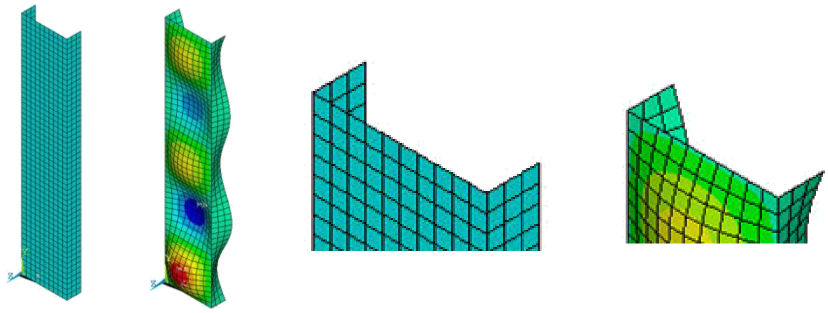

ELU e ELS – Exemplos Iniciais

Programa de Pós-Graduação em Engenharia Civil
PGECIV - Mestrado Acadêmico
Faculdade de Engenharia – FEN/UERJ
Disciplina: Tópicos Especiais em Estruturas (Chapa Dobrada)
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1. Exemplo 1

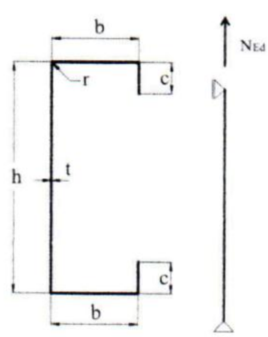
Example 3.3: Design of a cold-formed steel member in tension

Basic Data

Height of stud $H = 3.00$ m

The dimensions of the cross section and the material properties are:

Total height	$h = 150$ mm
Total width of flange	$b = 45$ mm
Total width of edge fold	$c = 16$ mm
Internal radius	$r = 3$ mm
Nominal thickness	$t_{nom} = 1.0$ mm
Steel core thickness (§§2.4.2.3, eqn. (2.10))	$t = 0.96$ mm
Basic yield strength	$f_{yb} = 350$ N/mm^2
Ultimate strength	$f_u = 420$ N/mm^2
Modulus of elasticity	$E = 210000$ N/mm^2
Poisson's ratio	$\nu = 0.3$
Shear modulus	$G = \dfrac{E}{2(1+\nu)} = 81000$ N/mm^2
Partial factors (§§2.3.1)	$\gamma_{M0} = 1.0$
	$\gamma_{M1} = 1.0$



Properties of the gross cross section

Gross cross section area: $A_g = 257$ mm^2

The design value of the tension force on the stud $N_{Ed} = 68.26$ kN

1. Exemplo 1

Resistance check of the cross section

The following criterion should be satisfied (§§3.8.2, eqn. (3.79)):

$$\frac{N_{Ed}}{N_{t,Rd}} \leq 1$$

where the design resistance of a cross section for uniform tension is (§§3.8.2, eqn. (3.78)):

$$N_{t,Rd} = \frac{f_{ya} A_g}{\gamma_{M0}}$$

f_{ya} – the average yield strength (§§2.4.2.2, eqn. (2.8))

$$f_{ya} = f_{yb} + (f_u - f_{yb})\frac{knt^2}{A_g} \quad \text{but} \quad f_{ya} \leq \frac{f_u + f_{yb}}{2} = \frac{420 + 350}{2} = 385 \text{ N/mm}^2$$

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1. Exemplo 1

where

- k – coefficient depending on the type of forming; $k = 7$ for roll forming
- n – the number of 90° bends in the cross section with an internal radius $r \leq 5t$; $n = 4$

$$f_{ya} = f_{yb} + (f_u - f_{yb})\frac{knt^2}{A_g} = 350 + (420 - 350) \times \frac{7 \times 4 \times 0.96^2}{257} =$$

$$= 357 \text{ N/mm}^2$$

$$f_{ya} = 357 \text{ N/mm}^2 < \frac{f_u + f_{yb}}{2} = 385 \text{ N/mm}^2 \qquad \text{– OK}$$

The design resistance will be:

$$N_{t,Rd} = \frac{f_{ya} A_g}{\gamma_{M0}} = \frac{357 \times 257}{1.0} = 91749 \text{ N} = 91.75 \text{ kN}$$

The resistance check is:

$$\frac{N_{Ed}}{N_{t,Rd}} = \frac{68.26}{91.75} = 0.744 < 1 \qquad \text{– OK}$$

2. Exemplo 2

Example 3.4: Design resistance of a cold-formed steel member in compression

Basic Data

Height of stud	$H = 3.10$ m
Spacing between studs	$S = 0.6$ m
Span of floor	$L = 5$ m
Spacing between floor joists	$S = 0.6$ m

Distributed loads applied to the floor:

dead load – lightweight slab	$1.20\ \text{kN/m}^2$
	$q_G = 1.20 \times 0.6 = 0.72\ \text{kN/m}$
imposed load	$2.50\ \text{kN/m}^2$
	$q_Q = 2.50 \times 0.6 = 1.50\ \text{kN/m}$

