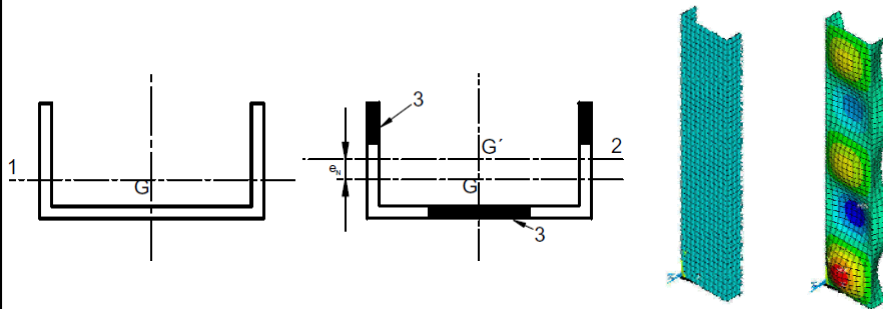




## Placas – Largura Efetiva

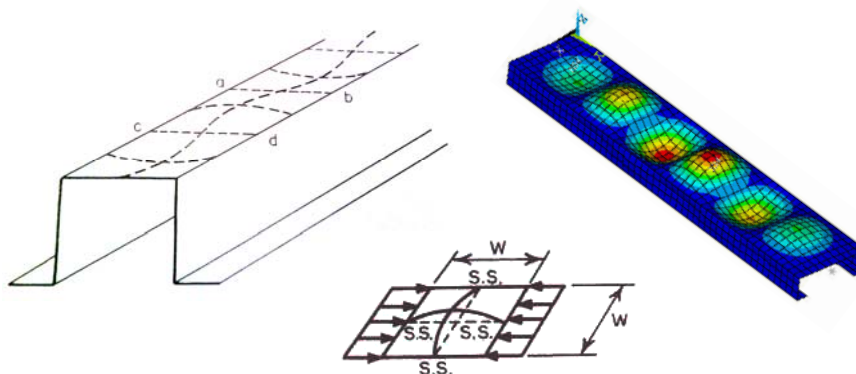
Programa de Pós-Graduação em Engenharia Civil  
PGECIV - Mestrado Acadêmico  
Faculdade de Engenharia – FEN/UERJ  
Disciplina: Tópicos Especiais em Estruturas (Chapa Dobrada)  
Professor: Luciano Rodrigues Ornelas de Lima



## 1. Introdução

### ■ ELU

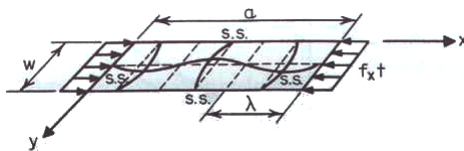
- ✓ plastificação (escoamento)  $\Rightarrow$  índice de esbeltez baixo
- ✓ flambagem local elástica  $\Rightarrow$  índice de esbeltez elevado





## 2. Flambagem Local Elástica

- Placa sujeita à compressão  $\Rightarrow$  simplesmente apoiada



tensão crítica de flambagem  $\Rightarrow$   
solução da eq. dif. de Bryan  $\Rightarrow$   
teoria de pequenas deformações  $\Rightarrow$   
 $\omega \approx t$

$$\frac{\partial^4 \omega}{\partial x^4} + 2 \frac{\partial^4 \omega}{\partial x^2 \partial y^2} + \frac{\partial^4 \omega}{\partial y^4} + \frac{f_x t}{D} \frac{\partial^2 \omega}{\partial x^2} = 0$$

onde  $D = \frac{Et^3}{12(1 - \mu^2)}$

$E$  = modulus of elasticity of steel =  $29.5 \times 10^3$  ksi (203 GPa)

$t$  = thickness of plate

$\mu$  = Poisson's ratio = 0.3 for steel in the elastic range

$\omega$  = deflection of plate perpendicular to surface

$f_x$  = compression stress in  $x$  direction



## 2. Flambagem Local Elástica

- Placa sujeita à compressão  $\Rightarrow$  simplesmente apoiada

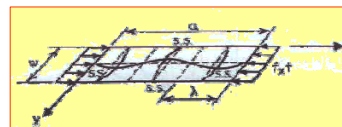
- Se  $m$  e  $n \Rightarrow$  nº de ondas  $\frac{1}{2}$  seno nas direções  $x$  e  $y \Rightarrow$  a forma deformada da placa devido a flambagem  $\Rightarrow$  duas séries

$$\omega = \sum_{m=1}^{\infty} \sum_{n=1}^{\infty} A_{mn} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{w}$$

- Condições de contorno

- $\omega(0;y) = \omega(a;y) = \omega_{,xx}(0;y) = \omega_{,xx}(a;y) = 0$

- $\omega(x;0) = \omega(x;b) = \omega_{,yy}(0;y) = \omega_{,yy}(a;y) = 0$



- 2ªs derivadas = 0 nas faces externas  $\Rightarrow \frac{\partial^2 \omega}{\partial x^2} = 0$  e  $\frac{\partial^2 \omega}{\partial y^2} = 0 \Rightarrow M = 0$

$$M_x = -D \left( \frac{\partial^2 \omega}{\partial x^2} + \mu \frac{\partial^2 \omega}{\partial y^2} \right) \quad M_y = -D \left( \frac{\partial^2 \omega}{\partial y^2} + \mu \frac{\partial^2 \omega}{\partial x^2} \right)$$



## 2. Flambagem Local Elástica

- Placa sujeita à compressão  $\Rightarrow$  simplesmente apoiada

✓ Resolvendo a eq. de Bryan usando  $\omega$

$$\sum_{m=1}^{\infty} \sum_{n=1}^{\infty} A_{mn} \left[ \pi^4 \left( \frac{m^2}{a^2} + \frac{n^2}{w^2} \right)^2 - \frac{f_x t}{D} \frac{m^2 \pi^2}{a^2} \right] \sin \frac{m \pi x}{a} \sin \frac{n \pi y}{w} = 0$$

✓ Obtenção da solução

- $A_{mn} = 0$  ou termo  $[ ] = 0 \Rightarrow$  flambagem não ocorre  $\Rightarrow$  solução trivial

$$\pi^4 \left( \frac{m^2}{a^2} + \frac{n^2}{w^2} \right)^2 - \frac{f_x t}{D} \frac{m^2 \pi^2}{a^2} = 0$$

- E isolando-se  $f_x$

$$f_{cr} = f_x = \frac{D \pi^2}{t w^2} \left[ m \left( \frac{w}{a} \right) + \frac{n^2}{m} \left( \frac{a}{w} \right) \right]^2$$



## 2. Flambagem Local Elástica

- Placa sujeita à compressão  $\Rightarrow$  simplesmente apoiada

✓ Termo  $[ ] \Rightarrow$  menor valor  $\Rightarrow n = 1 \Rightarrow$  onda de 1/2 seno ocorre na direção y

$$f_{cr} = f_x = \frac{D \pi^2}{t w^2} \left[ m \left( \frac{w}{a} \right) + \frac{n^2}{m} \left( \frac{a}{w} \right) \right]^2$$

✓ Logo  $f_{cr} = \frac{k D \pi^2}{t w^2}$  onde  $k = \left[ m \left( \frac{w}{a} \right) + \frac{1}{m} \left( \frac{a}{w} \right) \right]^2$

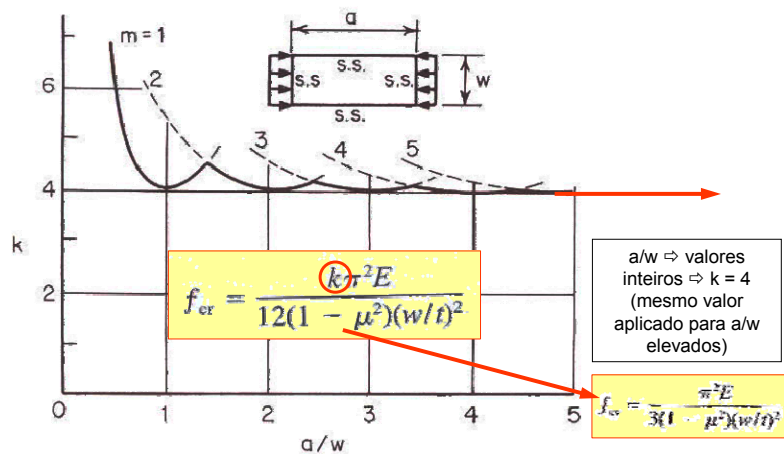
✓ E finalmente

$$f_{cr} = \frac{k \pi^2 E}{12(1 - \mu^2)(w/t)^2}$$



## 2. Flambagem Local Elástica

- Placa sujeita à compressão ⇒ simplesmente apoiada



## 2. Flambagem Local Elástica

- Placa sujeita à compressão ⇒ simplesmente apoiada

| Case | Boundary condition | Type of stress | Value of k for long plate |
|------|--------------------|----------------|---------------------------|
| (a)  |                    | Compression    | 4.0                       |
| (b)  |                    | Compression    | 6.97                      |
| (c)  |                    | Compression    | 0.425                     |
| (d)  |                    | Compression    | 1.277                     |



## 2. Flambagem Local Elástica

- Placa sujeita à compressão  $\Rightarrow$  simplesmente apoiada

| Case | Boundary condition | Type of stress | Value of k for long plate |
|------|--------------------|----------------|---------------------------|
| (e)  |                    | Compression    | 5.42                      |
| (f)  |                    | Shear          | 5.34                      |
| (g)  |                    | Shear          | 8.98                      |
| (h)  |                    | Bending        | 23.9                      |
| (i)  |                    | Bending        | 41.8                      |



## 3. Flambagem Plástica

- Placa sujeita à compressão  $\Rightarrow$  tensões em uma direção  $\Rightarrow f_y$
- Placa anisotrópica  $\Rightarrow$  propriedades  $\neq$  nas duas direções
- Bleich em 1924

$E_t$  é o módulo tangente

$$\left( \tau \frac{\partial^4 \omega}{\partial x^4} + 2\sqrt{\tau} \frac{\partial^4 \omega}{\partial x^2 \partial y^2} + \frac{\partial^4 \omega}{\partial y^4} \right) + \frac{f_x t}{D} \frac{\partial^2 \omega}{\partial x^2} = 0 \quad \text{onde} \quad \tau = E_t/E$$

(fator de redução devido a plasticidade)

- Aplicando-se as condições de contorno modificadas

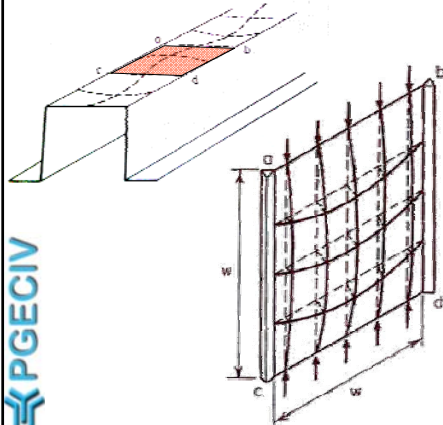
$$f_{cr} = \frac{k \pi^2 E \sqrt{\tau}}{12(1 - \mu^2)(w/t)^2} = \frac{k \pi^2 \sqrt{E E_t}}{12(1 - \mu^2)(w/t)^2} \quad \text{com comp. onda p/ placa longa}$$

$$\lambda = \sqrt[4]{\tau w}$$

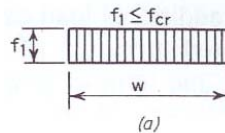


## 4. Resistência Pós-Flambagem

- Tensões em uma direção  $\Rightarrow f_y \Rightarrow$  flambagem plástica
- Cargas adicionais  $\Rightarrow$  redistribuição de tensões
- Valores elevados de  $w / t$

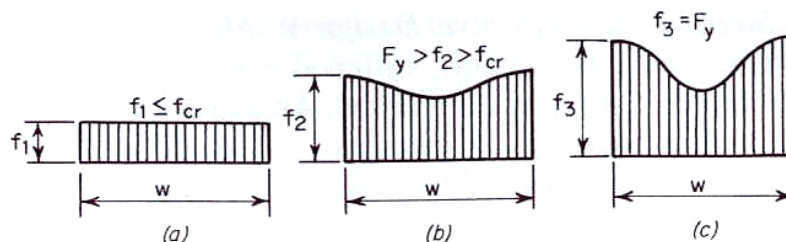


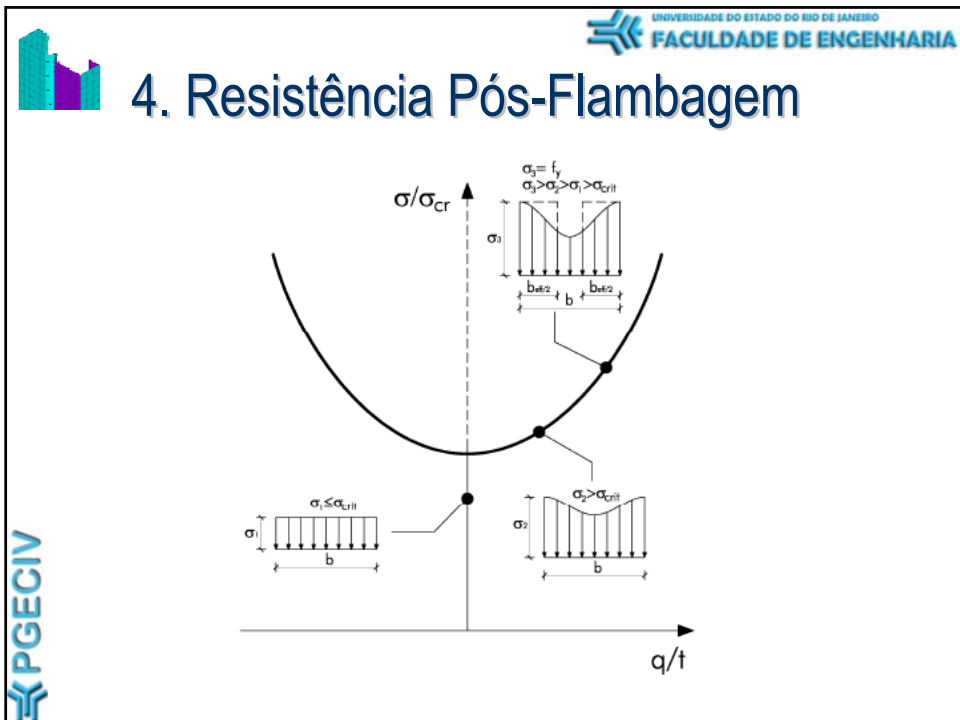
- Início da flambagem  $\Rightarrow$  barras horizontais  $\Rightarrow$  diminuir o aumento de deflexões
- Tensões uniformes até o momento da flambagem ( $f < f_{cr}$ )



## 4. Resistência Pós-Flambagem

- Após ocorrer a flambagem  $\Rightarrow$  parte da carga no centro da placa transfere-se para as extremidades  $\Rightarrow$  tensões não uniformes
- A redistribuição de tensões continua até que nas extremidades  $\Rightarrow f_y$  (escoamento)  $\Rightarrow$  falha da placa
- Necessidade  $\Rightarrow$  Teoria de Grandes Deslocamentos





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FACULDADE DE ENGENHARIA

## 4. Resistência Pós-Flambagem

- von Karman em 1910

$$\frac{\partial^4 \omega}{\partial x^4} + 2 \frac{\partial^4 \omega}{\partial x^2 \partial y^2} + \frac{\partial^4 \omega}{\partial y^4} = \frac{t}{D} \left( \frac{\partial^2 F}{\partial y^2} \frac{\partial^2 \omega}{\partial x^2} - 2 \frac{\partial^2 F}{\partial x \partial y} \frac{\partial^2 \omega}{\partial x \partial y} + \frac{\partial^2 F}{\partial x^2} \frac{\partial^2 \omega}{\partial y^2} \right)$$

onde F é a função de tensões na fibra média da placa e

$$f_x = \frac{\partial^2 F}{\partial y^2} \quad f_y = \frac{\partial^2 F}{\partial x^2} \quad \tau_{xy} = - \frac{\partial^2 F}{\partial x \partial y}$$

- Resolução complexa para aplicação em projeto  $\Rightarrow$  **introdução do conceito de largura efetiva**

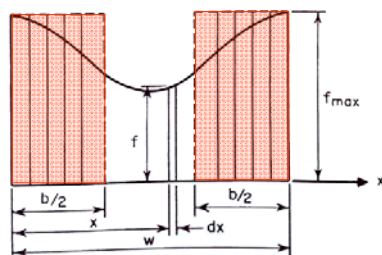
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## 4. Resistência Pós-Flambagem

