordinates of the shear centre location (x_O, y_O) are calculated with respect to the centroid. A calculation method is given by Galambos (1968).

Monosymmetry Constant

The monosymmetry constant, β_X, is used in calculating the buckling moment resistance of laterally unsupported monosymmetric beams loaded in the plane of symmetry (CSA 2000). In the case of a monosymmetric section that is symmetric about the vertical axis, the general formula is given by SSRC (1998):

$$\beta_X = \frac{1}{I_X} \int_A y \left(x^2 + y^2\right) dA - 2\, y_O \qquad [3]$$

where I_X is moment of inertia about the horizontal centroidal axis, dA is an area element and y_O is the vertical location of the shear centre with respect to the centroid. Integration is performed over the entire cross section. The value of β_X is zero for doubly-symmetric sections.

Shear Constant

The shear constant, C_{RT}, is used for determining the maximum shear stress in the cross section due to an applied shear force.

For hollow structural sections, the maximum shear stress in the cross section is given by:

$$\tau_{\max} = \frac{V\,Q}{2\,t\,I} \qquad [4]$$

where V is the applied shear force, Q is the statical moment of the portion of the section lying outside the neutral axis taken about the neutral axis, I is the moment of inertia, and t is the wall thickness.

The shear constant is expressed as the ratio of the applied shear force to the maximum shear stress (Stelco 1981):

$$C_{RT} = \frac{V}{\tau_{\max}} = \frac{2\,t\,I}{Q} \qquad [5]$$

A) Open Cross Sections

1. Doubly-Symmetric Wide-Flange Shapes (W-Shapes and I-Beams)

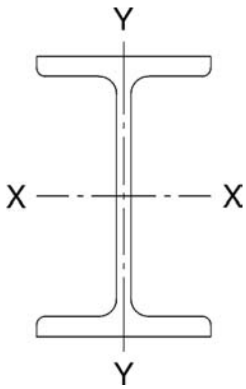

Fig. 1a

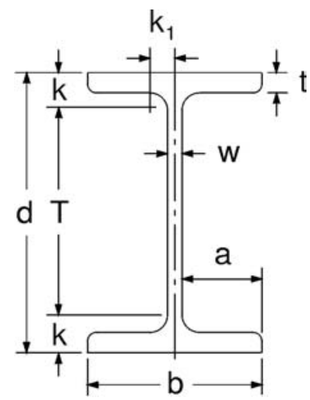

Fig. 1b

Torsional section properties (flange-to-web fillets neglected):

$$J = \frac{2bt^3 + d'w^3}{3}$$ (Galambos 1968) [6]

$$C_w = \frac{(d')^2 b^3 t}{24}$$ (Galambos 1968, Picard and Beaulieu 1991) [7]

$$d' = d - t$$ [8]

Example calculation: W610x125

d = 612 mm, b = 229 mm, t = 19.6 mm, w = 11.9 mm

d' = 592 mm

J = 1480 x 10^3 mm^4

C_w = 3440 x 10^9 mm^6

2. Channels

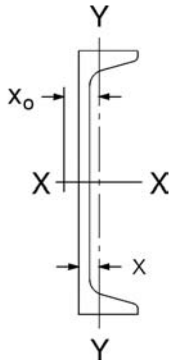

Fig. 2a

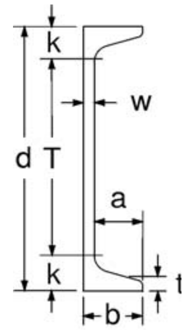

Fig. 2b

Torsional section properties (flange slope and flange-to-web fillets neglected):

$$J = \frac{2\,b'\,t^3 + d'\,w^3}{3} \quad \text{(SSRC 1998)} \qquad [9]$$

$$C_w = (d')^2 (b')^3\, t \left[\frac{1 - 3\alpha}{6} + \frac{\alpha^2}{2} \left(1 + \frac{d'\,w}{6\,b'\,t} \right) \right] \quad \text{(Galambos 1968, SSRC 1998)} \qquad [10]$$

$$\alpha = \frac{1}{2 + \dfrac{d'\,w}{3\,b'\,t}} \qquad [11]$$

$$d' = d - t\,, \quad b' = b - w/2 \qquad [12]$$

Shear centre location:

$$x_o = x + b'\,\alpha - \frac{w}{2} \quad \text{(Galambos 1968, Seaburg and Carter 1997)} \qquad [13]$$