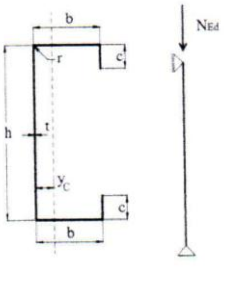
Ultimate Limit State concentrated load

from upper level and roof: $Q = 14.0$ kN

2. Exemplo 2

The dimensions of one cross section and the material properties are:

Total height	$h = 150$ mm
Total width of flange	$b = 45$ mm
Total width of edge fold	$c = 15$ mm
Internal radius	$r = 3$ mm
Nominal thickness	$t_{nom} = 1$ mm
Steel core thickness	$t = 0.96$ mm



Basic yield strength	$f_{yb} = 350\ \text{N/mm}^2$
Modulus of elasticity	$E = 210000\ \text{N/mm}^2$
Poisson's ratio	$\nu = 0.3$
Shear modulus	$G = \dfrac{E}{2(1+\nu)} = 81000\ \text{N/mm}^2$
Partial factors (§§2.3.1)	$\gamma_{M0} = 1.0$
	$\gamma_{M1} = 1.0$
	$\gamma_G = 1.35$ – permanent loads
	$\gamma_Q = 1.50$ – variable loads

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2. Exemplo 2

Derived data (section properties and action effects)

Properties of the gross cross section

Gross cross section area: $A = 255\ \text{mm}^2$

Position of *z-z* axis of the gross cross section with respect to the web:

$$y_c = 12.08\ \text{mm}$$

Properties of the effective cross section (§§3.7.3, Flowcharts 3.1 and 3.2)
The properties of effective cross section were calculated following the procedures presented in Examples 3.1 and 3.2.

Effective area of the cross section in compression:

$$A_{eff} = 115\ \text{mm}^2$$

Position of *z-z* axis of the effective cross section with respect to the web:

$$y_{c,eff} = 16.28\ \text{mm}$$

Effective section modulus for bending about weak axis:

$$W_{eff,z,com} = 1561\ \text{mm}^3$$

$$W_{eff,z,ten} = 4127\ \text{mm}^3$$

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2. Exemplo 2

The applied concentrated load on the stud (only compression) – §§2.3.1, Flowchart 2.1

$$N_{Ed} = \left(\gamma_G q_G + \gamma_Q q_Q\right) L + Q = \left(1.35 \times 0.72 + 1.50 \times 1.50\right) \times 5 + 14 = 30.11\ \text{kN}$$

Resistance check of the cross section (§§3.8.9)

The following condition should be satisfied:

$$\frac{N_{Ed}}{N_{c,Rd}} + \frac{M_{z,Ed} + \Delta M_{z,Ed}}{M_{cz,Rd,com}} \le 1$$

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2. Exemplo 2

where

$N_{c,Rd} = A_{eff} f_{yb} / \gamma_{M0}$ (§§3.8.3, eqn. (3.81))

$M_{cz,Rd,com} = W_{eff,com} f_{yb} / \gamma_{M0}$ (§§3.8.4, eqn. (3.83a))

$\Delta M_{z,Ed} = N_{Ed} e_{Nz}$ (§§3.8.9)

e_{Nz} – is the shift of centroidal z-z axis

$M_{z,Ed} = 0$

$e_{Nz} = y_{c,eff} - y_c = 16.28 - 12.08 = 4.2$ mm

The resistance check is:

$$\frac{30110}{115 \times 350/1.0} + \frac{0 + 30110 \times 4.2}{1561 \times 350/1.0} = 0.979 < 1 \text{ – OK}$$

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3. Exemplo 3

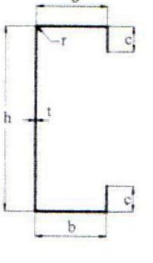
Example 3.5: Design of a cold-formed steel member in bending.

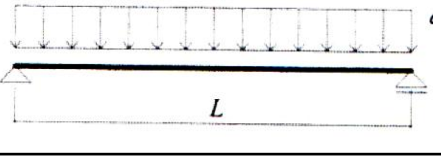
Basic Data

Span of joist	$L = 5.5$ m
Spacing between joists	$S = 0.6$ m

Distributed loads applied to the joist:

self-weight of the beam	$q_{G,beam} = 0.06$ kN/m
lightweight slab	0.75 kN/m^2
	$q_{G,slab} = 0.75 \times 0.6 = 0.45$ kN/m
dead load	$q_G = q_{G,beam} + q_{G,slab} = 0.51$ kN/m
imposed load	2.50 kN/m^2
	$q_Q = 2.50 \times 0.6 = 1.50$ kN/m





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3. Exemplo 3

The dimensions of the cross section and the material properties are:

Total height	$h = 200$ mm	
Total width of flange in compression	$b_1 = 74$ mm	
Total width of flange in tension	$b_2 = 66$ mm	
Total width of edge fold	$c = 20.8$ mm	
Internal radius	$r = 3$ mm	
Nominal thickness	$t_{nom} = 2$ mm	
Steel core thickness	$t = 1.96$ mm	
Basic yield strength	$f_{yb} = 350\ N/mm^2$	
Modulus of elasticity	$E = 210000\ N/mm^2$	
Poisson's ratio	$\nu = 0.3$	
Partial factors (§§2.3.1)	$\gamma_{M0} = 1.0$	
	$\gamma_{M1} = 1.0$	
	$\gamma_G = 1.35$	– permanent loads
	$\gamma_Q = 1.50$	– variable loads

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3. Exemplo 3

Design of the joist for Ultimate Limit State (see Flowchart 3.7)

Effective section properties at the ULS (§§3.7.3, Flowcharts 3.1 and 3.2)
The properties of effective cross section in bending were calculated following the procedure presented in Example 3.1.

Second moment of area of cold-formed lipped channel section subjected to bending about its major axis: $I_{eff,y} = 4139861\ mm^4$

Position of the neutral axis:

- from the flange in compression: $z_c = 102.3$ mm
- from the flange in tension: $z_t = 95.7$ mm

3. Exemplo 3

Effective section modulus:

- with respect to the flange in compression:

$$W_{eff,y,c} = \frac{I_{eff,y}}{z_c} = \frac{4139861}{102.3} = 40463 \text{ mm}^3$$

- with respect to the flange in tension:

$$W_{eff,y,t} = \frac{I_{eff,y}}{z_t} = \frac{4139861}{95.7} = 43264 \text{ mm}^3$$

$$W_{eff,y} = \min\left(W_{eff,y,c}, W_{eff,y,t}\right) = 40463 \text{ mm}^3$$

Applied loading on the joist at the ULS (§§2.3.1, Flowchart 2.1)

$$q_d = \gamma_G q_G + \gamma_Q q_Q = 1.35 \times 0.51 + 1.50 \times 1.50 = 2.94 \text{ kN/m}$$

Maximum applied bending moment (at mid-span) about the major axis *y-y*:

$$M_{Ed} = q_d L^2 / 8 = 2.94 \times 5.5^2 / 8 = 11.12 \text{ kNm}$$

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3. Exemplo 3

Check of bending resistance at ULS

Design moment resistance of the cross section for bending (§§3.8.4, eqn. (3.83)):

$$M_{c,Rd} = W_{eff,y} f_{yb} / \gamma_{M0} = 40463 \times 10^{-9} \times 350 \times 10^3 / 1.0 = 14.16 \text{ kNm}$$

Verification of bending resistance (§§3.8.4, eqn. (3.83)):

$$\frac{M_{Ed}}{M_{c,Rd}} = \frac{11.12}{14.16} = 0.785 < 1 \; - \text{OK}$$

Check of shear resistance at the ULS

Design to shear force

Maximum applied shear force

$$V_{Ed} = q_d L / 2 = 2.94 \times 5.5 / 2 = 8.085 \text{ kN}$$

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3. Exemplo 3

Design shear buckling resistance (§§3.8.5, eqn. (3.91))

$$V_{b,Rd} = \frac{\frac{h_w}{\sin\phi} t f_{bv}}{\gamma_{M0}}$$

where

f_{bv} is the shear strength, considering buckling

For a web with stiffening at the support:

$$f_{bv} = 0.58 f_{yb} \qquad \text{if} \qquad \bar{\lambda}_w \le 0.83$$

$$f_{bv} = 0.48 f_{yb} / \bar{\lambda}_w \qquad \text{if} \qquad \bar{\lambda}_w > 0.83$$

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3. Exemplo 3

The relative slenderness $\bar{\lambda}_w$ for webs without longitudinal stiffeners:

$$\bar{\lambda}_w = 0.346 \frac{s_w}{t} \sqrt{\frac{f_{yb}}{E}} = 0.346 \frac{h - t_{nom}}{t} \sqrt{\frac{f_{yb}}{E}} =$$

$$0.346 \times \frac{200-2}{1.96} \times \sqrt{\frac{350}{210000}} = 1.427$$

$\bar{\lambda}_w = 1.427 > 0.83$ so:

$$f_{bv} = 0.48 f_{yb} / \bar{\lambda}_w = 0.48 \times 350/1.427 = 117.73 \text{ N/mm}^2$$

$$V_{b,Rd} = \frac{\frac{(200-2)\times 10^{-3}}{\sin 90°} \times 1.96 \times 10^{-3} \times 117.73 \times 10^{3}}{1.0} = 45.7 \text{ kN}$$