### ■ Conceito de Largura Efetiva

- ✓ largura  $w \Rightarrow$  tensões não-uniformes
- ✓ largura fictícia “efetiva”  $b \Rightarrow$  tensões uniformes  $\Rightarrow f_{\max}$  na extremidade
- ✓ largura efetiva  $b \Rightarrow$  largura particular da placa que flamba qdo as tensões de compressão atingem  $f_y$  (escoamento)



$$\int_0^w f dx = b f_{\max}$$



## 4. Resistência Pós-Flambagem

### ■ Conceito de Largura Efetiva

- ✓ placas longas

$$f_{cr} = F_y = \frac{\pi^2 E}{3(1 - \mu^2)(b/t)^2} \Rightarrow b = Ct \sqrt{\frac{E}{F_y}} = 1.9t \sqrt{\frac{E}{F_y}}$$

$$C = \frac{\pi}{\sqrt{3(1 - \mu^2)}} = 1.9$$

$$\mu = 0.3$$

- ✓ para  $w > b$

$$f_{cr} = \frac{\pi^2 E}{3(1 - \mu^2)(w/t)^2} \Rightarrow w = Ct \sqrt{\frac{E}{f_{cr}}} \Rightarrow \frac{b}{w} = \sqrt{\frac{f_{cr}}{F_y}}$$

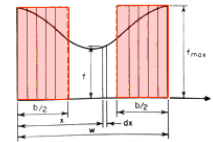


## 4. Resistência Pós-Flambagem

### Conceito de Largura Efetiva

✓ Winter  $\Rightarrow$  eq. de  $b$  para o elemento com tensões inferiores a  $f_y$

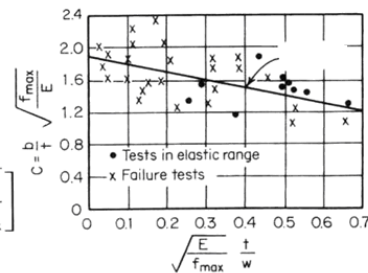
$$b = Ct \sqrt{\frac{E}{F_y}} = 1.9t \sqrt{\frac{E}{F_y}} \Rightarrow b = Ct \sqrt{\frac{E}{f_{\max}}}$$



onde  $f_{\max}$  é a máxima tensão na extremidade do elemento

✓ Resultados experimentais  $\Rightarrow$   
C depende do parâmetro  
adimensional

$$\sqrt{\frac{E}{f_{\max}}} \left( \frac{t}{w} \right) \Rightarrow C = 1.9 \left[ 1 - 0.475 \left( \frac{t}{w} \right) \sqrt{\frac{E}{f_{\max}}} \right]$$

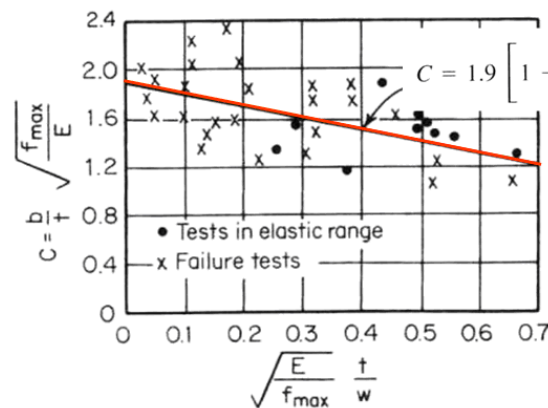


## 4. Resistência Pós-Flambagem

### Conceito de Largura Efetiva

✓ Linha cheia  $\Rightarrow C = 1.9$  em  $x = 0 \Rightarrow w/t$  elevado  $\Rightarrow$  semelhante com

$$b = Ct \sqrt{\frac{E}{F_y}} = 1.9t \sqrt{\frac{E}{F_y}}$$







## 4. Resistência Pós-Flambagem

- Conceito de Largura Efetiva
  - ✓ Winter em 1946  $\Rightarrow$  equação para cálculo da largura efetiva dependente de  $f_{max}$  e  $w/t$ 

$$b = 1.9t \sqrt{\frac{E}{f_{max}}} \left[ 1 - 0.475 \left( \frac{t}{w} \right) \sqrt{\frac{E}{f_{max}}} \right]$$
  - ✓ Reescrevendo em termos de  $f_{cr}/f_{max}$ 

$$\frac{b}{w} = \sqrt{\frac{f_{cr}}{f_{max}}} \left( 1 - 0.25 \sqrt{\frac{f_{cr}}{f_{max}}} \right)$$
  - ✓ Completamente efetiva  $\Rightarrow b = w \Rightarrow w/t \Rightarrow 1^a$  onda ocorre p/ tensão igual a  $f_{cr}/4$ 

$$\left( \frac{w}{t} \right)_{lim} = 0.95 \sqrt{\frac{E}{f_{max}}}$$





## 4. Resistência Pós-Flambagem

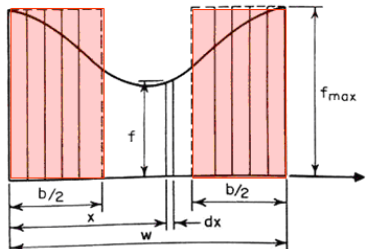
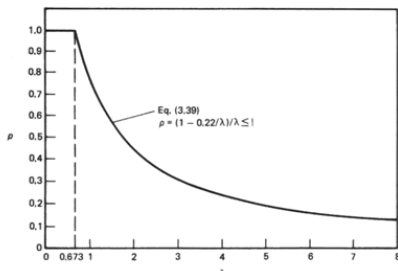
- Conceito de Largura Efetiva
 

$$\frac{b}{w} = \sqrt{\frac{f_{cr}}{f_{max}}} \left( 1 - 0.25 \sqrt{\frac{f_{cr}}{f_{max}}} \right) \Rightarrow b = \rho w$$

$\rho$  = reduction factor

$$= (1 - 0.22/\sqrt{f_{max}/f_{cr}})/\sqrt{f_{max}/f_{cr}}$$

$$= (1 - 0.22/\lambda)/\lambda \leq 1$$

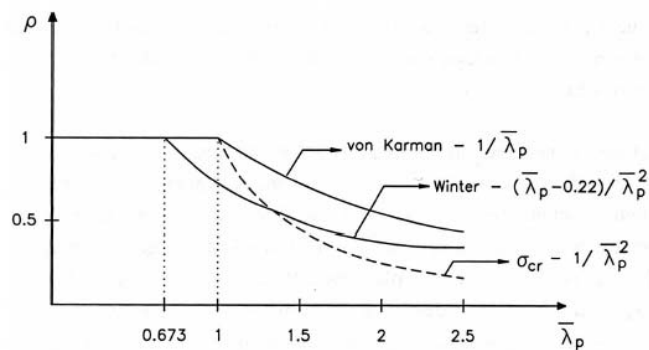





## 5. Eurocode 3

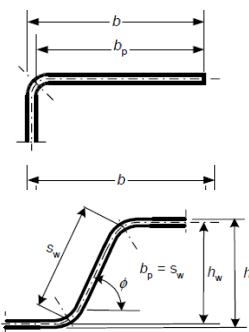
- Tensão crítica para flambagem elástica de placa ortotrópica

$$\sigma_{cr,p} = k_{\sigma,p} \sigma_E \quad \text{where} \quad \sigma_E = \frac{\pi^2 E t^2}{12 (1 - \nu^2) b^2}$$

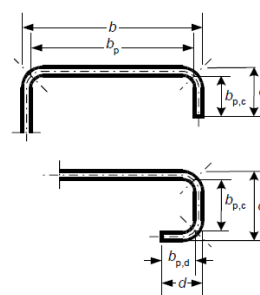


## 5. Eurocode 3

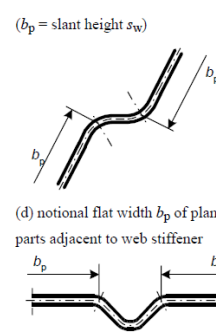
- Larguras efetivas
  - ✓ Elementos de placa sem enrijecedores longitudinais
  - ✓ § 5.5.2 EC3-1-3  $\Rightarrow$  § 4.4 EC3-1-5
    - $b_p = \bar{b}$  onde  $b_p$  é determinada conforme apresentado a seguir



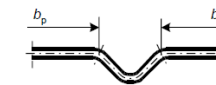
(c) notional flat width  $b_p$  for a web  
( $b_p = \text{slant height } s_w$ )



(b) notional flat width  $b_p$  of plane parts of flanges



(d) notional flat width  $b_p$  of plane parts adjacent to web stiffener



(e) notional flat width  $b_p$  of flat parts adjacent to flange stiffener



## 5. Eurocode 3

### ■ Larguras efetivas

✓ Elementos de placa sem enrijecedores longitudinais

✓ § 5.5.2 EC3-1-3 ⇒ § 4.4 EC3-1-5

- Ver anexo D EC3-1.3 para método de redução de espessura
- Com base no valor de  $\psi = \sigma_2 / \sigma_1$  (relação entre as tensões atuantes nas extremidades do elemento), calcula-se o valor do coeficiente de flambagem local da parede  $k_\sigma$
- Para tal, as Tabelas 4.1 (elementos internos) e 4.2 (elementos externos) apresentam expressões do tipo  $k_\sigma = k_\sigma(\psi)$
- Nota-se que o valor da tensão crítica de instabilidade local do elemento é obtida através da eq. do slide 21
- Recorde-se ainda que os elementos (internos e externos) se consideram simplesmente apoiados e, por isso,  $k_\sigma = 4$  quando o elemento é interno e está submetido à compressão uniforme



## 5. Eurocode 3

### ■ Larguras efetivas

✓ Elementos de placa sem enrijecedores longitudinais

✓ § 5.5.2 EC3-1-3 ⇒ § 4.4 EC3-1-5

- Com base no valor de  $k_\sigma$ , calcula-se o valor da esbelteza normalizada local do elemento (placa), a qual é dada por

$$\bar{\lambda}_p = \sqrt{\frac{f_y}{\sigma_{cr}}} = \frac{\bar{b}/t}{28,4 \varepsilon \sqrt{k_\sigma}}$$

- Com base no valor da esbelteza normalizada local do elemento  $\bar{\lambda}_p$ , calcula-se o valor do fator de redução de largura efetiva  $\rho$ , o qual é dado pelas eq. abaixo e depois calcula-se a área efetiva:  $A_{e,eff} = \rho A_e$

internal compression elements:

$$\rho = \frac{\bar{\lambda}_p - 0,055 (3 + \psi)}{\bar{\lambda}_p^2} \leq 1,0$$

outstand compression elements:

$$\rho = \frac{\bar{\lambda}_p - 0,188}{\bar{\lambda}_p^2} \leq 1,0$$

Ver Anexo C EC3-1-3



## 5. Eurocode 3

### ■ Larguras efetivas

✓ Elementos de placa sem enrijecedores longitudinais

✓ § 5.5.2 EC3-1-3 ⇒ § 4.4 EC3-1-5

$\psi$  is the stress ratio determined in accordance with 4.4(3) and 4.4(4)

$\bar{b}$  is the appropriate width as follows (for definitions, see Table 5.2 of EN 1993-1-1)

$b_w$  for webs;

$b$  for internal flange elements (except RHS);

$b - 3t$  for flanges of RHS;

$c$  for outstand flanges;

$h$  for equal-leg angles;

$h$  for unequal-leg angles;

$k_\sigma$  is the buckling factor corresponding to the stress ratio  $\psi$  and boundary conditions. For long plates  $k_\sigma$  is given in Table 4.1 or Table 4.2 as appropriate;

$t$  is the thickness;

$\sigma_{cr}$  is the elastic critical plate buckling stress see Annex A.1(2).



## 5. Eurocode 3

### ■ Larguras efetivas

✓ Elementos de placa sem enrijecedores longitudinais

✓ § 5.5.2 EC3-1-3 ⇒ § 4.4 EC3-1-5

Table 4.1: Internal compression elements

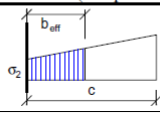
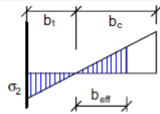
| Stress distribution (compression positive) |     |                       |      | Effective <sup>b</sup> width $b_{eff}$                                |      |                     |
|--------------------------------------------|-----|-----------------------|------|-----------------------------------------------------------------------|------|---------------------|
|                                            |     |                       |      | $\psi = 1$ :                                                          |      |                     |
|                                            |     |                       |      | $b_{eff} = \rho \bar{b}$                                              |      |                     |
|                                            |     |                       |      | $b_{e1} = 0,5 b_{eff} \quad b_{e2} = 0,5 b_{eff}$                     |      |                     |
|                                            |     |                       |      | $1 > \psi > 0$ :                                                      |      |                     |
|                                            |     |                       |      | $b_{eff} = \rho \bar{b}$                                              |      |                     |
|                                            |     |                       |      | $b_{e1} = \frac{2}{5 - \psi} b_{eff} \quad b_{e2} = b_{eff} - b_{e1}$ |      |                     |
|                                            |     |                       |      | $\psi < 0$ :                                                          |      |                     |
|                                            |     |                       |      | $b_{eff} = \rho b_c = \rho \bar{b} / (1 - \psi)$                      |      |                     |
|                                            |     |                       |      | $b_{e1} = 0,4 b_{eff} \quad b_{e2} = 0,6 b_{eff}$                     |      |                     |
| $\psi = \sigma_2 / \sigma_1$               | 1   | $1 > \psi > 0$        | 0    | $0 > \psi > -1$                                                       | -1   | $-1 > \psi > -3$    |
| Buckling factor $k_\sigma$                 | 4,0 | $8,2 / (1,05 + \psi)$ | 7,81 | $7,81 - 6,29\psi + 9,78\psi^2$                                        | 23,9 | $5,98 (1 - \psi)^2$ |



## 5. Eurocode 3

- Larguras efetivas
  - ✓ Elementos de placa sem enrijecedores longitudinais
  - ✓ § 5.5.2 EC3-1-3 ⇒ § 4.4 EC3-1-5

Table 4.2: Outstand compression elements

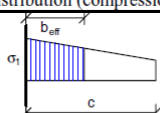
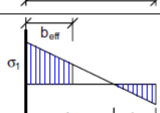
| Stress distribution (compression positive)                                        |      | Effective <sup>b</sup> width $b_{eff}$                                 |      |                                |               |
|-----------------------------------------------------------------------------------|------|------------------------------------------------------------------------|------|--------------------------------|---------------|
|  |      | $1 > \psi \geq 0$ :<br>$b_{eff} = \rho \cdot c$                        |      |                                |               |
|  |      | $\psi < 0$ :<br>$b_{eff} = \rho \cdot b_c = \rho \cdot c / (1 - \psi)$ |      |                                |               |
| $\psi = \sigma_2 / \sigma_1$                                                      | 1    | 0                                                                      | -1   | $\frac{1}{3}$                  | $\frac{1}{3}$ |
| Buckling factor $k_\sigma$                                                        | 0,43 | 0,57                                                                   | 0,85 | $0,57 - 0,21\psi + 0,07\psi^2$ |               |



## 5. Eurocode 3

- Larguras efetivas
  - ✓ Elementos de placa sem enrijecedores longitudinais
  - ✓ § 5.5.2 EC3-1-3 ⇒ § 4.4 EC3-1-5

Table 4.2: Outstand compression elements

| Stress distribution (compression positive)                                          |      | Effective <sup>b</sup> width $b_{eff}$                                 |      |                            |    |
|-------------------------------------------------------------------------------------|------|------------------------------------------------------------------------|------|----------------------------|----|
|  |      | $1 > \psi \geq 0$ :<br>$b_{eff} = \rho \cdot c$                        |      |                            |    |
|  |      | $\psi < 0$ :<br>$b_{eff} = \rho \cdot b_c = \rho \cdot c / (1 - \psi)$ |      |                            |    |
| $\psi = \sigma_2 / \sigma_1$                                                        | 1    | $1 > \psi > 0$                                                         | 0    | $0 > \psi > -1$            | -1 |
| Buckling factor $k_\sigma$                                                          | 0,43 | $0,578 / (\psi + 0,34)$                                                | 1,70 | $1,7 - 5\psi + 17,1\psi^2$ |    |
|                                                                                     |      |                                                                        |      | 23,8                       |    |

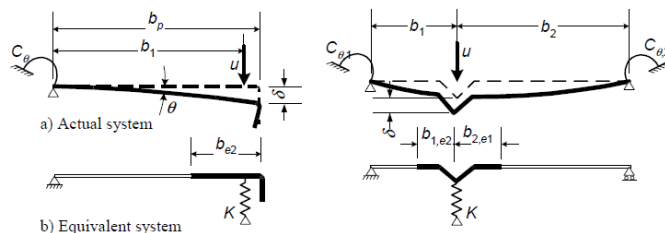


## 5. Eurocode 3

### ■ Larguras efetivas $\Rightarrow$ § 5.5.3 EC3-1-3

#### ✓ Elementos de placa com enrijecedores intermediários ou de borda

- O enrijecedor comporta-se como um membro em compressão com restrição parcial, ou seja, apoiado sobre uma mola cuja rigidez depende das condições de contorno dos elementos planos adjacentes
- A rigidez da mola é determinada aplicando-se uma carga unitária por comprimento unitário  $u$  conforme ilustrado abaixo



$$K = u/\delta$$

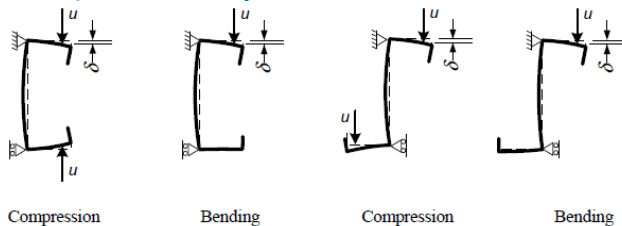
$\delta$  is the deflection of the stiffener due to the unit load  $u$  acting in the centroid ( $b_1$ ) of the effective part of the cross-section.