Example calculation: C310x31

d = 305 mm, b = 74 mm, t = 12.7 mm, w = 7.2 mm

(Actual flange slope = 1/6; zero slope assumed here for simplicity)

d' = 292 mm, b' = 70.4 mm

J = 132 x 10^3 mm^4

α = 0.359, C_w = 29.0 x 10^9 mm^6

x = 17.5 mm (formula not shown)

x_O = 39.2 mm

3. Angles

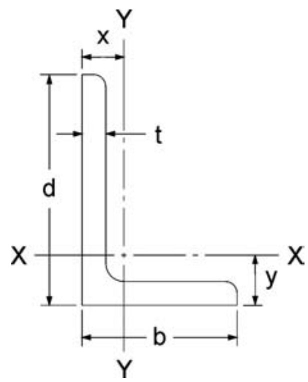

Fig. 3a

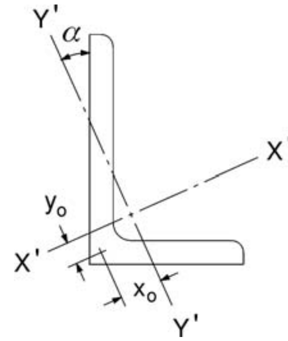

Fig. 3b

Torsional section properties (fillets neglected):

$$J = \frac{(d' + b')t^3}{3} \qquad [14]$$

$$C_w = \frac{t^3}{36}\left[(d')^3 + (b')^3\right] \text{ (Bleich 1952, Picard and Beaulieu 1991)} \qquad [15]$$

$$d' = d - \frac{t}{2}, \quad b' = b - \frac{t}{2} \qquad [16]$$

The warping constant of angles is small and often neglected. For double angles, the values of J and C_w can be taken equal to twice the value for single angles.

The shear centre (x_O, y_O) is located at the intersection of the angle leg axes.

Example calculation: L203x102x13

d = 203 mm, b = 102 mm, t = 12.7 mm

d' = 197 mm, b' = 95.7 mm

J = 200 x 10^3 mm^4

C_w = 0.485 x 10^9 mm^6

4. T-Sections

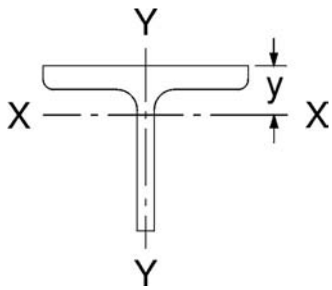

Fig. 4a

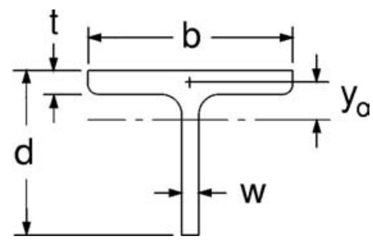

Fig. 4b

Torsional section properties (flange-to-web fillets neglected):

$$J = \frac{b\,t^3 + d'\,w^3}{3} \qquad [17]$$

$$C_w = \frac{b^3 t^3}{144} + \frac{(d')^3\,w^3}{36} \quad \text{(Bleich 1952, Picard and Beaulieu 1991)} \qquad [18]$$

$$d' = d - \frac{t}{2} \qquad [19]$$

The warping constant of T-sections is small and often neglected.

The shear centre is located at the intersection of the flange and stem plate axes.