Verification of shear resistance (§§3.8.5, eqn. (3.90))

$$\frac{V_{Ed}}{V_{c,Rd}} = \frac{8.085}{45.7} = 0.177 < 1 \text{ – OK}$$

3. Exemplo 3

Check of local transverse resistance at ULS

Support reaction:

$$F_{Ed} = q_d L/2 = 2.94 \times 5.5/2 = 8.085 \text{ kN}$$

To obtain the local transverse resistance of the web for a cross section with a single unstiffened web, the following criteria should be satisfied (§§3.8.7.2, eqn. (3.118)):

$h_w/t \le 200$ $\quad 198/1.96 = 101.02 < 200$ – OK

$r/t \le 6$ $\quad 3/1.96 = 1.53 < 6$ – OK

$45° \le \phi \le 90°$

where ϕ is the slope of the web relative to the flanges: $\phi = 90°$ – OK

The local transverse resistance of the web (§§3.8.7, eqn. (3.120), Figure 3.53)

The bearing length is: $s_s = 110$ mm

For $s_s/t = 80/1.96 = 40.816 < 60$ the local transverse resistance of the web $R_{w,Rd}$ is:

$$R_{w,Rd} = \frac{k_1 k_2 k_3 \left[5.92 - \frac{h_w/t}{132}\right]\left[1 + 0.01\frac{s_s}{t}\right] t^2 f_{yb}}{\gamma_{M1}}$$

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3. Exemplo 3

where (§§3.8.7)

$k_1 = 1.33 - 0.33k$ with $\quad k = f_{yb}/228 = 350/228 = 1.535$

$k_1 = 1.33 - 0.33 \times 1.535 = 0.823$

$k_2 = 1.15 - 0.15 r/t = 1.15 - 0.15 \times 3/1.96 = 0.92$

$k_3 = 0.7 + 0.3(\phi/90)^2 = 0.7 + 0.3 \times (90/90)^2 = 1$

$$R_{w,Rd} = \frac{0.823 \times 0.92 \times 1 \times \left[5.92 - \frac{198/1.96}{132}\right] \times \left[1 + 0.01 \times \frac{110}{1.96}\right] \times 1.96^2 \times 350}{1.0}$$

$= 8193$ N$=8.193$ kN

Verification of local transverse force (§§3.8.7.1, eqn. (3.117))

$$F_{Ed} = 8.085 \text{ kN} < R_{w,Rd} = 8.193 \text{ kN} \text{ – OK}$$

4. Exemplo 4

Example 3.6: Design of a wall stud in compression and uniaxial bending.

This example presents the design of an external wall stud in compression and uniaxial bending. The stud has pinned end conditions and is composed of two thin-walled cold-formed lipped channel sections. The connection between the channels is assumed to be rigid (a welded connection, for example). Boards are attached to both flanges to prevent the buckling of the wall stud.

Basic Data

Height of wall stud $H = 3.5\ \text{m}$

The applied concentrated load on the external wall stud (compression) at the Ultimate Limit State is: $N_{Ed} = 17.4\ \text{kN}$

Uniform wind pressure: $q_w = 0.504\ \text{kN/m}$

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4. Exemplo 4

The maximum applied bending moment:

$$M_{Ed} = \gamma_Q q_w H^2/8 = 1.5 \times 0.504 \times 3.50^2/8 = 1.16\ \text{kNm}$$

The dimensions of one lipped channel section and the material properties are:

Total height $h = 150\ \text{mm}$

Total width of flange $b = 40\ \text{mm}$

Total width of edge fold $c = 15\ \text{mm}$

Internal radius $r = 3\ \text{mm}$

Nominal thickness $t_{nom} = 1.2\ \text{mm}$

Steel core thickness $t = 1.16\ \text{mm}$

Basic yield strength $f_{yb} = 320\ \text{N/mm}^2$

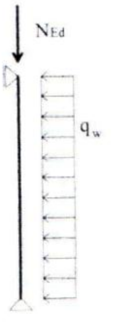
Modulus of elasticity $E = 210000\ \text{N/mm}^2$

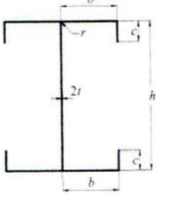
Poisson's ratio $\nu = 0.3$

Shear modulus $G = \dfrac{E}{2(1+\nu)} = 81000\ \text{N/mm}^2$

Partial factors $\gamma_{M0} = 1.0$

$\gamma_{M1} = 1.0$





4. Exemplo 4

Properties of the gross cross section

Area of gross cross section: $A = 522 \text{ mm}^2$

Second moment of area about strong axis y-y: $I_y = 1.143 \times 10^6 \text{ mm}^4$

Effective section properties of the cross section (§§3.7.3, Flowcharts 3.1 and 3.2)

The properties of effective cross section were calculated following the procedures presented in Examples 3.1 and 3.2.