## 5. Eurocode 3

### ■ Larguras efetivas $\Rightarrow$ § 5.5.3 EC3-1-3

#### ✓ Elementos de placa com enrijecedores intermediários ou de borda



c) Calculation of  $\delta$  for C and Z sections

- A determinação da rigidez rotacional de mola  $C_{\theta}$ ,  $C_{\theta1}$  e  $C_{\theta2}$ , deve-se considerar a possibilidade de existirem outros enrijecedores no elemento



## 5. Eurocode 3

### ■ Larguras efetivas ⇒ § 5.5.3 EC3-1-3

#### ✓ Elementos de placa com enrijecedores intermediários ou de borda

- Para um enrijecedor de face, o deslocamento é obtido através da equação

$$\delta = \theta b_p + \frac{u b_p^3}{3} \cdot \frac{12(1-\nu^2)}{Et^3} \quad \text{com} \quad \theta = u b_p / C_\theta$$

- No caso de enrijecedores de face de seções C ou Z (slide anterior),  $C_\theta$  deve ser calculada conforme apresentado. Isso fornece a rigidez  $K_1$  para a mesa 1

$$K_1 = \frac{Et^3}{4(1-\nu^2)} \cdot \frac{1}{b_1^2 h_w + b_1^3 + 0,5 b_1 b_2 h_w k_f}$$

where:

- $b_1$  is the distance from the web-to-flange junction to the gravity center of the effective area of the edge stiffener (including effective part  $b_{e1}$  of the flange) of flange 1, see figure 5.8(a);
  - $b_2$  is the distance from the web-to-flange junction to the gravity center of the effective area of the edge stiffener (including effective part of the flange) of flange 2;
  - $h_w$  is the web depth;
  - $k_f = 0$  if flange 2 is in tension (e.g. for beam in bending about the y-y axis);
  - $k_f = \frac{A_{e1}}{A_{e2}}$  if flange 2 is also in compression (e.g. for a beam in axial compression);
  - $k_f = 1$  for a symmetric section in compression.
- $A_{e1}$  and  $A_{e2}$  is the effective area of the edge stiffener (including effective part  $b_{e1}$  of the flange, see figure 5.8(b)) of flange 1 and flange 2 respectively.



## 5. Eurocode 3

### ■ Larguras efetivas ⇒ § 5.5.3 EC3-1-3

#### ✓ Elementos de placa com enrijecedores intermediários ou de borda

- Para um enrijecedor intermediário,  $C_{\theta 1}$  e  $C_{\theta 2}$  são tomadas iguais a zero e o deslocamento é obtido por

$$\delta = \frac{u b_1^2 b_2^2}{3(b_1 + b_2)} \cdot \frac{12(1-\nu^2)}{Et^3}$$

- E finalmente, o fator de redução da espessura do enrijecedor para a resistência a flambagem distorcional (flambagem por flexão do enrijecedor) é obtido em

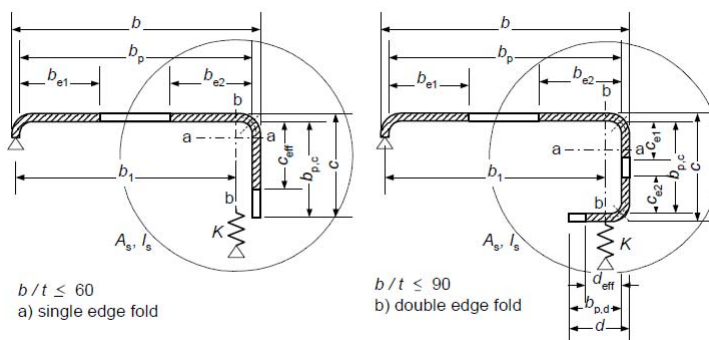
$$\chi_d = \begin{cases} 1,0 & \text{if } \bar{\lambda}_d \leq 0,65 \\ 1,47 - 0,723 \bar{\lambda}_d & \text{if } 0,65 < \bar{\lambda}_d < 1,38 \\ \frac{0,66}{\bar{\lambda}_d} & \text{if } \bar{\lambda}_d \geq 1,38 \end{cases} \quad \text{onde} \quad \bar{\lambda}_d = \sqrt{f_{yb} / \sigma_{cr,s}}$$

$\sigma_{cr,s}$  is the elastic critical stress for the stiffener(s)



## 5. Eurocode 3

- Larguras efetivas  $\Rightarrow$  § 5.5.3 EC3-1-3
  - ✓ Elementos de placa com enrijecedores de borda - § 5.5.3 EC3-1-3
    - ângulo entre 45° e 135°



## 5. Eurocode 3

- Larguras efetivas  $\Rightarrow$  § 5.5.3 EC3-1-3
  - ✓ Elementos de placa com enrijecedores de borda - § 5.5.3 EC3-1-3

(3) The procedure, which is illustrated in figure 5.10, should be carried out in steps as follows:

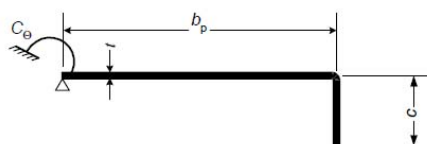
- **Step 1:** Obtain an initial effective cross-section for the stiffener using effective widths determined by assuming that the stiffener gives full restraint and that  $\alpha_{\text{com,Ed}} = f_{yb}/p_{b0}$ , see (3) to (5);
- **Step 2:** Use the initial effective cross-section of the stiffener to determine the reduction factor for distortional buckling (flexural buckling of a stiffener), allowing for the effects of the continuous spring restraint, see (6), (7) and (8);
- **Step 3:** Optionally iterate to refine the value of the reduction factor for buckling of the stiffener, see (9) and (10).

(4) Initial values of the effective widths  $b_{e1}$  and  $b_{e2}$  shown in figure 5.9 should be determined from clause 5.5.2 by assuming that the plane element  $b_p$  is doubly supported, see table 5.3.

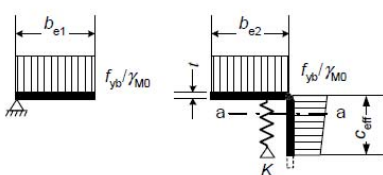


## 5. Eurocode 3

- Larguras efetivas  $\Rightarrow$  § 5.5.3 EC3-1-3
  - ✓ Elementos de placa com enrijecedores de borda - § 5.5.3 EC3-1-3



a) Gross cross-section and boundary conditions

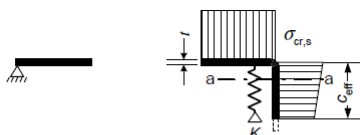


b) Step 1: Effective cross-section for  $K = \infty$   
based on  $\sigma_{com,Ed} = f_{yb} / \gamma_{M0}$

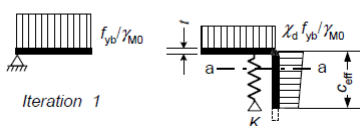


## 5. Eurocode 3

- Larguras efetivas  $\Rightarrow$  § 5.5.3 EC3-1-3
  - ✓ Elementos de placa com enrijecedores de borda - § 5.5.3 EC3-1-3



c) Step 2: Elastic critical stress  $\sigma_{cr,s}$  for  
effective area of stiffener  $A_s$  from step 1



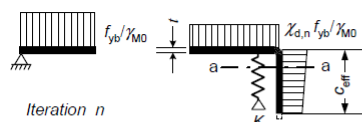
d) Reduced strength  $\chi_d f_{yb} / \gamma_{M0}$  for effective  
area of stiffener  $A_s$ , with reduction factor  $\chi_d$   
based on  $\sigma_{cr,s}$



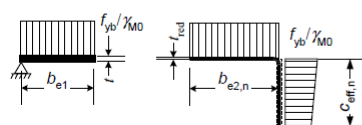
## 5. Eurocode 3

### ■ Larguras efetivas $\Rightarrow$ § 5.5.3 EC3-1-3

#### ✓ Elementos de placa com enrijecedores de borda - § 5.5.3 EC3-1-3



e) **Step 3:** Optionally repeat step 1 by calculating the effective width with a reduced compressive stress  $\sigma_{\text{com.Edi}} = \chi_a f_{yb} / \gamma_{M0}$  with  $\chi_a$  from previous iteration, continuing until  $\chi_{dn} \approx \chi_{d(n-1)}$  but  $\chi_{dn} \leq \chi_{d(n-1)}$ .



f) Adopt an effective cross-section with  $b_{e2}$ ,  $c_{\text{eff}}$  and reduced thickness  $t_{\text{red}}$  corresponding to  $\chi_{dn}$



## 5. Eurocode 3

### ■ Larguras efetivas $\Rightarrow$ § 5.5.3 EC3-1-3

#### ✓ Elementos de placa com enrijecedores de borda - § 5.5.3 EC3-1-3

(5) Initial values of the effective widths  $c_{\text{eff}}$  and  $d_{\text{eff}}$  shown in figure 5.9 should be obtained as follows:

a) for a single edge fold stiffener:

$$c_{\text{eff}} = \rho b_{p,c} \quad \dots (5.13a)$$

with  $\rho$  obtained from 5.5.2, except using a value of the buckling factor  $k_{\sigma}$  given by the following:

- if  $b_{p,c}/b_p \leq 0,35$ :

$$k_{\sigma} = 0,5 \quad \dots (5.13b)$$

- if  $0,35 < b_{p,c}/b_p \leq 0,6$ :

$$k_{\sigma} = 0,5 + 0,83 \sqrt[3]{(b_{p,c}/b_p - 0,35)^2} \quad \dots (5.13c)$$





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## 5. Eurocode 3

- **Larguras efetivas** ⇒ § 5.5.3 EC3-1-3
  - ✓ **Elementos de placa com enrijecedores de borda** - § 5.5.3 EC3-1-3

b) for a double edge fold stiffener:

$$c_{eff} = \rho b_{p,c} \quad \dots (5.13d)$$

with  $\rho$  obtained from 5.5.2 with a buckling factor  $k_\sigma$  for a doubly supported element from table 5.3;

$$d_{eff} = \rho b_{p,d} \quad \dots (5.13e)$$

with  $\rho$  obtained from 5.5.2 with a buckling factor  $k_\sigma$  for an outstand element from table 5.4.

(6) The effective cross-sectional area of the edge stiffener  $A_s$  should be obtained from:

$$A_s = t(b_{e2} + c_{eff}) \quad \text{or} \quad \dots (5.14a)$$

$$A_s = t(b_{e2} + c_{e1} + c_{e2} + d_{eff}) \quad \dots (5.14b)$$

respectively.

NOTE: The rounded comers should be taken into account if needed, see 5.1.

PGECIV





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## 5. Eurocode 3

- **Larguras efetivas** ⇒ § 5.5.3 EC3-1-3
  - ✓ **Elementos de placa com enrijecedores de borda** - § 5.5.3 EC3-1-3

(7) The elastic critical buckling stress  $\sigma_{cr,s}$  for an edge stiffener should be obtained from:

$$\sigma_{cr,s} = \frac{2 \sqrt{K E I_s}}{A_s} \quad \dots (5.15)$$

where:

$K$  is the spring stiffness per unit length, see 5.5.3.1(2).

$I_s$  is the effective second moment of area of the stiffener, taken as that of its effective area  $A_s$  about the centroidal axis  $a - a$  of its effective cross-section, see figure 5.9.

(8) Alternatively, the elastic critical buckling stress  $\sigma_{cr,s}$  may be obtained from elastic first order buckling analyses using numerical methods, see 5.5.1(8).

(9) The reduction factor  $\chi_d$  for the distortional buckling (flexural buckling of a stiffener) resistance of an edge stiffener should be obtained from the value of  $\sigma_{cr,s}$  using the method given in 5.5.3.1(7).

(10) If  $\chi_d < 1$  it may be refined iteratively, starting the iteration with modified values of  $\rho$  obtained using 5.5.2(5) with  $\sigma_{com,Ed,i}$  equal to  $\chi_d f_{yb} / \gamma_{M0}$ , so that:

$$\bar{\lambda}_{p,red} = \bar{\lambda}_p \sqrt{\chi_d} \quad \dots (5.16)$$

PGECIV



## 5. Eurocode 3

- Larguras efetivas  $\Rightarrow$  § 5.5.3 EC3-1-3
  - ✓ Elementos de placa com enrijecedores de borda - § 5.5.3 EC3-1-3

(11) The reduced effective area of the stiffener  $A_{s,red}$  allowing for flexural buckling should be taken as:

$$A_{s,red} = \chi_d A_s \frac{f_{yb} / \gamma_{M0}}{\sigma_{com,Ed}} \quad \text{but } A_{s,red} \leq A_s \quad \dots (5.17)$$

where

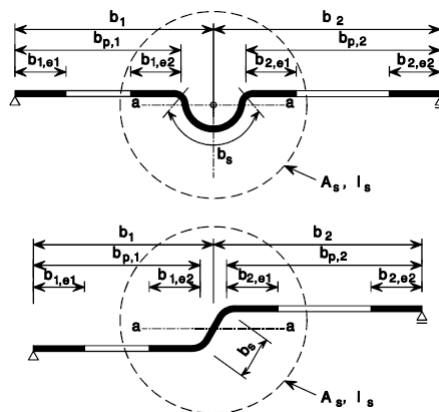
$\sigma_{com,Ed}$  is compressive stress at the centreline of the stiffener calculated on the basis of the effective cross-section.

(12) In determining effective section properties, the reduced effective area  $A_{s,red}$  should be represented by using a reduced thickness  $t_{red} = t A_{s,red} / A_s$  for all the elements included in  $A_s$ .



## 5. Eurocode 3

- Larguras efetivas  $\Rightarrow$  § 5.5.3 EC3-1-3
  - ✓ Elementos de placa c/ enrijecedores intermediários - § 5.5.3.3 EC3-1-3





## 5. Eurocode 3

### ■ Larguras efetivas $\Rightarrow$ § 5.5.3 EC3-1-3

#### ✓ Elementos de placa c/ enrijecedores intermediários - § 5.5.3.3 EC3-1-3

(2) The cross-section of an intermediate stiffener should be taken as comprising the stiffener itself plus the adjacent effective portions of the adjacent plane elements  $b_{p,1}$  and  $b_{p,2}$  shown in figure 5.11.

(3) The procedure, which is illustrated in figure 5.12, should be carried out in steps as follows:

- **Step 1:** Obtain an initial effective cross-section for the stiffener using effective widths determined by assuming that the stiffener gives full restraint and that  $\sigma_{com,Ed} = f_{yb}/\gamma_{M0}$ , see (4) and (5);
- **Step 2:** Use the initial effective cross-section of the stiffener to determine the reduction factor for distortional buckling (flexural buckling of an intermediate stiffener), allowing for the effects of the continuous spring restraint, see (6), (7) and (8);
- **Step 3:** Optionally iterate to refine the value of the reduction factor for buckling of the stiffener, see (9) and (10).

(4) Initial values of the effective widths  $b_{1,e2}$  and  $b_{2,e1}$  shown in figure 5.11 should be determined from 5.5.2 by assuming that the plane elements  $b_{p,1}$  and  $b_{p,2}$  are doubly supported, see table 5.3.

(5) The effective cross-sectional area of an intermediate stiffener  $A_s$  should be obtained from:

$$A_s = t(b_{1,e2} + b_{2,e1} + b_s) \quad \dots (5.18)$$

in which the stiffener width  $b_s$  is as shown in figure 5.11.

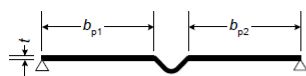
**NOTE:** The rounded corners should be taken into account if needed, see 5.1.