Example calculation: WT180x67

d = 178 mm, *b* = 369 mm, *t* = 18.0 mm, *w* = 11.2 mm

d′ = 169 mm

J = 796 x 10^3 mm^4

C_w = 2.22 x 10^9 mm^6

5. Monosymmetric Wide-Flange Shapes

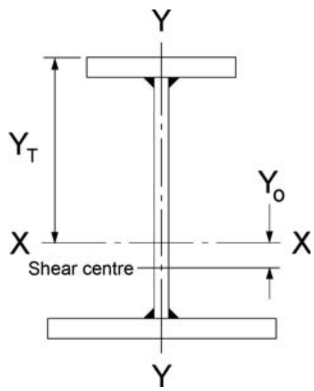

Fig. 5a

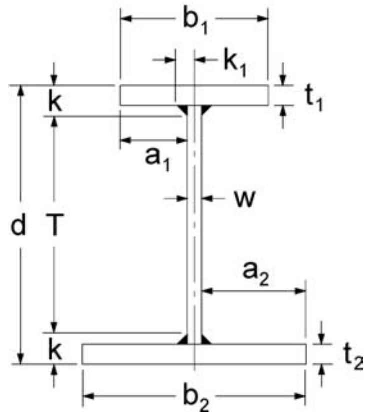

Fig. 5b

Torsional section properties (fillet welds neglected):

$$J = \frac{b_1\, t_1^3 + b_2\, t_2^3 + d'\, w^3}{3} \quad \text{(SSRC 1998)} \qquad [20]$$

$$C_w = \frac{(d')^2\, b_1^3\, t_1\, \alpha}{12} \quad \text{(SSRC 1998, Picard and Beaulieu 1991)} \qquad [21]$$

$$\alpha = \frac{1}{1 + (b_1/b_2)^3 (t_1/t_2)} \qquad [22]$$

$$d' = d - \frac{(t_1 + t_2)}{2} \qquad [23]$$

Subscripts "1" and "2" refer to the top and bottom flanges, respectively, as shown on Fig. 5b.

Shear centre location:

$$Y_O = Y_T - \frac{t_1}{2} - \alpha\, d' \quad \text{(Galambos 1968)} \qquad [24]$$

The sign of Y_O calculated from Eq. 24 indicates whether the shear centre is located above ($Y_O > 0$) or below ($Y_O < 0$) the centroid. The shear centre is generally located between the centroid and the wider of the two flanges. For doubly-symmetric sections, Y_O is equal to zero since the centroid and shear centre coincide.

Monosymmetry constant:

$$\beta_X \approx \delta\, 0.9\left(2\rho - 1\right)d'\left[1 - \left(\frac{I_y}{I_x}\right)^2\right], \quad \frac{I_y}{I_x} \le 0.5 \quad \text{(Kitipornchai and Trahair 1980)} \qquad [25]$$

$$\rho = \frac{I_{y\,TOP}}{I_y} = 1 - \alpha \qquad [26]$$

Eq. 25 is an approximate formula and is only valid if $I_Y \le 0.5\, I_X$, where I_Y and I_X are the moments of inertia of the section about the vertical and horizontal centroidal axes, respectively. A more accurate expression is given by SSRC (1998).

The value of δ depends on which flange is in compression:

$$\delta = \begin{cases} +1 & \text{If the top flange is in compression} \\ -1 & \text{If the bottom flange is in compression} \end{cases} \qquad [27]$$

Generally, the value of β_X obtained from Eq. 25 will be positive when the wider flange is in compression and negative when in tension.

Example calculation: WRF1200x244 The top flange is in compression.
d = 1200 mm, b_1 = 300 mm, b_2 = 550 mm, $t_1 = t_2$ = 20.0 mm, w = 12.0 mm
d' = 1180 mm
α = 0.860, ρ = 1 - α = 0.140
J = 2950 x 10^3 mm^4
C_w = 53 900 x 10^9 mm^6
Y_T = 695 mm (formula not shown)
Y_O = -330 mm
Since Y_O is negative, the shear centre is located between the centroid and the bottom flange.
I_X = 7240 x 10^6 mm^4, I_Y = 322 x 10^6 mm^4 (formulas not shown)
I_Y / I_X = 0.0445 < 0.5 OK
δ = +1
β_X = -763 mm (The top flange is narrower and in compression.)