Effective area of the cross section when subjected to compression only:

$$A_{eff.c} = 336.28 \text{ mm}^2$$

Effective section modulus in bending:

with respect to the flange in compression: $W_{eff.y.c} = 18308 \text{ mm}^3$

with respect to the flange in tension: $W_{eff.y.t} = 19040 \text{ mm}^3$

$$W_{eff.y.min} = \min\left(W_{eff.y.c}, W_{eff.y.t}\right) = 18308 \text{ mm}^3$$

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4. Exemplo 4

Resistance check of the cross section

The following criterion should be met (§§3.8.9, eqn. (3.128a)):

$$\frac{N_{Ed}}{N_{c,Rd}} + \frac{M_{y,Ed} + \Delta M_{y,Ed}}{M_{cy,Rd,com}} \leq 1$$

where

$$N_{c,Rd} = A_{eff} f_{yb} / \gamma_{M0}$$

$$M_{cz,Rd,com} = W_{eff,com} f_{yb} / \gamma_{M0}$$

$$\Delta M_{y,Ed} = N_{Ed} e_{Ny}$$

e_{Ny} – is the shift of the centroidal y-y axis; the cross section is doubly symmetric $\Rightarrow$ $e_{Ny} = 0$.

The resistance check is:

$$\frac{17.4 \times 10^3}{336.28 \times 320/1.0} + \frac{1.16 \times 10^6 + 0}{18308 \times 320/1.0} = 0.360 < 1 \quad \text{– OK}$$

5. Exemplo 5

Example 4.1: This example deals with the design of an internal wall stud in compression. The stud has pinned end conditions and is composed of two thin-walled cold-formed back-to-back lipped channel sections. The connection between the channels is assumed to be rigid (a welded connection, for example). No restraints against buckling are applied between the ends.

Basic Data

Height of column	$H = 3.00$ m
Span of floor	$L = 6.00$ m
Spacing between floor joists	$S = 0.6$ m

Distributed loads applied to the floor:

- dead load – lightweight slab: $1.5\ \text{kN/m}^2$
  $q_G = 1.5\times 0.6 = 0.9\ \text{kN/m}$
- imposed load: $3.00\ \text{kN/m}^2$
  $q_Q = 3.00\times 0.6 = 1.80\ \text{kN/m}$

5. Exemplo 5

Ultimate Limit State concentrated load from upper level and roof:

$$Q = 7.0\ \text{kN}$$

The dimensions of a lipped channel section and the material properties are:

Total height	$h = 150$ mm
Total width of flange	$b = 40$ mm
Total width of edge fold	$c = 15$ mm
Internal radius	$r = 3$ mm
Nominal thickness	$t_{nom} = 1.2$ mm
Steel core thickness (§§2.4.2.3)	$t = 1.16$ mm
Steel grade	S350GD+Z
Basic yield strength	$f_{yb} = 350\ \text{N/mm}^2$
Modulus of elasticity	$E = 210000\ \text{N/mm}^2$
Poisson's ratio	$\nu = 0.3$
Shear modulus	$G = \dfrac{E}{2(1+\nu)} = 81000\ \text{N/mm}^2$
Partial factors	$\gamma_{M0} = 1.0$ (§§2.3.1)
	$\gamma_{M1} = 1.0$
	$\gamma_G = 1.35$, – permanent loads (§§2.3.1)
	$\gamma_Q = 1.50$ – variable loads

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5. Exemplo 5

The applied concentrated load on the external column (compression) at the Ultimate Limit State is (§§2.3.1, Flowchart 2.1):

$$N_{Ed} = (\gamma_G q_G + \gamma_Q q_Q) L/2 + Q = (1.35 \times 0.9 + 1.50 \times 1.80) \times 5/2 + 7 = 16.79\text{kN}$$

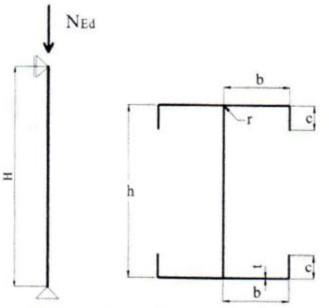

Figure 4.16 – Overall dimensions of internal wall stud and cross section

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5. Exemplo 5

Properties of the gross cross section

Area of gross cross section:	$A = 592\ \text{mm}^2$
Radii of gyration:	$i_y = 57.2\ \text{mm}$; $i_z = 18\ \text{mm}$
Second moment of area about strong axis *y-y*:	$I_y = 1.936 \times 10^6\ \text{mm}^4$
Second moment of area about weak axis *z-z*:	$I_z = 19.13 \times 10^4\ \text{mm}^4$
Warping constant:	$I_w = 4.931 \times 10^8\ \text{mm}^6$
Torsion constant:	$I_t = 266\ \text{mm}^4$

Effective section properties of the cross section (§§3.7.3, Flowchart 3.1 and Flowchart 3.2)

The properties of effective cross section were calculated following the procedures presented in Example 3.2 in Chapter 3.