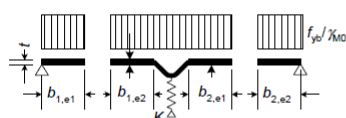
## 5. Eurocode 3

### ■ Larguras efetivas $\Rightarrow$ § 5.5.3 EC3-1-3

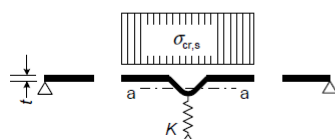
#### ✓ Elementos de placa c/ enrijecedores intermediários - § 5.5.3.3 EC3-1-3



a) Gross cross-section and boundary conditions



b) **Step 1:** Effective cross-section for  $K = \infty$  based on  $\sigma_{com,Ed} = f_{yb}/\gamma_{M0}$



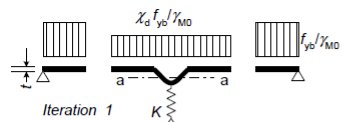
c) **Step 2:** Elastic critical stress  $\sigma_{cr,s}$  for effective area of stiffener  $A_s$  from step 1



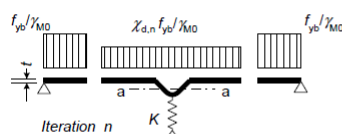
## 5. Eurocode 3

### ■ Larguras efetivas ⇒ § 5.5.3 EC3-1-3

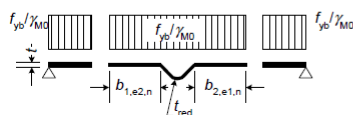
#### ✓ Elementos de placa c/ enrijecedores intermediários - § 5.5.3.3 EC3-1-3



d) Reduced strength  $\chi_d f_{yb} / \gamma_{M0}$  for effective area of stiffener  $A_s$ , with reduction factor  $\chi_d$  based on  $\sigma_{cr,s}$



e) **Step 3:** Optionally repeat step 1 by calculating the effective width with a reduced compressive stress  $\sigma_{com,Edi} = \chi_d f_{yb} / \gamma_{M0}$  with  $\chi_d$  from previous iteration, continuing until  $\chi_{d,n} \approx \chi_{d,(n-1)}$  but  $\chi_{d,n} \leq \chi_{d,(n-1)}$ .



f) Adopt an effective cross-section with  $b_{1,e2}$ ,  $b_{2,e1}$  and reduced thickness  $t_{red}$  corresponding to  $\chi_{d,n}$



## 5. Eurocode 3

### ■ Larguras efetivas ⇒ § 5.5.3 EC3-1-3

#### ✓ Elementos de placa c/ enrijecedores intermediários - § 5.5.3.3 EC3-1-3

(6) The critical buckling stress  $\sigma_{cr,s}$  for an intermediate stiffener should be obtained from:

$$\sigma_{cr,s} = \frac{2\sqrt{KEI_s}}{A_s} \quad \dots (5.19)$$

where:

$K$  is the spring stiffness per unit length, see 5.5.3.1(2).

$I_s$  is the effective second moment of area of the stiffener, taken as that of its effective area  $A_s$  about the centroidal axis  $a-a$  of its effective cross-section, see figure 5.11.

(7) Alternatively, the elastic critical buckling stress  $\sigma_{cr,s}$  may be obtained from elastic first order buckling analyses using numerical methods, see 5.5.1(11).

(8) The reduction factor  $\chi_d$  for the distortional buckling resistance (flexural buckling of an intermediate stiffener) should be obtained from the value of  $\sigma_{cr,s}$  using the method given in 5.5.3.1(7).

(9) If  $\chi_d < 1$  it may optionally be refined iteratively, starting the iteration with modified values of  $\rho$  obtained using 5.5.2(5) with  $\sigma_{com,Edi}$  equal to  $\chi_d f_{yb} / \gamma_{M0}$ , so that:

$$\bar{\lambda}_{p,red} = \bar{\lambda}_p \sqrt{\chi_d} \quad \dots (5.20)$$



## 5. Eurocode 3

### ■ Larguras efetivas $\Rightarrow$ § 5.5.3 EC3-1-3

#### ✓ Elementos de placa c/ enrijecedores intermediários - § 5.5.3.3 EC3-1-3

(10) The reduced effective area of the stiffener  $A_{s,red}$  allowing for distortional buckling (flexural buckling of a stiffener) should be taken as:

$$A_{s,red} = \chi_d A_s \frac{f_{yb} / \gamma_{M0}}{\sigma_{com,Ed}} \quad \text{but } A_{s,red} \leq A_s \quad \dots (5.21)$$

where

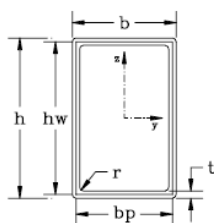
$\sigma_{com,Ed}$  is compressive stress at the centreline of the stiffener calculated on the basis of the effective cross-section.

(11) In determining effective section properties, the reduced effective area  $A_{s,red}$  should be represented by using a reduced thickness  $t_{red} = t A_{s,red} / A_s$  for all the elements included in  $A_s$ .

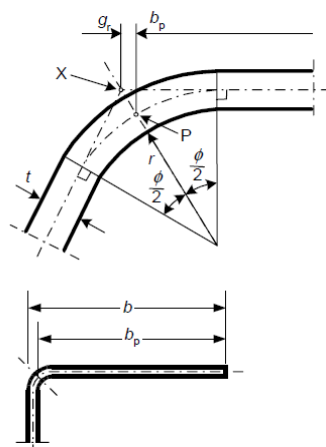


## 6. Exemplo 1

### ■ Avaliação de Propriedades Geométricas Efetivas de PFF



|       |         |                 |
|-------|---------|-----------------|
| $f_y$ | 350     | MPa             |
| $b$   | 50      | mm              |
| $h$   | 150     | mm              |
| $t$   | 1       | mm              |
| $A$   | 392     | mm <sup>2</sup> |
| $I_y$ | 1070900 | mm <sup>4</sup> |
| $I_z$ | 195900  | mm <sup>4</sup> |





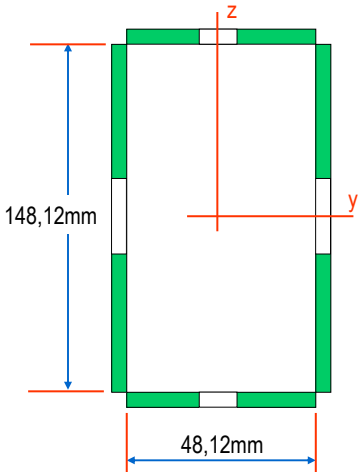


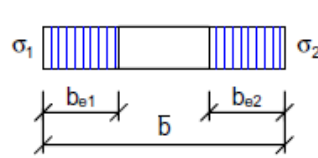


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## 6. Exemplo 1

■ Compressão





$\sigma_1$   $\sigma_2$

$b_{e1}$   $b$   $b_{e2}$

$\psi = 1$ :

$b_{eff} = \rho \bar{b}$

$b_{e1} = 0,5 b_{eff}$        $b_{e2} = 0,5 b_{eff}$

|                            |     |
|----------------------------|-----|
| $\psi = \sigma_2/\sigma_1$ | 1   |
| Buckling factor $k_\sigma$ | 4,0 |

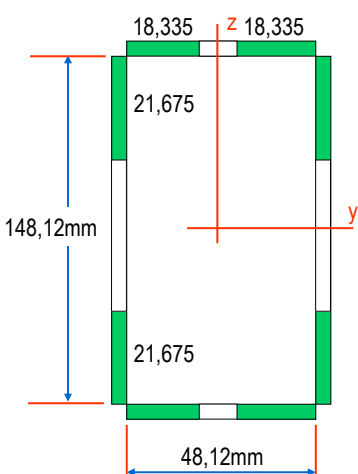




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## 6. Exemplo 1

■ Compressão



**Almas**

$$\bar{\lambda}_p = \sqrt{\frac{f_y}{\sigma_{cr}}} = \frac{\bar{b}/t}{28,4 \cdot \varepsilon \cdot \sqrt{k_\sigma}} = \frac{148,12/1}{28,4 \cdot \sqrt{\frac{235}{f_y}} \cdot \sqrt{4}} = 3,18$$

$$\rho = \frac{\bar{\lambda}_p - 0,055 \cdot (3 + \psi)}{\bar{\lambda}_p^2} = 0,293$$

$$h_{eff} = \rho \cdot h_w = 0,293 \cdot 148,12 = 43,35 \text{ mm}$$

**Mesas**

$$\bar{\lambda}_p = \sqrt{\frac{f_y}{\sigma_{cr}}} = \frac{\bar{b}/t}{28,4 \cdot \varepsilon \cdot \sqrt{k_\sigma}} = \frac{48,12/1}{28,4 \cdot \sqrt{\frac{235}{f_y}} \cdot \sqrt{4}} = 1,033$$

$$\rho = \frac{\bar{\lambda}_p - 0,055 \cdot (3 + \psi)}{\bar{\lambda}_p^2} = 0,762$$

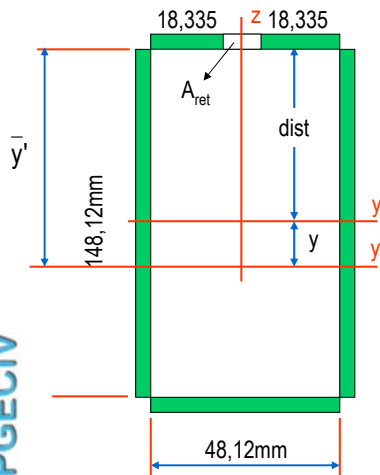
$$b_{eff} = \rho \cdot b_p = 0,762 \cdot 48,12 = 36,67 \text{ mm}$$

**Área Efetiva**       $A_{eff} = 2 \cdot 43,35 + 2 \cdot 36,67 = 160 \text{ mm}^2$



## 6. Exemplo 1

- Flexão em torno de y



### Mesas

$$\bar{\lambda}_p = \sqrt{\frac{f_y}{\sigma_{cr}}} = \frac{\bar{b}/t}{28,4 \cdot \epsilon \cdot \sqrt{k_\sigma}} = \frac{48,12/1}{28,4 \cdot \sqrt{\frac{235}{f_y}} \cdot \sqrt{4}} = 1,033$$

$$\rho = \frac{\bar{\lambda}_p - 0,055 \cdot (3 + \psi)}{\bar{\lambda}_p^2} = 0,762$$

$$b_{eff} = \rho \cdot b_p = 0,762 \cdot 48,12 = 36,67 \text{ mm}$$

$$A_{eff'} = A - A_{ret} = 392,48 - (48,12 - 36,67) = 381,03 \text{ mm}$$

### Rebaixamento do centróide

$$\frac{A_{ret}}{A_{eff'}} = \frac{y}{dist} \Rightarrow \frac{(48,12 - 36,67)}{381,03} = \frac{y}{74,06} \Rightarrow$$

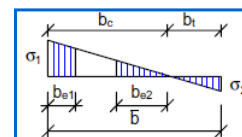
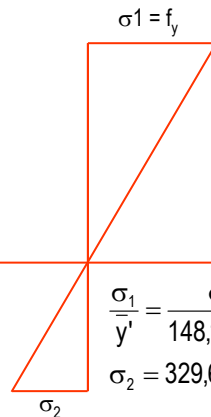
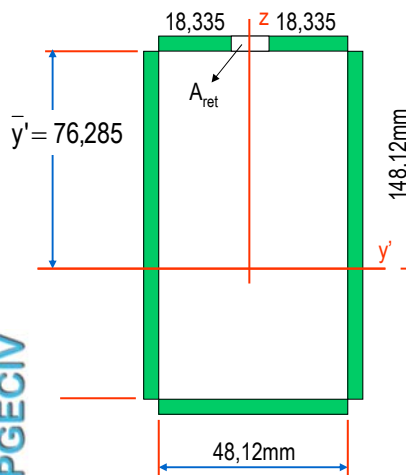
$$y = 2,225 \text{ mm}$$

$$\bar{y}' = 74,06 + 2,225 = 76,285 \text{ mm}$$



## 6. Exemplo 1

- Flexão em torno de y



$$\psi \leq 0:$$

$$b_{eff} = \rho \cdot b_c = \rho \cdot \bar{b} / (1 - \psi)$$

$$b_{e1} = 0,4 \cdot b_{eff} \quad b_{e2} = 0,6 \cdot b_{eff}$$

$$\frac{\sigma_1}{\bar{y}'} = \frac{\sigma_2}{148,12 - \bar{y}'} \Rightarrow \frac{350}{76,285} = \frac{\sigma_2}{71,835} \Rightarrow$$

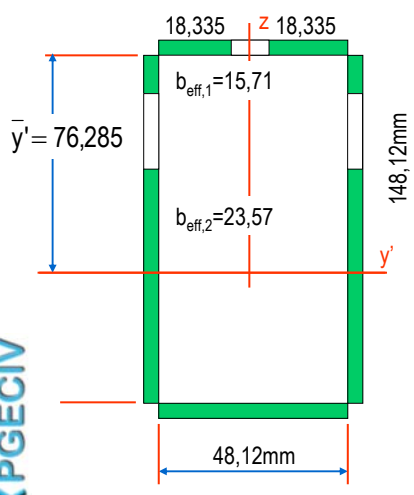
$$\sigma_2 = 329,6 \text{ MPa}$$

$$\psi = \frac{\sigma_2}{\sigma_1} = \frac{329,6}{-350} = -0,942$$



## 6. Exemplo 1

▪ Flexão em torno de y





$$k_{\sigma} = 7,81 - 6,29\psi + 9,78\psi^2$$

$$k_{\sigma} = 7,81 - 6,29(-0,942) + 9,78(-0,942)^2$$

$$k_{\sigma} = 22,41$$

$$\bar{\lambda}_p = \frac{\sqrt{f_y}}{\sqrt{\sigma_{cr}}} = \frac{\bar{b}/t}{28,4 \cdot \epsilon \cdot \sqrt{k_{\sigma}}} = \frac{148,12/1}{28,4 \cdot \sqrt{235} \cdot \sqrt{22,41}} = 1,344$$

$$\rho = \frac{\bar{\lambda}_p - 0,055(3 + \psi)}{\bar{\lambda}_p^2} = 0,612$$

$$b_{eff} = \frac{\rho \cdot \bar{b}}{1 - \psi} = \frac{0,612 \cdot 148,12}{1 - (-0,942)} = 46,68\text{mm}$$

$$b_{eff,1} = 0,4 \cdot b_{eff} = 18,67\text{mm}$$

$$b_{eff,2} = 0,6 \cdot b_{eff} = 28,00\text{mm}$$

$$k_{\sigma} = 7,81 - 6,29\psi + 9,78\psi^2$$

$$k_{\sigma} = 7,81 - 6,29(-0,942) + 9,78(-0,942)^2$$

$$k_{\sigma} = 22,41$$

$$\bar{\lambda}_p = \frac{\sqrt{f_y}}{\sqrt{\sigma_{cr}}} = \frac{\bar{b}/t}{28,4 \cdot \epsilon \cdot \sqrt{k_{\sigma}}} = \frac{148,12/1}{28,4 \cdot \sqrt{235} \cdot \sqrt{22,41}} = 1,344$$

$$\rho = \frac{\bar{\lambda}_p - 0,055(3 + \psi)}{\bar{\lambda}_p^2} = 0,612$$

$$b_{eff} = \frac{\rho \cdot \bar{b}}{1 - \psi} = \frac{0,612 \cdot 148,12}{1 - (-0,942)} = 46,68\text{mm}$$

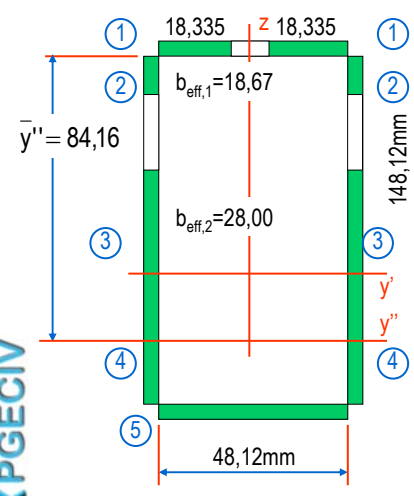
$$b_{eff,1} = 0,4 \cdot b_{eff} = 18,67\text{mm}$$

$$b_{eff,2} = 0,6 \cdot b_{eff} = 28,00\text{mm}$$



## 6. Exemplo 1

▪ Flexão em torno de y





$$\bar{y}'' = 84,16$$

Novo centróide

| ID | Área  | y      | Qtde. | A . y    |
|----|-------|--------|-------|----------|
| 1  | 18,34 | 148,12 | 2     | 5433,0   |
| 2  | 18,67 | 138,79 | 2     | 5182,4   |
| 3  | 28,00 | 85,84  | 2     | 4807,0   |
| 4  | 71,84 | 35,92  | 2     | 5161,0   |
| 5  | 48,12 | 0      | 1     | 0        |
| Σ  | 321,8 | -      | -     | 20583,49 |

$$\bar{y}'' = 148,12 - \frac{20583,49}{321,8} = 84,16\text{mm}$$

**6. Exemplo 1**

▪ Flexão em torno de y

Diagram showing the channel section dimensions and effective flange widths:

- Top flange width: 48,12mm
- Web height: 148,12mm
- Vertical offset: 63,96mm
- Distances from y'' axis to flange centerlines: 18,335mm
- Effective flange widths:  $b_{eff,1} = 15,71$ mm,  $b_{eff,2} = 23,57$ mm

Stress distribution diagram shows  $\sigma_1 = f_y$  and  $\sigma_2$  at the extreme fibers.