6. Wide-Flange Shapes with Channel Cap

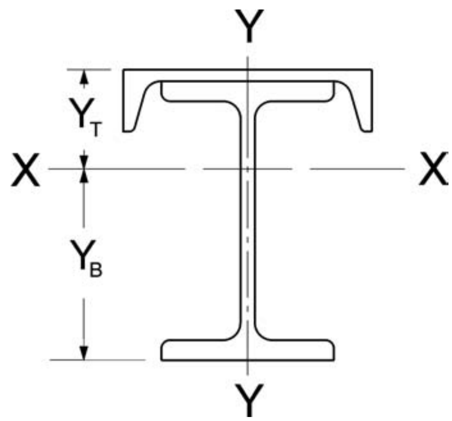

Fig. 6a

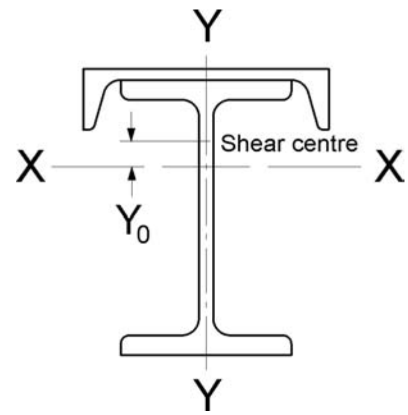

Fig. 6b

Torsional section properties (flange-to-web fillets neglected):

A simple and conservative estimate of the St. Venant torsional constant is given by:

$$J \approx J_w + J_c \qquad [28]$$

The "*w*" and "*c*" subscripts refer to the W-shape and channel, respectively. A more refined expression for *J* is proposed by Ellifritt and Lue (1998).

Shear centre location:

$$Y_O = Y_T - \frac{t_w + w_c}{2} - a + e \quad \text{(Kitipornchai and Trahair 1980)} \qquad [29]$$

$$a = (1 - \rho)h\,, \quad b = \rho h \qquad [30]$$

$$\rho = \frac{I_{y\,TOP}}{I_{y\,TOP} + I_{y\,BOT}} = \frac{I_{y\,TOP}}{I_y} \qquad [31]$$

where $I_{y\,TOP}$, $I_{y\,BOT}$, and I_y are the moments of inertia of the built-up top flange (channel + top flange of the W-shape), the bottom flange, and the entire built-up section about the vertical axis, respectively. With the channel cap on the top flange, as shown on Fig. 6, the value of Y_O obtained from Eq. 29 will be positive, indicating that the shear centre is located above the centroid.

The distance between the shear centres of the top and bottom flanges is given by:

$$h = d_W - t_W + \frac{w_C}{2} + e \qquad [32]$$

The distance between the shear centre of the built-up top flange and the centre line of the channel web and W-shape top flange, taken together as a single plate, is given by:

$$e = \frac{b_C^2\, d_C^2\, t_C}{4\, \rho\, I_y} \qquad [33]$$

Warping constant of the built-up section:

$$C_w = a^2\, I_{y\,TOP} + b^2\, I_{y\,BOT} \quad \text{(Kitipornchai and Trahair 1980)} \qquad [34]$$

A simplified formula for C_w is also given by Ellifritt and Lue (1998).

Monosymmetry constant:

$$\beta_X \approx \delta\, 0.9\,(2\,\rho - 1)\,h \left[1 - \left(\frac{I_y}{I_x}\right)^2\right]\left(1 + \frac{b_C}{2\,d}\right), \qquad \frac{I_y}{I_x} \le 0.5 \quad \text{(Kitipornchai and Trahair 1980)} \qquad [35]$$

where d is the built-up section depth:

$$d = d_W + w_C \qquad [36]$$

Eq. 35 is an approximation which is only valid if $I_Y \le 0.5\, I_X$, where I_x is the moment of inertia of the built-up section about the horizontal centroidal axis. See page 11 for the value of δ and the sign of β_X. A further simplified expression is given by Ellifritt and Lue (1998).