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5. Exemplo 5

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Effective area of the cross section when subjected to compression only:

$$A_{eff} = 322 \text{ mm}^2$$

Resistance check of the cross section

The following criterion should be met (§§3.8.3, eqn. (3.80), Flowchart 3.4):

$$\frac{N_{Ed}}{N_{c,Rd}} \leq 1$$

where

$$N_{c,Rd} = A_{eff} f_{yb} / \gamma_{M0}$$ (§§3.8.3, eqn. (3.81), Flowchart 3.5)

The cross section is doubly symmetric and so the shift of the centroidal y–y axis is $e_{Ny} = 0$ (§§3.8.3, Figure 3.43).

The resistance check is:

$$\frac{16.79 \times 10^3}{322 \times 350/1.0} = 0.149 < 1 - \text{OK}$$

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5. Exemplo 5

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Buckling resistance check

Members which are subjected to axial compression should satisfy (§§4.2.2, eqn. (4.45), Flowchart 4.1):

$$\frac{N_{Ed}}{N_{b,Rd}} \leq 1$$

$$N_{b,Rd} = \frac{\chi A_{eff} f_y}{\gamma_{M1}}$$, where χ is the reduction factor for the relevant buckling mode.

$$\chi = \frac{1}{\phi + \sqrt{\phi^2 - \bar{\lambda}^2}}$$ but $\chi \leq 1.0$ (§§4.2.2, eqn. (4.47))

$$\phi = 0.5\left[1 + \alpha\left(\bar{\lambda} - 0.2\right) + \bar{\lambda}^2\right]$$

α – imperfection factor

The non-dimensional slenderness is:

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$$\bar{\lambda} = \sqrt{\frac{A_{eff} f_{yb}}{N_{cr}}}$$

N_{cr} – the elastic critical force for the relevant buckling mode.

Determination of the reduction factors χ_y, χ_z, χ_T

Flexural buckling (§§4.2.2.1, eqn. (4.48), Flowchart 4.2)

$$\bar{\lambda}_F = \sqrt{\frac{A_{eff} f_{yb}}{N_{cr}}} = \frac{L_{cr}}{i} \frac{\sqrt{A_{eff}/A}}{\lambda_1}$$

The buckling length:

$$L_{cr,y} = L_{cr,z} = H = 3000 \text{ mm}$$

$$\lambda_1 = \pi \sqrt{\frac{E}{f_{yb}}} = \pi \times \sqrt{\frac{210000}{350}} = 76.95$$

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5. Exemplo 5

Buckling about y–y axis (§§4.2.2.1, Table 4.2):

$$\bar{\lambda}_y = \frac{L_{cr,y}}{i_y} \frac{\sqrt{A_{eff}/A}}{\lambda_1} = \frac{3000}{57.2} \times \frac{\sqrt{322/592}}{76.95} = 0.503$$

$\alpha_y = 0.21$ – buckling curve *a* (§§4.2.1.2, Table 4.1)

$$\phi_y = 0.5\left[1 + \alpha_y\left(\bar{\lambda}_y - 0.2\right) + \bar{\lambda}_y^2\right] =$$
$$= 0.5 \times \left[1 + 0.21 \times (0.503 - 0.2) + 0.503^2\right] = 0.658$$

$$\chi_y = \frac{1}{\phi_y + \sqrt{\phi_y^2 - \bar{\lambda}_y^2}} = \frac{1}{0.658 + \sqrt{0.658^2 - 0.503^2}} = 0.924$$

Buckling about z–z axis (§§4.2.2.1, Table 4.2):

$$\bar{\lambda}_z = \frac{L_{cr,z}}{i_z} \frac{\sqrt{A_{eff}/A}}{\lambda_1} = \frac{3000}{18} \times \frac{\sqrt{322/592}}{76.95} = 1.597$$

$\alpha_z = 0.34$ – buckling curve *b* (§§4.2.1.2, Table 4.1)

$$\phi_z = 0.5\left[1 + \alpha_z\left(\bar{\lambda}_z - 0.2\right) + \bar{\lambda}_z^2\right] =$$

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$$= 0.5 \times \left[1 + 0.34 \times (1.597 - 0.2) + 1.597^2\right] = 2.013$$

$$\chi_z = \frac{1}{\phi_z + \sqrt{\phi_z^2 - \bar{\lambda}_z^2}} = \frac{1}{2.013 + \sqrt{2.013^2 - 1.597^2}} = 0.309$$

Torsional buckling (§§4.2.1.1, eqn. (4.14))

$$N_{cr,T} = \frac{1}{i_o^2}\left(GI_t + \frac{\pi^2 EI_w}{l_T^2}\right)$$

where

$i_o^2 = i_y^2 + i_z^2 + y_o^2 + z_o^2$

y_o, z_o – the shear centre coordinates with respect to the centroid of the gross cross section: $y_o = z_o = 0$