Calculation for  $\psi < 0$ :

$$b_{eff} = \rho \cdot b_c = \rho \cdot \bar{b} / (1 - \psi)$$

$$b_{e1} = 0,4 \cdot b_{eff} \quad b_{e2} = 0,6 \cdot b_{eff}$$

Stress ratio calculation:

$$\frac{\sigma_1}{y''} = \frac{\sigma_2}{148,12 - y''} \Rightarrow \frac{350}{84,16} = \frac{\sigma_2}{63,96} \Rightarrow \sigma_2 = 266,0 \text{MPa}$$

Warping constant calculation:

$$\psi = \frac{\sigma_2}{\sigma_1} = \frac{266}{-350} = -0,76$$

**6. Exemplo 1**

▪ Flexão em torno de y

Diagram showing the channel section dimensions and effective flange widths:

- Top flange width: 48,12mm
- Web height: 148,12mm
- Vertical offset: 84,16mm
- Distances from y'' axis to flange centerlines: 18,335mm
- Effective flange widths:  $b_{eff,1} = 18,85$ mm,  $b_{eff,2} = 28,29$ mm

Calculation for  $k_\sigma$ :

$$k_\sigma = 7,81 - 6,29\psi + 9,78\psi^2$$

$$k_\sigma = 7,81 - 6,29(-0,76) + 9,78(-0,76)^2$$

$$k_\sigma = 18,23$$

Calculation for  $\bar{\lambda}_p$ :

$$\bar{\lambda}_p = \sqrt{\frac{f_y}{\sigma_\sigma}} = \frac{\bar{b}/t}{28,4 \cdot \varepsilon \cdot \sqrt{k_\sigma}} = \frac{148,12/1}{28,4 \cdot \sqrt{235} \cdot \sqrt{18,23}} = 1,49$$

Calculation for  $\rho$ :

$$\rho = \frac{\bar{\lambda}_p - 0,055 \cdot (3 + \psi)}{\bar{\lambda}_p^2} = 0,56$$

Calculation for  $b_{eff}$ :

$$b_{eff} = \frac{\rho \cdot \bar{b}}{1 - \psi} = \frac{0,56 \cdot 148,12}{1 - (-0,76)} = 47,13 \text{mm}$$

Calculation for  $b_{eff,1}$  and  $b_{eff,2}$ :

$$b_{eff,1} = 0,4 \cdot b_{eff} = 18,85 \text{mm}$$

$$b_{eff,2} = 0,6 \cdot b_{eff} = 28,29 \text{mm}$$

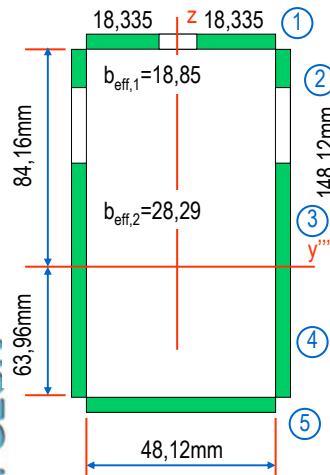
Calculation for the ratio of actual to anterior effective widths:

$$\frac{(b_{eff,1} + b_{eff,2})_{atual}}{(b_{eff,1} + b_{eff,2})_{anterior}} = \frac{18,85 + 28,29}{18,67 + 28} = 1,01 = 1\% \quad \checkmark$$



## 6. Exemplo 1

### Flexão em torno de y



| ID       | Qtde. | $I_0$          | Área  | d     | Qtde( $I_0 + A \cdot d^2$ ) |
|----------|-------|----------------|-------|-------|-----------------------------|
| 1        | 2     | -              | 18,34 | 84,16 | 259800.98                   |
| 2        | 2     | $(18,85)^3/12$ | 18,85 | 74,73 | 211654.70                   |
| 3        | 2     | $(28,29)^3/12$ | 28,29 | 14,5  | 15669.47                    |
| 4        | 2     | $(63,96)^3/12$ | 63,96 | 31,98 | 174435.19                   |
| 5        | 1     | -              | 48,12 | 62,41 | 187427.79                   |
| $\Sigma$ | -     |                | 307   | -     | 848988.13                   |

$$I_{\text{eff},y} = 848988,13 \text{ mm}^4$$

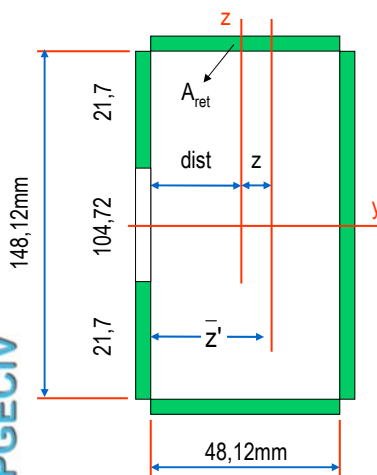
$$W_{\text{eff},y}^{\text{comp}} = \frac{848988,13}{84,16} = 10087,8 \text{ mm}^3$$

$$W_{\text{eff},y}^{\text{tração}} = \frac{848988,13}{148,12 - 84,16} = 13273,7 \text{ mm}^3$$



## 6. Exemplo 1

### Flexão em torno de z



#### Mesas

$$\bar{\lambda}_p = \sqrt{\frac{f_y}{\sigma_{cr}}} = \frac{\bar{b}/t}{28,4 \cdot \varepsilon \cdot \sqrt{k_\sigma}} = \frac{148,12/1}{28,4 \cdot \sqrt{\frac{235}{f_y}} \cdot \sqrt{4}} = 3,182$$

$$\rho = \frac{\bar{\lambda}_p - 0,055 \cdot (3 + \psi)}{\bar{\lambda}_p^2} = 0,293$$

$$b_{\text{eff}} = \rho \cdot b_p = 0,293 \cdot 148,12 = 43,40 \text{ mm}$$

$$A_{\text{eff}'} = A - A_{\text{ret}} = 392,48 - (104,72) = 287,76 \text{ mm}$$

#### Rebaixamento do centróide

$$\frac{A_{\text{ret}}}{A_{\text{eff}'}} = \frac{z}{\text{dist}} \Rightarrow \frac{104,72}{287,76} = \frac{z}{24,06} \Rightarrow$$

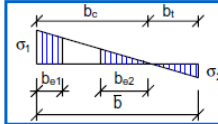
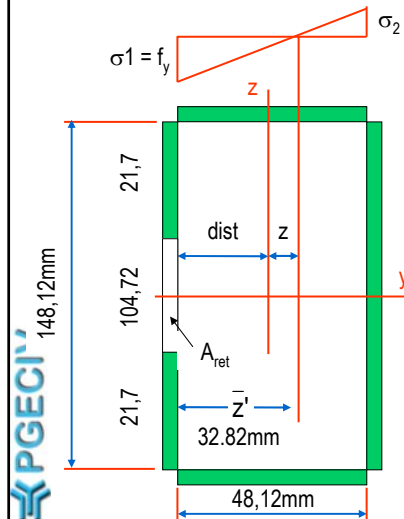
$$z = 8,76 \text{ mm}$$

$$\bar{z}' = 24,06 + 8,76 = 32,82 \text{ mm}$$



## 6. Exemplo 1

- Flexão em torno de z



$$\psi < 0:$$

$$b_{eff} = \rho b_e = \rho \bar{b} / (1 - \psi)$$

$$b_{e1} = 0,4 b_{eff} \quad b_{e2} = 0,6 b_{eff}$$

$$\frac{\sigma_1}{z'} = \frac{\sigma_2}{48,12 - z'} \Rightarrow \frac{350}{32,82} = \frac{\sigma_2}{15,3} \Rightarrow$$

$$\sigma_2 = 163,16 \text{ MPa}$$

$$\psi = \frac{\sigma_2}{\sigma_1} = \frac{163,16}{-350} = -0,466$$

$$k_\sigma = 7,81 - 6,29\psi + 9,78\psi^2$$

$$k_\sigma = 7,81 - 6,29(-0,466) + 9,78(-0,466)^2$$

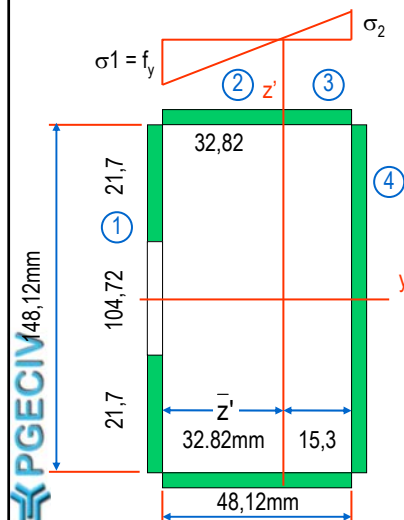
$$k_\sigma = 12,86$$

$$\bar{\lambda}_p = \sqrt{\frac{f_y}{\sigma_{cr}}} = \frac{\bar{b}/t}{28,4 \cdot \varepsilon \cdot \sqrt{k_\sigma}} = \frac{48,12/1}{28,4 \cdot \sqrt{\frac{235}{f_y} \cdot \sqrt{12,86}}} = 0,577$$



## 6. Exemplo 1

- Flexão em torno de z



$$\rho = \frac{\bar{\lambda}_p - 0,055(3 + \psi)}{\bar{\lambda}_{p,2}} = 1,142 > 1 \Rightarrow \rho = 1$$

$$b_{eff} = \frac{\rho \bar{b}}{1 - \psi} = \frac{1 \cdot 48,12}{1 - (-0,466)} = 32,82 \text{ mm}$$

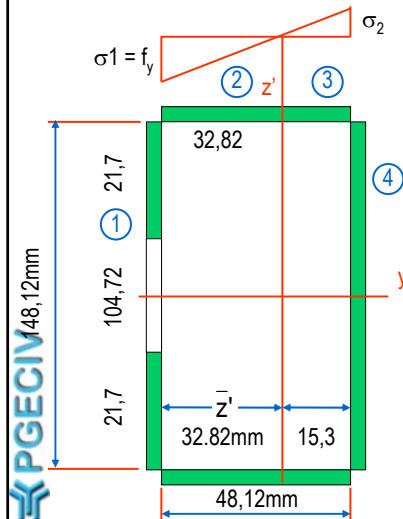
(toda a alma é efetiva)

| ID       | Qtde. | $I_0$          | Área   | d     | Qtde( $I_0 + A \cdot d^2$ ) |
|----------|-------|----------------|--------|-------|-----------------------------|
| 1        | 2     | -              | 21,7   | 32,82 | 46748,41                    |
| 2        | 2     | $(32,82)^3/12$ | 16,41  | 29,52 | 23568,09                    |
| 3        | 2     | $(15,3)^3/12$  | 15,3   | 7,65  | 2387,72                     |
| 4        | 1     | -              | 148,12 | 15,3  | 34673,41                    |
| $\Sigma$ | -     |                | 287,8  | -     | 107377,6                    |



## 6. Exemplo 1

- Flexão em torno de z



$$I_{\text{eff},z} = 107377,63 \text{ mm}^4$$

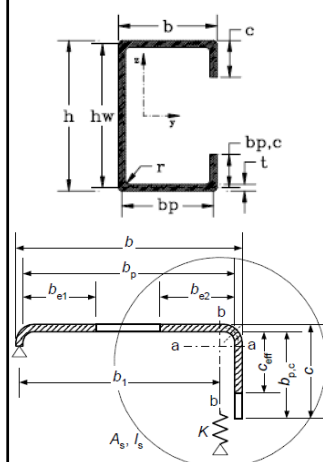
$$W_{\text{eff},z}^{\text{comp}} = \frac{107377,63}{32,82} = 3271,71 \text{ mm}^3$$

$$W_{\text{eff},z}^{\text{comp}} = \frac{107377,63}{15,3} = 7018,15 \text{ mm}^3$$



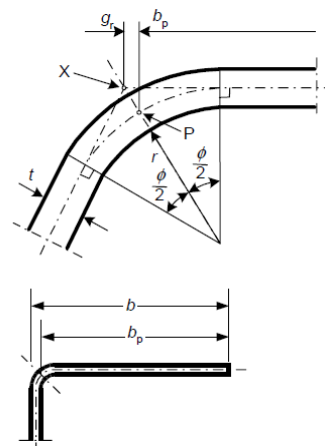
## 7. Exemplo 2

- Avaliação de Propriedades Geométricas Efetivas de PFF



|       |     |                 |
|-------|-----|-----------------|
| $f_y$ | 350 | MPa             |
| $b$   | 75  | mm              |
| $h$   | 200 | mm              |
| $t$   | 2   | mm              |
| $c$   | 20  | mm              |
| $A$   | 262 | mm <sup>2</sup> |

$$\varepsilon = \sqrt{235 / f_{yb}} \Rightarrow \varepsilon = 0.819$$



$b/t \leq 60$   
a) single edge fold



## 7. Exemplo 2

### Seção Real

Calculation thickness ( $t_{cor} = t$ )

Z275  $t_{coatings} = 0.04$  mm,  $t = t_n - 0.04$  mm

$t = 1.96$  mm

Corner radius (inner/outer)

$$r_i = 2 \cdot t_n$$

$$r_u = r_i + t_n$$

Average corner radius:

$$r_m = r_i + 0.5 \cdot t_n$$

$$r_m = 5$$
 mm

|           | External dimensions | Centreline dimensions $c$ | Interior dimensions $i$ |
|-----------|---------------------|---------------------------|-------------------------|
| Height    | $h = 200$ mm        | $h_c = h - t$             | $h_i = h - t - 2 r_m$   |
| Width     | $b = 75$ mm         | $b_c = b - t$             | $b_i = b - t - 2 r_m$   |
| Stiffener | $c = 20$ mm         | $c_c = c - t/2$           | $c_i = b - 0.5 t - r_m$ |

Influence of rounded corners (clause 5.1.4):

Rounding length:

$$u = 1.57 \cdot r_m = 7.85$$
 mm

Rounding gravity center:

$$w = 0.363 \cdot r_m = 1.815$$
 mm

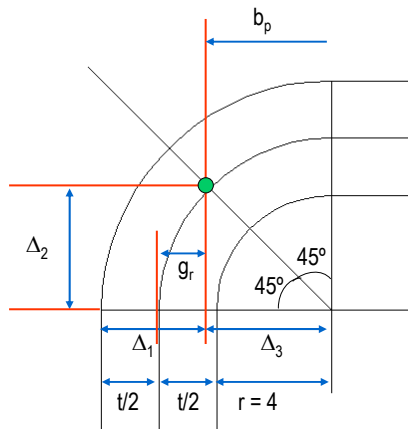
Notional dimensions:

$$\phi = 0.5 \cdot \pi \quad v = 0.637 \cdot r_m = 3.185$$
 mm



## 7. Exemplo 2

### Seção Real



$$\sin 45^\circ = \frac{\Delta_2}{r + t/2}$$

$$\Delta_2 = (4 + 1) \sin 45^\circ = 3,536 \text{ mm}$$

$$\tan 45^\circ = \frac{\Delta_2}{\Delta_3} \Rightarrow \Delta_3 = \frac{3,536}{\tan 45^\circ} = 3,536 \text{ mm}$$

$$\Delta_1 = (r + t) - \Delta_3 = 6 - 3,536 = 2,464 \text{ mm}$$

$$g_r = \Delta_1 - t/2 = 2,464 - 1 = 1,464 \text{ mm}$$

$$b_p = b_c - 2g_r = 73,04 - 2 \cdot 1,464 = 70,112 \text{ mm}$$

$$h_p = h_c - 2g_r = 198,04 - 2 \cdot 1,464 = 195,112 \text{ mm}$$

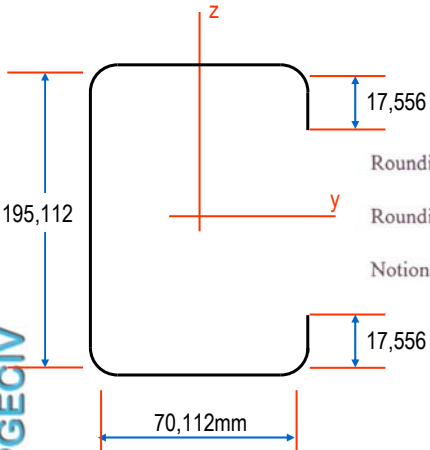
$$c_p = c_c - g_r = 19,02 - 1,464 = 17,556 \text{ mm}$$



## 7. Exemplo 2



▪ Seção Real



Rounding length:  $u = 1.57 \cdot r_m = 7.85 \text{ mm}$

Rounding gravity center:  $w = 0.363 \cdot r_m = 1.815 \text{ mm}$

Notional dimensions:  $\phi = 0.5 \cdot \pi$   $v = 0.637 \cdot r_m = 3.185 \text{ mm}$



## 7. Exemplo 2



▪ Seção Real

$r_i < 5 \cdot 2 \Rightarrow 4 < 10 \Rightarrow \text{OK}$   
 $r_i < 0,1 \cdot 17,556 \Rightarrow 4 < 1,7556 \Rightarrow \text{não OK!}$

Corners =  $\begin{cases} \text{"Sharp edges"} & \text{if } r_i < 5 \cdot t_n \wedge r_i < 0,1 \cdot \min(h_p, b_p, c_p) \\ \text{"Rounded corners"} & \text{otherwise} \end{cases}$

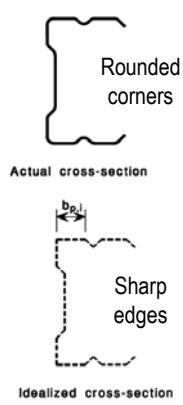
Corners = "Rounded corners"

$$\delta = 0,43 \frac{8 \cdot r_i}{2 \cdot h_c + 4 \cdot b_c + 4 \cdot c_c}$$

$\delta = \begin{cases} 0 & \text{if } r_i < 5 \cdot t_n \wedge r_i < 0,1 \cdot \min(h_p, b_p, c_p) \\ \delta & \text{otherwise} \end{cases}$