Example calculation: W610x125 and C310x31

W-shape: W610x125

d_W = 612 mm, b_W = 229 mm, t_W = 19.6 mm, w_W = 11.9 mm

$J_W = 1480 \times 10^3$ mm^4 (previously calculated, p. 6)

Channel cap: C310x31

d_C = 305 mm, b_C = 74 mm, t_C = 12.7 mm, w_C = 7.2 mm

$J_C = 132 \times 10^3$ mm^4 (previously calculated, p. 7)

Built-up section, with the top flange in compression:

$J = 1610 \times 10^3$ mm^4

Y_T = 255 mm (formula not shown)

$I_{y\ TOP} = 73.1 \times 10^6$ mm^4 (formula not shown)

$I_{y\ BOT} = 19.6 \times 10^6$ mm^4 (formula not shown)

$I_y = 92.7 \times 10^6$ mm^4

$\rho = 0.789$

e = 22.1 mm

h = 618 mm

a = 130 mm

b = 488 mm

Y_O = 134 mm

Since the calculated value of Y_O is positive, the shear centre is located above the centroid (see Fig. 6b).

$C_W = 5900 \times 10^9$ mm^6

$I_x = 1260 \times 10^6$ mm^4 (formula not shown)

d = 619 mm

$I_y / I_x = 0.0736 < 0.5$ OK

$\delta = +1$ (top flange in compression)

β_X = 339 mm (wider top flange in compression)

B) Closed Cross Sections

7. Hollow Structural Sections (HSS), Round

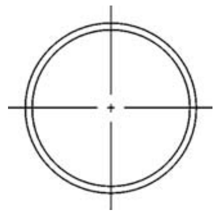

Fig. 7

St. Venant torsional constant:

$$J = 2I = \frac{\pi}{32}\left[d^4 - (d-2t)^4\right]$$ (Stelco 1981, Seaburg and Carter 1997) [37]

where *d* is the outer diameter, *I* is the moment of inertia, and *t* is the wall thickness.

The warping constant C_w is taken equal to zero.

Shear constant:

$$C_{RT} = \frac{2tI}{Q}$$ (Stelco 1981) [38]

$$I = \frac{\pi}{64}\left[d^4 - (d-2t)^4\right]$$ [39]

$$Q = \frac{t}{6}\left(3d^2 - 6dt + 4t^2\right)$$ (Stelco 1981) [40]

Example calculation: HSS610x9.5

d = 610 mm, *t* = 9.53 mm

J = 1 620 000 x 10^3 mm^4

I = 810 x 10^6 mm^4

Q = 1720 x 10^3 mm^3

C_{RT} = 8980 mm^2

8. Hollow Structural Sections (HSS), Square and Rectangular

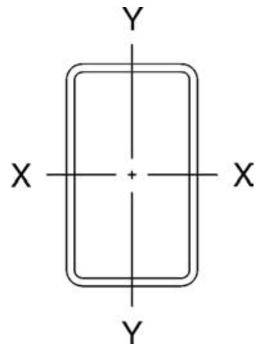

Fig. 8

A conservative estimate of the St. Venant torsional constant is given by:

$$J \approx \frac{4\,A_P^2\,t}{p} \quad \text{(Salmon and Johnson 1980)} \qquad [41]$$

Mid-contour length:

$$p = 2\left[(d-t)+(b-t)\right]-2\,R_C\,(4-\pi) \qquad [42]$$

Enclosed area:

$$A_P = (d-t)(b-t)-R_C^2\,(4-\pi) \qquad [43]$$

Mean corner radius:

$$R_C = \frac{R_O + R_i}{2} \approx 1.5\,t \qquad [44]$$

where *d* and *b* are the outside dimensions, and *t* is the wall thickness. R_O and R_i are the outer and inner corner radii taken equal to *2t* and *t*, respectively.

The warping constant C_w is usually taken equal to zero.

An approximate expression for the shear constant is given by Stelco (1981):

$$C_{RT} \approx 2t(h - 4t) \qquad [45]$$

where h is the outer section dimension in the direction of the applied shear force.

Example calculation: HSS230x102x6.4

d = 203 mm, b = 102 mm, t = 6.35 mm

R_O = 12.7 mm

R_i = 6.35 mm

R_C = 9.53 mm

p = 568 mm

A_p = 18 700 mm^2

J = 15 600 x 10^3 mm^4

It is assumed that the shear force acts in a direction parallel to the longer dimension, d.

$h = d$ = 203 mm

C_{RT} = 2260 mm^2

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