$i_o^2 = 57.2^2 + 18^2 + 0 + 0 = 3594 \text{ mm}^2$

$l_T = H = 3000 \text{ mm}$

5. Exemplo 5

The elastic critical force for torsional buckling is:

$$N_{cr,T} = \frac{1}{3594} \times \left(81000 \times 266 + \frac{\pi^2 \times 210000 \times 4.931 \times 10^8}{3000^2}\right) =$$

$$= 37.59 \times 10^3 \text{ N}$$

The elastic critical force will be:

$$N_{cr} = N_{cr,T} = 37.59 \text{ kN}$$

The non-dimensional slenderness is:

$$\bar{\lambda}_T = \sqrt{\frac{A_{eff} f_{yb}}{N_{cr}}} = \sqrt{\frac{322 \times 350}{37.59 \times 10^3}} = 1.731$$

$\alpha_T = 0.34$ – buckling curve *b* (§§4.2.2.1, Table 4.2, §§4.2.1.2, Table 4.1)

$$\phi_T = 0.5\left[1 + \alpha_T\left(\bar{\lambda}_T - 0.2\right) + \bar{\lambda}_T^2\right] =$$

$$= 0.5 \times \left[1 + 0.34 \times (1.731 - 0.2) + 1.731^2\right] = 2.258$$

The reduction factor for torsional buckling is:

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$$\chi_T = \frac{1}{\phi_T + \sqrt{\phi_T^2 - \bar{\lambda}_T^2}} = \frac{1}{2.258 + \sqrt{2.258^2 - 1.731^2}} = 0.270$$

$$\chi = \min(\chi_y, \chi_z, \chi_T) = \min(0.924, 0.309, 0.270) = 0.270$$

$$N_{b,Rd} = \frac{\chi A_{eff} f_y}{\gamma_{M1}} = \frac{0.270 \times 322 \times 350}{1.00} = 30429\text{N} = 30.429\text{kN}$$

$$\frac{N_{Ed}}{N_{b,Rd}} = \frac{16.79}{30.429} = 0.552 \leq 1 - \text{OK}$$

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6. Exemplo 6

Example 4.3: Design of an unrestrained cold-formed steel beam in bending at the Ultimate Limit State (see Figure 4.32). The beam has pinned end conditions and is composed of two thin-walled cold-formed steel back-to-back lipped channel sections. The connection between the channels is assumed to be rigid.

Basic Data

Span of beam	$L = 4.5$ m
Spacing between beams	$S = 3.0$ m

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Distributed loads applied to the joist:

self-weight of the beam	$q_{G,beam} = 0.14\ \text{kN/m}$
weight of the floor and finishing	$0.6\ \text{kN/m}^2$
	$q_{G,slab} = 0.55 \times 3.0 = 1.65\ \text{kN/m}$
total dead load	$q_G = q_{G,beam} + q_{G,slab} = 1.79\ \text{kN/m}$
imposed load	$1.50\ \text{kN/m}^2$
	$q_Q = 1.50 \times 3.0 = 4.50\ \text{kN/m}$

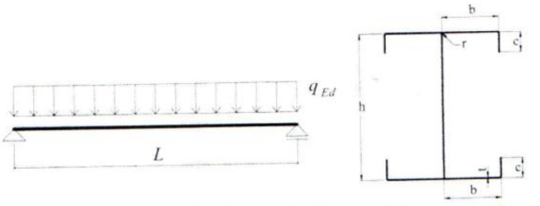

Figure 4.32 – Overall dimensions of beam and of the cross section

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The dimensions of the cross section and the material properties are:

Total height	$h = 250$ mm
Total width of flanges	$b = 70$ mm
Total width of edge fold	$c = 25$ mm
Internal radius	$r = 3$ mm
Nominal thickness	$t_{nom} = 3.0$ mm
Steel core thickness (§§2.4.2.3)	$t = 2.96$ mm
Steel grade	S350GD+Z
Basic yield strength	$f_{yb} = 350\ \text{N/mm}^2$
Modulus of elasticity	$E = 210000\ \text{N/mm}^2$
Poisson's ratio	$\nu = 0.3$

Partial factors $\gamma_{M0} = 1.0$ (§§2.3.1)

$\gamma_{M1} = 1.0$

$\gamma_G = 1.35$ – permanent loads (§§2.3.1, Table 2.3)

$\gamma_Q = 1.50$ – variable loads

Design of the beam for Ultimate Limit State

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Properties of the gross cross section

Second moment of area about strong axis *y–y*: $I_y = 2302.15\times10^4\ \text{mm}^4$

Second moment of area about weak axis *z–z*: $I_z = 244.24\times10^4\ \text{mm}^4$

Radii of gyration: $i_y = 95.3\ \text{mm}$; $i_z = 31\ \text{mm}$

Warping constant: $I_w = 17692.78\times10^6\ \text{mm}^6$

Torsion constant: $I_t = 7400\ \text{mm}^4$

Effective section properties at the ultimate limit state (§§3.7.3, Flowchart 3.1 and Flowchart 3.2)

The properties of effective cross section were calculated following the procedures presented in Example 3.1 in Chapter 3.