$\delta = 0,018$

página 19 – EC3 – 1.3





## 7. Exemplo 2

### Seção Real

página 22 – EC3 – 1.3

Maximum width-to-thickness ratios (clause 5.2, Table 5.1):

$$\begin{aligned} h/t_n &= 100.00 < 500 \\ b/t_n &= 37.50 < 60 \\ c/t_n &= 10.00 < 50 \\ c/b &= 0.27 & 0.2 \text{ to } 0.6 \end{aligned}$$

Ratios =  $\left\{ \begin{array}{ll} \text{"EN 1993-1-3 covers"} & \text{if } h/t \leq 500 \wedge b/t \leq 90 \wedge c/b \leq 0.6 \wedge \\ & c/b \geq 0.2 \wedge c/t_n \leq 50 \\ \text{"EN 1993-1-3 does not cover"} & \text{otherwise} \end{array} \right.$

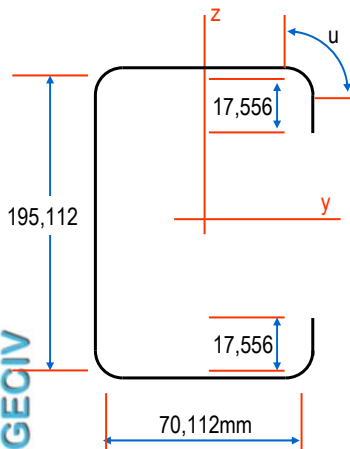
Ratios = "EN 1993-1-3 covers"



## 7. Exemplo 2

### Seção Real

Gross cross-sectional properties should be calculated with rounded corners:



$$A_g = (h_p + 2 \cdot b_p + 2 \cdot c_p + 4 \cdot u) \cdot t = 787.6 \text{ mm}^2$$

$$I_{yg} = h_p^3 \frac{t}{12} + 2 \left[ \frac{b_p \cdot t^3}{12} + b_p \cdot t \left( \frac{h_p}{2} \right)^2 \right] + 2 \left[ \frac{c_p^3 \cdot t}{12} + c_p \cdot t \left( \frac{h_p}{2} - c_p + \frac{c_p}{2} \right)^2 \right] + 4 \cdot u \cdot t \left( \frac{h_p}{2} - w \right)^2$$

$$I_{yg} = 5.034 \cdot 10^6 \text{ mm}^4$$

$$W'_{ef} = I_{yg} \left( \frac{h_p}{2} \right)^{-1} = 5.083 \cdot 10^4 \text{ mm}^4$$



## 7. Exemplo 2

### ■ Funções Gerais

Slenderness ratio

$$\bar{\lambda}_p(w, k_\sigma) = \frac{w}{t} \frac{1}{28.4 \cdot \varepsilon \cdot \sqrt{k_\sigma}}$$

Reduction factor for doubly supported element

$$\rho_\chi(\bar{\lambda}_p, \psi) = \begin{cases} \min \left[ \frac{\bar{\lambda}_p - 0.055(3 + \psi)}{\bar{\lambda}_p^2}, 1 \right] & \text{if } \bar{\lambda}_p \geq 0.673 \\ 1 & \text{otherwise} \end{cases}$$

$$\rho_\phi(\bar{\lambda}_p) = \begin{cases} \min \left[ \left( 1 - \frac{0.188}{\bar{\lambda}_p} \right) \frac{1}{\bar{\lambda}_p}, 1 \right] & \text{if } \bar{\lambda}_p \geq 0.748 \\ 1 & \text{otherwise} \end{cases}$$

Buckling curve for distortional buckling

$$\chi_{rd}(\bar{\lambda}_d) = \begin{cases} 1 & \text{if } \bar{\lambda}_d \leq 0.65 \\ 1.47 - 0.723 \bar{\lambda}_d & \text{if } \bar{\lambda}_d > 0.65 \wedge \bar{\lambda}_d < 1.38 \\ 0.66 / \bar{\lambda}_d & \text{if } \bar{\lambda}_d \geq 1.38 \end{cases}$$



## 7. Exemplo 2

### ■ Compressão

Compression stress (clause 5.5.3.2(3)):

$$\sigma_{comp, Ed} = f_{yk} / \gamma_{M0} = 350 \text{ N/mm}^2$$

Clause 5.5.3.2 (3) Step 1:

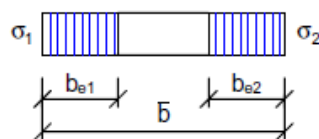
Wely with width  $b$ :

$$\psi = 1 \quad k_\sigma = 4$$

$$\bar{\lambda}_{pb} = \frac{b_p}{t} \frac{1}{28.4 \cdot \varepsilon \cdot \sqrt{k_\sigma}} = 2.139$$

$$\rho = \min \left[ \frac{\bar{\lambda}_{pb} - 0.055(3 + \psi)}{\bar{\lambda}_{pb}^2}, 1 \right] = 0.419$$

$$b_{ef} = 0.5 \cdot \rho \cdot b_p + g_r = 42.384 \text{ mm}$$



$$\psi = 1:$$

$$b_{eff} = \rho \cdot b$$

$$b_{e1} = 0.5 b_{eff} \quad b_{e2} = 0.5 b_{eff}$$

$$\frac{\psi = \sigma_2 / \sigma_1}{\text{Buckling factor } k_\sigma} \quad \frac{1}{4.0}$$



## 7. Exemplo 2

### ■ Compressão

Flange with width  $b$ :

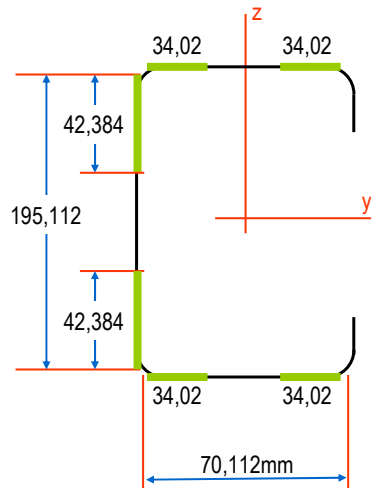
$$\psi = 1 \quad k_{\sigma} = 4$$

$$\bar{\lambda}_{pb} = \frac{b_p}{t} \frac{1}{28.4 \cdot \varepsilon \cdot \sqrt{k_{\sigma}}} = 0.769$$

$$\rho = \min \left[ \frac{\bar{\lambda}_{pb} - 0.055(3 + \psi)}{\bar{\lambda}_{pb}^2}, 1 \right] = 0.929$$

$$b_{ef1} = 0.5 \cdot \rho \cdot b_p + g_r = 34.02 \text{ mm}$$

$$b_{ef1} = b_{ef2}$$



## 7. Exemplo 2

página 30 – EC3 – 1.3

### ■ Compressão

Edge stiffener with width  $c$

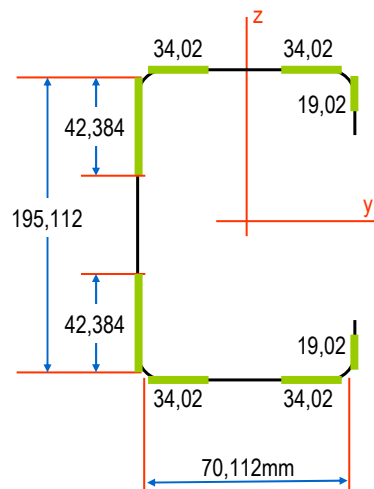
$$k_{\sigma c} = \begin{cases} 0.5 & \text{if } \frac{c_p}{b_p} \leq 0.35 \\ 0.5 + 0.83 \sqrt{\left( \frac{c_p}{b_p} - 0.35 \right)^2} & \text{if } \frac{c_p}{b_p} \leq 0.6 \wedge \frac{c_p}{b_p} > 0.35 \end{cases}$$

$$k_{\sigma c} = 0.5$$

$$\bar{\lambda}_{pc} = \frac{c_p}{t} \frac{1}{28.4 \cdot \varepsilon \cdot \sqrt{k_{\sigma c}}} = 0.544$$

$$\rho = \min \left[ \left( 1 - \frac{0.188}{\bar{\lambda}_{pc}} \right) \frac{1}{\bar{\lambda}_{pc}}, 1 \right] = 1.00$$

$$c_{ef} = 0.5 \cdot \rho \cdot c_p + g_r = 19.02 \text{ mm}$$



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página 30 – EC3 – 1.3

## 7. Exemplo 2

■ Compressão

Clause 5.5.3.2 (3) Step 2:  
Effective area of edge stiffener

Cross-sectional properties of the edge stiffener (Sharp corner):

$$A_s = (b_{ef2} + c_{ef}) \cdot t = 103.985 \text{ mm}^2$$

$$y_s = \frac{t \cdot c_{ef}^2}{2 \cdot A_s} = 3.41 \text{ mm}$$

$$z_s = \frac{t \cdot b_{ef2}^2}{2 \cdot A_s} = 10.91 \text{ mm}$$

$$I_s = \frac{1}{12} b_{ef2} \cdot t^3 + b_{ef2} \cdot t \cdot y_s^2 + \frac{1}{12} t \cdot c_{ef}^3 + c_{ef} \cdot t (0.5 \cdot c_{ef} - y_s)^2$$

$$I_s = 3.308 \cdot 10^3 \text{ mm}^4$$

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## 7. Exemplo 2

■ Compressão

Cross-sectional properties of the edge stiffener (Rounded corner):

$$A_s = [(b_{ef2} - g_r) + (c_{ef} - g_r) + u] \cdot t = 113.603 \text{ mm}^2$$

$$y_s = \frac{t (c_{ef} - g_r) \left( g_r + \frac{c_{ef} - g_r}{2} \right) + t \cdot u \cdot w}{A_s} = 3.348 \text{ mm}$$

$$z_s = \frac{t (b_{ef2} - g_r) \left( g_r + \frac{b_{ef2} - g_r}{2} \right) + t \cdot u \cdot w}{A_s} = 10.211 \text{ mm}$$

$$I_s = \frac{1}{12} (b_{ef2} - g_r) \cdot t^3 + (b_{ef2} - g_r) \cdot t \cdot y_s^2 + \frac{1}{12} t \cdot (c_{ef} - g_r)^3 + \dots$$

$$\dots + (c_{ef} - g_r) \cdot t \left( g_r + \frac{c_{ef} - g_r}{2} - y_s \right)^2 + u \cdot t (y_s - w)^2 + \frac{r_m^3 \cdot t}{12}$$

$$I_s = 3.311 \cdot 10^3 \text{ mm}^4$$



## 7. Exemplo 2

página 28 – EC3 – 1.3

### ■ Compressão

Spring stiffness for the stiffener as beam on the elastic foundation (clause 5.5.3.1 (5)):

$$K = \frac{E \cdot t^3}{4(1-\nu^2)} \cdot \frac{1}{(b_c - z_s)^2 \cdot h_c + (b_c - z_s)^3 + 0.5[(b_c - z_s)^2 \cdot h_c] \cdot 1}$$

$$K = 0.3058 \text{ N/mm}^2$$

Elastic critical buckling stress of the edge stiffener (clause 5.5.3.2(7)):

$$\sigma_{cr,s} = \frac{2 \cdot \sqrt{K \cdot E \cdot I_s}}{A_s} = 256.7 \text{ N/mm}^2$$



## 7. Exemplo 2

página 28 – EC3 – 1.3

### ■ Compressão

Reduction factor for the distortional buckling flexural buckling of a stiffener (clause 5.5.3.2(9)):

$$\bar{\lambda}_d = \sqrt{\frac{f_{yb}}{\sigma_{cr,s}}} = 1.168$$

$$\chi_d = \begin{cases} 1 & \text{if } \bar{\lambda}_d \leq 0.65 \\ 1.47 - 0.723 \bar{\lambda}_d & \text{if } \bar{\lambda}_d > 0.65 \wedge \bar{\lambda}_d < 1.38 \\ 0.66 / \bar{\lambda}_d & \text{if } \bar{\lambda}_d \geq 1.38 \end{cases}$$

$$\chi_d = 0.626$$



## 7. Exemplo 2

### ■ Compressão

Reduced effective area of the stiffener (clause 5.5.3.2(11)):

$$A_{s,red} = \chi_d \cdot A_s \cdot \frac{f_{yb} / \gamma_{M0}}{\sigma_{com,Ed}} = 71.093 \text{ mm}^2$$

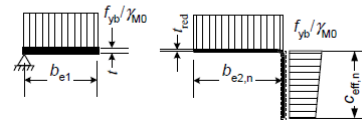
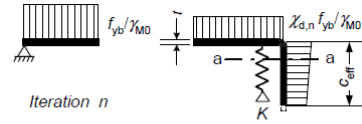
$$t_{s,red} = \frac{A_{s,red}}{(b_{ef2} - g_r) + (c_{ef} - g_r) + 2t} = 1.227 \text{ mm}$$

Area of effective cross-section sharp edges and rounded

$$A_{eff} = 2(b_{ef} + b_{ef1}) \cdot t + 2(b_{ef2} - c_{ef}) \cdot t_{s,red} = 429.6 \text{ mm}^2$$

$$A_{eff} = 2(b_{ef} - g_r) \cdot t + 2(b_{ef1} - g_r) \cdot t + 2 \cdot 2t + 2(b_{ef2} - g_r) \cdot t_{s,red} + \dots$$

$$\dots + 2(c_{ef} - g_r) \cdot t_{s,red} + 2 \cdot 2t \cdot t_{s,red} = 461 \text{ mm}^2$$



## 7. Exemplo 2

### ■ Compressão

Clause 5.5.3.2 (3) Step 3 - Iteration 1:

In the optional iteration the effective widths of the edge stiffener are calculated with reduced compression stress

$$\sigma_{com,Ed} = \chi_d \cdot f_{yb} / \gamma_{M0} = 219 \text{ N/mm}^2$$

$$\bar{\lambda}_{p,red,p}(\bar{\lambda}_p) = \bar{\lambda}_p \sqrt{\frac{\sigma_{com,Ed}}{f_{yb} / \gamma_{M0}}}$$

$$\bar{\lambda}_{p,red}(\bar{\lambda}_p) = \bar{\lambda}_p \sqrt{\chi_d}$$

Flange with width  $b$

$$\bar{\lambda}_{pb,l} = \bar{\lambda}_{p,red}(\bar{\lambda}_{pb}) = 0.608$$

$$\rho = \rho_w(\bar{\lambda}_{pb,l}, 1) = 1$$

$$b_{ef2} = 0.5 \cdot \rho \cdot b_p + g_r = 36.52 \text{ mm}$$



## 7. Exemplo 2

### ■ Compressão

Edge stiffener with width  $c$

$$\bar{\lambda}_{pc,t} = \bar{\lambda}_{p,red}(\lambda_{pc}) = 0.431 \quad \rho = \rho_0(\lambda_{pb,t}) = 1$$

$$c_{ef} = 0.5 \cdot \rho \cdot b_p + g_r = 19.02 \text{ mm}$$

Effective area of edge stiffener

Cross-sectional properties of the edge stiffener:

$$A_s = (b_{ef2} + c_{ef}) \cdot t = 108.858 \text{ mm}^2$$

$$y_s = \frac{t \cdot c_{ef}^2}{2A_s} = 3.257 \text{ mm} \quad z_s = \frac{t \cdot b_{ef2}^2}{2A_s} = 12.007 \text{ mm}$$

$$I_s = \frac{1}{12} b_{ef2} \cdot t^3 + b_{ef2} \cdot t \cdot y_s^2 + \frac{1}{12} t \cdot c_{ef}^3 + c_{ef} \cdot t (0.5 \cdot c_{ef} - y_s)^2$$

$$I_s = 3.364 \cdot 10^3 \text{ mm}^4$$



## 7. Exemplo 2

### ■ Compressão

Cross-sectional properties of the edge stiffener (Rounded corner):

$$A_s = \left[ (b_{ef2} - g_r) + (c_{ef} - g_r) + u \right] \cdot t = 118.504 \text{ mm}^2$$

$$y_s = \frac{t \left( c_{ef} - g_r \right) \left( g_r + \frac{c_{ef} - g_r}{2} \right) + t \cdot u \cdot w}{A_s} = 3.21 \text{ mm}$$

$$z_s = \frac{t \left( b_{ef2} - g_r \right) \left( g_r + \frac{b_{ef2} - g_r}{2} \right) + t \cdot u \cdot w}{A_s} = 11.247 \text{ mm}$$

$$I_s = \frac{1}{12} (b_{ef2} - g_r) \cdot t^3 + (b_{ef2} - g_r) \cdot t \cdot y_s^2 + \frac{1}{12} t \cdot (c_{ef} - g_r)^3 + \dots$$

$$\dots + (c_{ef} - g_r) \cdot t \left( g_r + \frac{c_{ef} - g_r}{2} - y_s \right)^2 + u \cdot t (y_s - w)^2 + \frac{r_m^3 \cdot t}{12}$$

$$I_s = 3.366 \cdot 10^3 \text{ mm}^4$$



## 7. Exemplo 2

### ■ Compressão

Spring stiffness for the stiffener as beam on the elastic foundation (clause 5.5.3.1 (5)):

$$K = \frac{E \cdot t^3}{4(1-\nu^2)} \cdot \frac{1}{(b_e - z_s)^2 \cdot h_e + (b_e - z_s)^3 + 0.5[(b_e - z_s)^2 \cdot h_e] \cdot 1}$$

$$K = 0.3170 \text{ N/mm}^2$$

Elastic critical buckling stress of the edge stiffener (clause 5.5.3.2(7)):

$$\sigma_{cr,s} = \frac{2 \cdot \sqrt{K \cdot E \cdot I_s}}{A_s} = 252.6 \text{ N/mm}^2$$

Reduction factor for the distortional buckling-flexural buckling of a stiffener (clause 5.5.3.2(9)):

$$\bar{\lambda}_{d,s} = \sqrt{\frac{f_{yb}}{\sigma_{cr,s}}} = 1.177$$

$$\chi_{d,s} = \chi_{rel}(\bar{\lambda}_{d,s})$$

$$\chi_{d,s} = 0.619 < \chi_d = 0.626$$

OK!