Second moment of area of cold-formed lipped channel section subjected to bending about its major axis: $I_{eff,y} = 22688890\ \text{mm}^4$

Position of the neutral axis:

- from the flange in compression: $z_c = 124.6\ \text{mm}$
- from the flange in tension: $z_t = 122.4\ \text{mm}$

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Effective section modulus:

- with respect to the flange in compression:

$$W_{eff,y,c} = \frac{I_{eff,y}}{z_c} = \frac{22688890}{124.6} = 182094\ \text{mm}^3$$

- with respect to the flange in tension:

$$W_{eff,y,t} = \frac{I_{eff,y}}{z_t} = \frac{22688890}{122.4} = 185367\ \text{mm}^3$$

$$W_{eff,y} = \min\left(W_{eff,y,c}, W_{eff,y,t}\right) = 182094\ \text{mm}^3$$

Applied loading on the beam at ULS (§§2.3.1, Flowchart 2.1)

$$q_d = \gamma_G q_G + \gamma_Q q_Q = 1.35\times1.79 + 1.50\times4.5 = 9.17\ \text{kN/m}$$

Maximum applied bending moment (at mid-span) about the major axis *y-y*:

$$M_{Ed} = q_d L^2/8 = 9.17\times4.5^2/8 = 23.21\ \text{kNm}$$

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6. Exemplo 6

Check of bending resistance at ULS

Design moment resistance of the cross section for bending (§§3.8.4, eqn. (3.83), Flowchart 3.6):

$$M_{c,Rd} = W_{eff,y} f_{yb} / \gamma_{M0} = 182094 \times 10^{-9} \times 350 \times 10^{3} / 1.0 = 63.73 \text{ kNm}$$

Verification of bending resistance (§§3.8.4, eqn. (3.82), Flowchart 3.6):

$$\frac{M_{Ed}}{M_{c,Rd}} = \frac{23.21}{63.73} = 0.364 < 1 \ - \text{OK}$$

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6. Exemplo 6

Determination of the reduction factor χ_{LT}

Lateral-torsional buckling (§§4.3.2.1, eqn. (4.57), Flowchart 4.4)

$$\chi_{LT} = \frac{1}{\phi_{LT} + \sqrt{\phi_{LT}^{2} - \bar{\lambda}_{LT}^{2}}} \quad \text{but} \quad \chi_{LT} \le 1.0$$

$$\phi_{LT} = 0.5\left[1 + \alpha_{LT}\left(\bar{\lambda}_{LT} - 0.2\right) + \bar{\lambda}_{LT}^{2}\right]$$

$$\alpha_{LT} = 0.34 \ - \text{buckling curve } b$$

The non-dimensional slenderness is (§§4.3.2.1):

$$\bar{\lambda}_{LT} = \sqrt{\frac{W_{eff,y,min} f_{yb}}{M_{cr}}}$$

M_{cr} – the elastic critical moment for lateral-torsional buckling

$$M_{cr} = C_1 \frac{\pi^2 E I_z}{L^2} \sqrt{\frac{I_w}{I_z} + \frac{L^2 G I_t}{\pi^2 E I_z}}$$

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where $C_1 = 1.127$ for a simply supported beam under uniform loading

$$M_{cr} = 1.127 \times \frac{\pi^2 \times 210000 \times 244.24 \times 10^4}{4500^2} \times$$
$$\times \sqrt{\frac{17692.78 \times 10^6}{244.24 \times 10^4} + \frac{4500^2 \times 81000 \times 7400}{\pi^2 \times 210000 \times 244.24 \times 10^4}}$$

$$M_{cr} = 27.66 \text{ kNm}$$

$$\bar{\lambda}_{LT} = \sqrt{\frac{W_{eff,y,min} f_{yb}}{M_{cr}}} = \sqrt{\frac{182094 \times 350}{27.66 \times 10^6}} = 1.518$$

$$\phi_{LT} = 0.5\left[1 + \alpha_{LT}\left(\bar{\lambda}_{LT} - 0.2\right) + \bar{\lambda}_{LT}^{\ 2}\right] =$$
$$= 0.5 \times \left[1 + 0.34 \times (1.437 - 0.2) + 1.437^2\right] = 1.743$$

$$\chi_{LT} = \frac{1}{\phi_{LT} + \sqrt{\phi_{LT}^{\ 2} - \bar{\lambda}_{LT}^{\ 2}}} = \frac{1}{1.743 + \sqrt{1.734^2 - 1.437^2}} = 0.369$$

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6. Exemplo 6

Check of buckling resistance at ULS

Design moment resistance of the cross section for bending (§§4.3.2.1, eqn. (4.56), Flowchart 4.4):

$$M_{b,Rd} = \chi_{LT} W_{eff,y} f_{yb} / \gamma_{M1} = 0.369 \times 182091 \times 10^{-9} \times 350 \times 10^3 / 1.0 = 23.52 \text{ kNm}$$

Verification of buckling resistance (§§4.3.2.1, eqn. (4.55), Flowchart 4.4):

$$\frac{M_{Ed}}{M_{b,Rd}} = \frac{23.21}{23.52} = 0.987 < 1 \ - \text{OK}$$

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