## 7. Exemplo 2

### ■ Compressão

Iteration should be ended when  $\chi_{(n)} = \chi_{(n-1)}$  but  $\chi_{(n)} \leq \chi_{(n-1)}$

$$\sigma_{com,Ed,1} = \chi_{d,1} \cdot f_{yb} / \gamma_{M0} = 217 \text{ N/mm}^2$$

$$\sigma_{com,Ed} = 219.031 \text{ N/mm}^2$$

Reduced effective area of the stiffener (clause 5.5.3.2(11)):

$$A_{s,red} = \chi_{d,s} \cdot A_s \cdot \frac{f_{yb} / \gamma_{M0}}{\sigma_{com,Ed}} = 117.217 \text{ mm}^2$$

$$t_{s,red} = \frac{A_{s,red}}{(b_{ef2} - g_r) + (c_{ef} - g_r) + u} = 1.939 \text{ mm}$$

Area of effective cross-section rounded corners

$$A_{eff,1} = 2(h_{ef} - g_r) \cdot t + 2(b_{ef1} - g_r) \cdot t + 2 \cdot u \cdot t + 2(b_{ef2} - g_r) \cdot t_{s,red} + \dots$$

$$\dots + 2(c_{ef} - g_r) \cdot t_{s,red} + 2 \cdot u \cdot t_{s,red} = 553.2 \text{ mm}^2$$



## 7. Exemplo 2

### ■ Compressão

Clause 5.5.3.2 (3) Step 3 - Iteration 2:

In the optional iteration the effective widths of the edge stiffener are calculated with reduced compression stress

$$\sigma_{com,Ed} = \chi_{d,1} \cdot f_{yb} / \gamma_{M0} = 217 \text{ N/mm}^2$$

$$\bar{\lambda}_{p,red,p}(\bar{\lambda}_p) = \bar{\lambda}_p \sqrt{\frac{\sigma_{com,Ed}}{f_{yb}}} \quad \bar{\lambda}_{p,red}(\bar{\lambda}_p) = \bar{\lambda}_p \sqrt{\chi_d}$$

Flange with width  $b$

$$\bar{\lambda}_{pb,l} = \bar{\lambda}_{p,red}(\bar{\lambda}_{pb}) = 0.608 \quad \rho = \rho_w(\bar{\lambda}_{pb,l}, 1) = 1$$

$$b_{ef2} = 0.5 \cdot \rho \cdot b_p + g_r = 36.52 \text{ mm}$$

Edge stiffener with width  $c$

$$\bar{\lambda}_{pc,l} = \bar{\lambda}_{p,red}(\bar{\lambda}_{pc}) = 0.431 \quad \rho = \rho_0(\bar{\lambda}_{pb,l}) = 1$$

$$c_{ef} = 0.5 \cdot \rho \cdot b_p + g_r = 19.02 \text{ mm}$$



## 7. Exemplo 2

### ■ Compressão

Effective area of edge stiffener

Cross-sectional properties of the edge stiffener (Rounded corner):

$$A_s = [(b_{ef2} - g_r) + (c_{ef} - g_r) + u] \cdot t = 118.504 \text{ mm}^2$$

$$y_s = \frac{t(c_{ef} - g_r) \left( g_r + \frac{c_{ef} - g_r}{2} \right) + t \cdot u \cdot w}{A_s} = 3.21 \text{ mm}$$

$$z_s = \frac{t(b_{ef2} - g_r) \left( g_r + \frac{b_{ef2} - g_r}{2} \right) + t \cdot u \cdot w}{A_s} = 11.247 \text{ mm}$$

$$I_s = \frac{1}{12} (b_{ef2} - g_r) \cdot t^3 + (b_{ef2} - g_r) \cdot t \cdot y_s^2 + \frac{1}{12} t \cdot (c_{ef} - g_r)^3 + \dots$$

$$\dots + (c_{ef} - g_r) \cdot t \left( g_r + \frac{c_{ef} - g_r}{2} - y_s \right)^2 + u \cdot t (y_s - w)^2 + \frac{t_w^3 \cdot t}{12}$$

$$I_s = 3.366 \cdot 10^3 \text{ mm}^4$$



## 7. Exemplo 2

### ■ Compressão

Spring stiffness for the stiffener as beam on the elastic foundation (clause 5.5.3.1 (5)):

$$K = \frac{E \cdot t^3}{4(1-\nu^2)} \cdot \frac{1}{(b_e - z_s)^2 \cdot h_e + (b_e - z_s)^3 + 0.5 \left[ (b_e - z_s)^2 \cdot h_e \right] \cdot 1}$$

$$K = 0.3170 \text{ N/mm}^2$$

Elastic critical buckling stress of the edge stiffener (clause 5.5.3.2(7)):

$$\sigma_{cr,s} = \frac{2 \cdot \sqrt{K \cdot E \cdot I_s}}{A_s} = 252.6 \text{ N/mm}^2$$

Reduction factor for the distortional buckling-flexural buckling of a stiffener (clause 5.5.3.2(9)):



## 7. Exemplo 2

### ■ Compressão

$$\bar{\lambda}_d = \sqrt{\frac{f_{yb}}{\sigma_{cr,s}}} = 1.177$$

$$\chi_{d,2} = \chi_{ed}(\bar{\lambda}_d)$$

$$\chi_{d,2} = 0.619 = \chi_{d,1} = 0.619$$

OK!

Iteration should be ended when  $\chi_{(n)} = \chi_{(n-1)}$  but  $\chi_{(n)} \leq \chi_{(n-1)}$

$$\sigma_{com,Ed,1} = \chi_{d,2} \cdot f_{yb} / \gamma_{M0} = 217 \text{ N/mm}^2$$

$$\sigma_{com,Ed} = 216.652 \text{ N/mm}^2$$

Reduced effective area of the stiffener (clause 5.5.3.2(11)):

$$A_{s,red} = \chi_{d,2} \cdot A_s \cdot \frac{f_{yb} / \gamma_{M0}}{\sigma_{com,Ed}} = 118.504 \text{ mm}^2$$

$$t_{s,red} = \frac{A_{s,red}}{(b_{d2} - g_r) + (c_d - g_r) + u} = 1.96 \text{ mm}$$

Area of effective cross-section rounded corners

$$A_{eff,2} = 2(h_d - g_r) \cdot t + 2(b_{d1} - g_r) \cdot t + 2 \cdot u \cdot t + 2(b_{d2} - g_r) \cdot t_{s,red} + \dots$$

$$\dots + 2(c_d - g_r) \cdot t_{s,red} + 2 \cdot u \cdot t_{s,red} = 555.8 \text{ mm}^2$$



## 7. Exemplo 2

### Flexão em torno de y

The bending moment resistance is calculated about the axis of symmetry. The stress distribution of web is determined after the determination of effective widths for the flange and edge stiffener (Clause 6.1.4.1(5)).

Compression stress (clause 5.5.3.2(3)):

$$\sigma_{com} = f_{yb} / \gamma_{M0} = 350 \text{ N/mm}^2$$

Clause 5.5.3.2(3) Step 1:

Flange with width  $b$

$$\psi = 1 \quad k_\sigma = 4$$

$$\bar{\lambda}_{pb} = \frac{b_p}{t} \frac{1}{28.4 \cdot \epsilon \cdot \sqrt{k_\sigma}} = 0.769$$

$$\rho = \min \left[ \frac{\bar{\lambda}_{pb} - 0.055(3 + \psi)}{\bar{\lambda}_{pb}^2}, 1 \right] = 0.929$$

$$b_{ef1} = 0.5 \cdot \rho \cdot b_p + g_r = 34.02 \text{ mm}$$

$$b_{ef1} = b_{ef2}$$



## 7. Exemplo 2

### Flexão em torno de y

Edge stiffener with width  $c$

$$k_{\sigma c} = \begin{cases} 0.5 & \text{if } \frac{c_p}{b_p} \leq 0.35 \\ 0.5 + 0.83 \sqrt{\left( \frac{c_p}{b_p} - 0.35 \right)^2} & \text{if } \frac{c_p}{b_p} \leq 0.6 \wedge \frac{c_p}{b_p} > 0.35 \end{cases}$$

$$k_{\sigma c} = 0.5$$

$$\bar{\lambda}_{pc} = \frac{c_p}{t} \frac{1}{28.4 \cdot \epsilon \cdot \sqrt{k_{\sigma c}}} = 0.544$$

$$\rho = \min \left[ \left( 1 - \frac{0.188}{\bar{\lambda}_{pc}} \right) \frac{1}{\bar{\lambda}_{pc}}, 1 \right] = 1.000$$

$$c_{ef} = 0.5 \cdot \rho \cdot c_p + g_r = 19.02 \text{ mm}$$



## 7. Exemplo 2

### Flexão em torno de y

Stress distribution in the web  $\sigma_2/\sigma_1$ :

Change in the location of centre of gravity in z-direction (Rounded corners)

$$z_{nr} = - \frac{(b_c - b_{ef1} - b_{ef2})t \cdot \frac{h_c}{2} + (c_c - c_{ef})t \left( \frac{h_c}{2} - \frac{c_{ef}}{2} \right)}{t \left[ h_p + b_p + c_p + (b_{ef1} - g_r) + (b_{ef2} - g_r) + (c_{ef} - g_r) + 4 \cdot u \right]}$$

$$z_{nr} = -1.248 \text{ mm}$$

Change in the location of centre of gravity in z-direction (Sharp corners)

$$z_{ns} = - \frac{(b_c - b_{ef1} - b_{ef2})t \cdot \frac{h_c}{2} + (c_c - c_{ef})t \left( \frac{h_c}{2} - \frac{c_{ef}}{2} \right)}{t (h_p + b_p + c_p + b_{ef1} + b_{ef2} + c_{ef})}$$

$$z_{ns} = -1.313 \text{ mm}$$



## 7. Exemplo 2

### Flexão em torno de y

$$\psi = - \left( \frac{h_c}{2} + z_{nr} \right) \cdot \left( \frac{h_c}{2} - z_{nr} \right)^{-1} = -0.975$$

Web width  $b$

$$k_{cr} = 7.81 - 6.29 \cdot \psi + 9.78 \cdot \psi^2 = 23.243$$

$$\lambda_{cr} = \frac{h_p}{t} \cdot \frac{1}{28.4 \cdot E \cdot \sqrt{k_{cr}}} = 0.887$$

$$\rho' = \min \left[ \frac{\lambda_{cr} - 0.055(3 + \psi)}{\lambda_{cr}^2}, 1 \right] = 0.986$$

$$h_{eff} = \rho' \frac{h_p}{1 - \psi} = 97.359 \text{ mm}$$

$$h_{ef1} = 0.4 h_{eff} + g_r = 40.408 \text{ mm}$$

$$h_{ef2} = 0.6 h_{eff} + g_r + \left( h_p - \frac{h_c}{1 - \psi} \right) = 156.206 \text{ mm}$$

Clause 5.5.3.2 (3) Step 2:

Effective area of edge stiffener

Cross-sectional properties of the edge stiffener (Rounded corner):

$$A_s = [(b_{ef2} - g_r) + (c_{ef} - g_r) + u] \cdot t = 113.603 \text{ mm}^2$$

$$y_s = \frac{t(c_{ef} - g_r) \left( g_r + \frac{c_{ef} - g_r}{2} \right) + t \cdot u \cdot w}{A_s} = 3.348 \text{ mm}$$

$$z_s = \frac{t(b_{ef2} - g_r) \left( g_r + \frac{b_{ef2} - g_r}{2} \right) + t \cdot u \cdot w}{A_s} = 10.211 \text{ mm}$$

$$I_s = \frac{1}{12} (b_{ef2} - g_r) \cdot t^3 + (b_{ef2} - g_r) \cdot t \cdot y_s^2 + \frac{1}{12} t \cdot (c_{ef} - g_r)^3 + \dots$$

$$\dots + (c_{ef} - g_r) \cdot t \left( g_r + \frac{c_{ef} - g_r}{2} - y_s \right)^2 + u \cdot t (y_s - w)^2 + \frac{r_m^3 \cdot t}{12}$$

$$I_s = 3.311 \cdot 10^3 \text{ mm}^4$$



## 7. Exemplo 2

### Flexão em torno de y

Spring stiffness for the stiffener as beam on the elastic foundation (clause 5.5.3.1 (5)). If other flange is in tension factor  $k_f = 0$ :

$$K = \frac{E \cdot I^3}{4(1-\nu^2)} \cdot \frac{1}{(b_c - z_s)^2 \cdot h_c + (b_c - z_s)^3 + 0.5[(b_c - z_s)^2 \cdot h_c] \cdot 0}$$

$$K = 0.4218 \text{ N/mm}^2$$

Elastic critical buckling stress of the edge stiffener (clause 5.5.3.2(7)):

$$\sigma_{cr,s} = \frac{2 \cdot \sqrt{K \cdot E \cdot I_s}}{A_s} = 301.5 \text{ N/mm}^2$$

Reduction factor for the distortional buckling-flexural buckling of a stiffener (clause 5.5.3.2(9)):

$$\bar{\lambda}_d = \sqrt{\frac{f_{yb}}{\sigma_{cr,s}}} = 1.077$$



## 7. Exemplo 2

### Flexão em torno de y

$$\chi_{ds} = \begin{cases} 1 & \text{if } \bar{\lambda}_{ds} \leq 0.65 \\ 1.47 - 0.723 \cdot \bar{\lambda}_{ds} & \text{if } \bar{\lambda}_{ds} > 0.65 \wedge \bar{\lambda}_{ds} < 1.38 \\ 0.66 / \bar{\lambda}_{ds} & \text{if } \bar{\lambda}_{ds} \geq 1.38 \end{cases}$$

$$\chi_{ds} = 0.691$$

Reduced effective area of the stiffener (clause 5.5.3.2(11)):

$$A_{s,red} = \chi_{ds} \cdot A_s \cdot \frac{f_{yb} \cdot \gamma_{M0}}{\sigma_{adm,Ed}} = 78.506 \text{ mm}^2$$

$$i_{s,red} = \sqrt{\frac{A_{s,red}}{\{b_{d2} - g_r\} + \{c_d - g_r\} + \pi}} = 1.254 \text{ mm}$$

Calculation of section modulus for the effective section:

$$A_{eff,b} = I \left[ c_f + b_f + \{b_{d2} - g_r\} + \{b_{d1} - g_r\} + 3u \right] + \dots$$

$$\dots + i_{s,red} \left[ u + \{b_{d1} - g_r\} + \{c_d - g_r\} \right] = 676.114 \text{ mm}^2$$



## 7. Exemplo 2

### ■ Flexão em torno de y

C.G. of effective section from the compressed flange

$$z_1 = \frac{1}{A_{eff,b}} \left[ c_p \cdot t \left( h_c - \frac{c}{2} \right) + b_p \cdot t \cdot h_c + 2u \cdot t (h_c - w) + (h_{ef2} - g_r) t \left( h_c - \frac{h_{ef2}}{2} \right) \dots \right. \\ \left. + (h_{ef1} - g_r) t \left( \frac{h_{ef1}}{2} \right) + (c_{ef} - g_r) t_{s,red} \left( \frac{c_{ef}}{2} \right) + u \cdot t \cdot w + u \cdot t_{s,red} \cdot w \right]$$

$$z_1 = 115.263 \text{ mm}$$

$$z_2 = h_c - z_1 = 82.777 \text{ mm}$$



## 7. Exemplo 2

### ■ Flexão em torno de y

$$I_{eff,y} = \frac{c_p^3 \cdot t}{12} + c_p \cdot t \left( \frac{h_c}{2} - c_c + \frac{c_p}{2} \right)^2 + \frac{b_p \cdot t^3}{12} + b_p \cdot t \cdot \left( \frac{h_c}{2} \right)^2 + 2u \cdot t \left( \frac{h_c}{2} - w \right)^2 \dots \\ + \frac{(h_{ef2} - g_r)^3 t}{12} + (h_{ef2} - g_r) t \left( \frac{h_c}{2} - \frac{h_{ef2}}{2} \right)^2 + \frac{(h_{ef1} - g_r)^3 t}{12} \dots \\ + (h_{ef1} - g_r) t \left( \frac{h_c}{2} - \frac{h_{ef1}}{2} \right)^2 + u \cdot t \left( \frac{h_c}{2} - w \right)^2 + \frac{(b_{ef1} - g_r) t^3}{12} \dots \\ + (b_{ef1} - g_r) t \left( \frac{h_c}{2} \right)^2 + \frac{(b_{ef2} - g_r) t_{s,red}^3}{12} + (b_{ef2} - g_r) t_{s,red} \left( \frac{h_c}{2} \right)^2 \dots \\ + \frac{(c_{ef} - g_r)^3 t_{s,red}}{12} + \left[ (c_{ef} - g_r) t_{s,red} \left( \frac{h_c}{2} - c_c + \frac{c_{ef} - g_r}{2} \right)^2 \right]$$

$$I_{eff,y} = 4.523 \cdot 10^6 \text{ mm}^4$$

$$W_{eff,y} = \frac{I_{eff,y}}{z_1} = 3.924 \cdot 10^4 \text{ mm}^3$$



## 7. Exemplo 2

### Flexão em torno de y

Clause 5.5.3.2 (3) Step 3- Iteration 1:

In the optional iteration the effective widths of the edge stiffener are calculated with reduced compression stress:

$$\sigma_{com,Ed} = \chi_{d,1} \cdot f_{yb} / \gamma_{M0} = 242 \text{ N/mm}^2$$

$$\bar{\lambda}_{p,red,p}(\bar{\lambda}_p) = \bar{\lambda}_p \sqrt{\frac{\sigma_{com,Ed}}{f_{yb}}} \quad \bar{\lambda}_{p,red}(\bar{\lambda}_p) = \bar{\lambda}_p \sqrt{\chi_d}$$

Flange with width  $b$

$$\bar{\lambda}_{pb,l} = \bar{\lambda}_{p,red}(\bar{\lambda}_{pb}) = 0.639 \quad \rho = \rho_w(\bar{\lambda}_{pb,l}, 1) = 1$$

$$b_{ef2} = 0.5 \cdot \rho \cdot b_p + g_r = 36.52 \text{ mm}$$



## 7. Exemplo 2

### Flexão em torno de y

Edge stiffener with width  $c$

$$\bar{\lambda}_{pc,l} = \bar{\lambda}_{p,red}(\bar{\lambda}_{pc}) = 0.452 \quad \rho = \rho_0(\bar{\lambda}_{pb,l}) = 1$$

$$c_{ef} = 0.5 \cdot \rho \cdot b_p + g_r = 19.02 \text{ mm}$$

Effective area of edge stiffener

Cross-sectional properties of the edge stiffener:

$$A_s = [(b_{ef2} - g_r) + (c_{ef} - g_r) + u] \cdot t = 118.504 \text{ mm}^2$$

$$y_s = \frac{t(c_{ef} - g_r) \left( g_r + \frac{c_{ef} - g_r}{2} \right) + t \cdot u \cdot w}{A_s} = 3.21 \text{ mm}$$

$$z_s = \frac{t(b_{ef2} - g_r) \left( g_r + \frac{b_{ef2} - g_r}{2} \right) + t \cdot u \cdot w}{A_s} = 11.247 \text{ mm}$$



## 7. Exemplo 2

### Flexão em torno de y

$$I_x = \frac{1}{12} (b_{ef2} - g_r) \cdot t^3 + (b_{ef2} - g_r) \cdot t \cdot y_s^2 + \frac{1}{12} t \cdot (c_{ef} - g_r)^3 + \dots$$

$$\dots + (c_{ef} - g_r) \cdot t \left( g_r + \frac{c_{ef} - g_r}{2} - y_s \right)^2 + u \cdot t (y_s - w)^2 + \frac{r_m^3 \cdot t}{12}$$

$$I_x = 3.366 \cdot 10^3 \text{ mm}^4$$

Spring stiffness for the stiffener as beam on the elastic foundation (clause 5.5.3.1(5)). If other flange is in tension factor  $k_f = 0$ :

$$K = \frac{E \cdot t^3}{4(1-\nu^2)} \cdot \frac{1}{(b_c - z_s)^2 \cdot h_t + (b_c - z_s)^3 + 0.5[(b_c - z_s)^2 \cdot h_t] \cdot 0}$$

$$K = 0.4378 \text{ N/mm}^2$$

Elastic critical buckling stress of the edge stiffener (clause 5.5.3.2(7)):

$$\sigma_{cr,s} = \frac{2 \cdot \sqrt{K \cdot E \cdot I_x}}{A_k} = 296.9 \text{ N/mm}^2$$



## 7. Exemplo 2

### Flexão em torno de y

Reduction factor for the distortional buckling-flexural buckling of a stiffener (clause 5.5.3.2(9)):

$$\bar{\lambda}_d = \sqrt{\frac{f_{yb}}{\sigma_{cr,s}}} = 1.086$$

$$\chi_{d,1} = \chi_{rel}(\bar{\lambda}_d) \quad \chi_{d,1} = 0.685 > \chi_d = 0.691$$

OK!

Iteration should be ended when  $\chi_{(n)} = \chi_{(n-1)}$  but  $\chi_{(n)} \leq \chi_{(n-1)}$

$$\sigma_{com,Ed,1} = \chi_{d,1} \cdot f_{yb} / \gamma_{M0} = 240 \text{ N/mm}^2$$

$$\sigma_{com,Ed} = 241.869 \text{ N/mm}^2$$

Reduced effective area of the stiffener (clause 5.5.3.2(11)):

$$A_{s,red} = \chi_{d,1} \cdot A_s \cdot \frac{f_{yb} / \gamma_{M0}}{\sigma_{com,Ed}} = 117.465 \text{ mm}^2$$

$$t_{s,red} = \frac{A_{s,red}}{(b_{ef2} - g_r) + (c_{ef} - g_r) + u} = 1.943 \text{ mm}$$



## 7. Exemplo 2

### Flexão em torno de y

Calculation of section modulus for the effective section:

$$A_{eff,b} = t \left[ c_p + b_p + (h_{d2} - g_r) + (h_{d1} - g_r) + 3u \right] + \dots$$

$$\dots + t_{s,red} \left[ u + (b_{d2} - g_r) + (c_{d2} - g_r) \right] = 715.074 \text{ mm}^2$$

C.G. of effective section from the compressed flange

$$z_1 = \frac{1}{A_{eff,b}} \left[ c_p \cdot t \left( h_c - \frac{c_c}{2} \right) + b_p \cdot t \cdot h_c + 2u \cdot t \left( h_c - w \right) + (h_{d2} - g_r) t \left( h_c - \frac{h_{d2}}{2} \right) \dots \right. \\ \left. + (h_{d1} - g_r) t \left( \frac{h_{d1}}{2} \right) + (c_{d2} - g_r) t_{s,red} \left( \frac{c_{d2}}{2} \right) + u \cdot t \cdot w + u \cdot t_{s,red} \cdot w \right]$$

$$z_1 = 109.133 \text{ mm}$$

$$z_2 = h_c - z_1 = 88.907 \text{ mm}$$



## 7. Exemplo 2

### Flexão em torno de y

$$I_{eff,y} = \frac{c_p^3 \cdot t}{12} + c_p \cdot t \left( \frac{h_c}{2} - c_c + \frac{c_p}{2} \right)^2 + \frac{b_p \cdot t^3}{12} + b_p \cdot t \cdot \left( \frac{h_c}{2} \right)^2 + 2u \cdot t \left( \frac{h_c}{2} - w \right)^2 \dots \\ + \frac{(h_{d2} - g_r)^3 \cdot t}{12} + (h_{d2} - g_r) t \left( \frac{h_c}{2} - \frac{h_{d2}}{2} \right)^2 + \frac{(h_{d1} - g_r)^3 \cdot t}{12} \dots \\ + (h_{d1} - g_r) t \left( \frac{h_c}{2} - \frac{h_{d1}}{2} \right)^2 + u \cdot t \left( \frac{h_c}{2} - w \right)^2 + \frac{(b_{d1} - g_r) t^3}{12} \dots \\ + (b_{d1} - g_r) t \left( \frac{h_c}{2} \right)^2 + \frac{(b_{d2} - g_r) t_{s,red}^3}{12} + (b_{d2} - g_r) t_{s,red} \left( \frac{h_c}{2} \right)^2 \dots \\ + \frac{(c_{d2} - g_r)^3 \cdot t_{s,red}}{12} + \left[ (c_{d2} - g_r) t_{s,red} \left( \frac{h_c}{2} - c_c + \frac{c_{d2} - g_r}{2} \right)^2 \right]$$

$$I_{eff,y} = 4.84 \cdot 10^6 \text{ mm}^4$$

$$W_{eff,y} = \frac{I_{eff,y}}{z_1} = 4.435 \cdot 10^4 \text{ mm}^3$$

Anterior

$$I_{eff,y} = 4.523 \cdot 10^6 \text{ mm}^4$$

$$W_{eff,y} = \frac{I_{eff,y}}{z_1} = 3.924 \cdot 10^4 \text{ mm}^3$$

$$4,84 / 4,523 \Rightarrow 7\%$$



## 7. Exemplo 2

### Flexão em torno de y

Clause 5.5.3.2 (3) Step 3- Iteration 2:

In the optional iteration the effective widths of the edge stiffener are calculated with reduced compression stress:

$$\sigma_{com,Ed} = \chi_{d,1} \cdot f_{yb} / \gamma_{M0} = 240 \text{ N/mm}^2$$

$$\bar{\lambda}_{p,red,p}(\bar{\lambda}_p) = \bar{\lambda}_p \sqrt{\frac{\sigma_{com,Ed}}{f_{yb}}} \quad \bar{\lambda}_{p,red}(\bar{\lambda}_p) = \bar{\lambda}_p \sqrt{\chi_d}$$

Flange with width  $b$

$$\bar{\lambda}_{pb,l} = \bar{\lambda}_{p,red}(\bar{\lambda}_{pb}) = 0.636 \quad \rho = \rho_w(\bar{\lambda}_{pb,l}, 1) = 1$$

$$b_{ef2} = 0.5 \cdot \rho \cdot b_p + g_r = 36.52 \text{ mm}$$



## 7. Exemplo 2

### Flexão em torno de y

Edge stiffener with width  $c$

$$\bar{\lambda}_{pe,t} = \bar{\lambda}_{p,red}(\bar{\lambda}_{pe}) = 0.451 \quad \rho = \rho_w(\bar{\lambda}_{pe,t}, 1) = 1$$

$$c_{ef} = 0.5 \cdot \rho \cdot b_p + g_r = 19.02 \text{ mm}$$

Effective area of edge stiffener

Cross-sectional properties of the edge stiffener:

$$A_s = [(b_{ef2} - g_r) + (c_{ef} - g_r) + t] \cdot t = 118.504 \text{ mm}^2$$

$$y_s = \frac{t(c_{ef} - g_r) \left( g_r + \frac{c_{ef} - g_r}{2} \right) + t \cdot t \cdot w}{A_s} = 3.21 \text{ mm}$$

$$z_s = \frac{t(b_{ef2} - g_r) \left( g_r + \frac{b_{ef2} - g_r}{2} \right) + t \cdot t \cdot w}{A_s} = 11.247 \text{ mm}$$



## 7. Exemplo 2

### Flexão em torno de y

$$I_y = \frac{1}{12} (b_{d2} - g_r) \cdot t^3 + (b_{d2} - g_r) \cdot t \cdot y_s^2 + \frac{1}{12} t \cdot (c_d - g_r)^3 + \dots$$

$$\dots + (c_d - g_r) \cdot t \left( g_r + \frac{c_d - g_r}{2} - y_s \right)^2 + u \cdot t (y_s - w)^2 + \frac{t_m^3 \cdot t}{12}$$

$$I_y = 3.366 \cdot 10^3 \text{ mm}^4$$

Spring stiffness for the stiffener as beam on the elastic foundation (clause 5.5.3.1(5)). If other flange is in tension factor  $k_f = 0$ :

$$K = \frac{E \cdot t^3}{4(1-\nu^2)} \cdot \frac{1}{(b_c - z_s)^2 \cdot h_c + (b_c - z_s)^3 + 0.5[(b_c - z_s)^2 \cdot h_c] \cdot 0}$$

$$K = 0.4378 \text{ N/mm}^2$$

Elastic critical buckling stress of the edge stiffener (clause 5.5.3.2(7)):

$$\sigma_{cr,s} = \frac{2 \cdot \sqrt{K \cdot E \cdot I_y}}{A_s} = 296.9 \text{ N/mm}^2$$



## 7. Exemplo 2

### Flexão em torno de y

Reduction factor for the distortional buckling-flexural buckling of a stiffener (clause 5.5.3.2(9)):

$$\bar{\lambda}_{d1} = \sqrt{\frac{f_{yb}}{\sigma_{cr,s}}} = 1.086$$

$$\chi_{d,2} = \chi_{cd}(\bar{\lambda}_{d1})$$

$$\chi_{d,2} = 0.685 > \chi_{d,1} = 0.685$$

OK!

Iteration should be ended when  $\chi_{(n)} = \chi_{(n-1)}$  but  $\chi_{(n)} \leq \chi_{(n-1)}$

$$\sigma_{com,Ed,1} = \chi_{d,1} \cdot f_{yb} / \gamma_{M0} = 240 \text{ N/mm}^2$$

$$\sigma_{com,Ed} = 239.749 \text{ N/mm}^2$$



## 7. Exemplo 2

### Flexão em torno de y

Reduced effective area of the stiffener (clause 5.5.3.2(11)):

$$A_{s,red} = \chi_{d,2} \cdot A_s \frac{f_{yb} / \gamma_{M0}}{\sigma_{com,Ed}} = 118.504 \text{ mm}^2$$

$$t_{s,red} = \frac{A_{s,red}}{(b_{d2} - g_r) + (c_{d2} - g_r) + u} = 1.96 \text{ mm}$$

Calculation of section modulus for the effective section:

$$A_{eff,b} = t \left[ c_p + b_p + (h_{d2} - g_r) + (h_{d1} - g_r) + 3u \right] + \dots$$

$$\dots + t_{s,red} \left[ u + (b_{d2} - g_r) + (c_{d2} - g_r) \right] = 716.112 \text{ mm}^2$$

C.G. of effective section from the compressed flange

$$z_1 = \frac{1}{A_{eff,b}} \left[ c_p \cdot t \left( h_c - \frac{c_c}{2} \right) + b_p \cdot t \cdot h_c + 2u \cdot t \left( h_c - w \right) + (h_{d2} - g_r) t \left( h_c - \frac{h_{d2}}{2} \right) \dots \right. \\ \left. + (h_{d1} - g_r) t \left( \frac{h_{d1}}{2} \right) + (c_{d2} - g_r) t_{s,red} \left( \frac{c_{d2}}{2} \right) + u \cdot t \cdot w + u \cdot t_{s,red} \cdot w \right]$$



## 7. Exemplo 2

### Flexão em torno de y

$$z_1 = 108.979 \text{ mm}$$

$$z_2 = h_c - z_1 = 89.061 \text{ mm}$$

$$I_{eff,y} = \frac{c_p^3 \cdot t}{12} + c_p \cdot t \left( \frac{h_c}{2} - c_c + \frac{c_p}{2} \right)^2 + \frac{b_p \cdot t^3}{12} + b_p \cdot t \cdot \left( \frac{h_c}{2} \right)^2 + 2u \cdot t \left( \frac{h_c}{2} - w \right)^2 \dots \\ + \frac{(h_{d2} - g_r)^3 t}{12} + (h_{d2} - g_r) t \left( \frac{h_c}{2} - \frac{h_{d2}}{2} \right)^2 + \frac{(h_{d1} - g_r)^3 t}{12} \dots \\ + (h_{d1} - g_r) t \left( \frac{h_c}{2} - \frac{h_{d1}}{2} \right)^2 + u \cdot t \left( \frac{h_c}{2} - w \right)^2 + \frac{(b_{d2} - g_r)^3 t}{12} \dots \\ + (b_{d2} - g_r) t \left( \frac{h_c}{2} \right)^2 + \frac{(b_{d2} - g_r) t_{s,red}^3}{12} + (b_{d2} - g_r) t_{s,red} \left( \frac{h_c}{2} \right)^2 \dots \\ + \frac{(c_{d2} - g_r)^3 t_{s,red}}{12} + \left[ (c_{d2} - g_r) t_{s,red} \left( \frac{h_c}{2} - c_c + \frac{c_{d2} - g_r}{2} \right)^2 \right]$$

Anterior

$$I_{eff,y} = 4.84 \cdot 10^6 \text{ mm}^4$$

$$W_{eff,y} = \frac{I_{eff,y}}{z_1} = 4.435 \cdot 10^4 \text{ mm}^3$$

$$4,849 / 4,84 \Rightarrow 1\% \quad \checkmark$$

$$I_{eff,y} = 4.849 \cdot 10^6 \text{ mm}^4$$

$$W_{eff,y} = \frac{I_{eff,y}}{z_1} = 4.449 \cdot 10^4 \text{ mm}^3$$