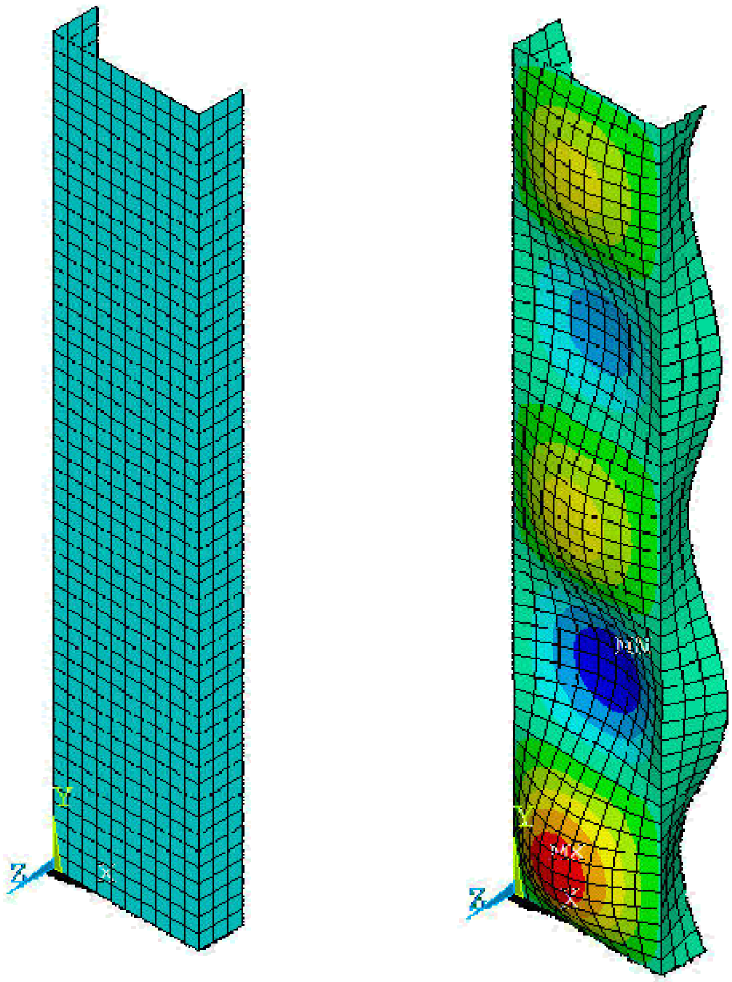

Exemplo Pórtico Completo

Programa de Pós-Graduação em Engenharia Civil
PGECIV - Mestrado Acadêmico
Faculdade de Engenharia – FEN/UERJ
Disciplina: Tópicos Especiais em Estruturas (Chapa Dobrada)
Professor: Luciano Rodrigues Ornelas de Lima

2.5 Example D: Portal frames

2.5.1 Statement of the problem

This example considers the design of the main portal frame. The frame is composed of double C sections (with lips). The eaves connection is made rigid by means of diagonal bracing members. The bases are assumed to be pinned.

Note: If not else specified, chapter and clause references are to Part 1.3 of EN 1993 [1].

The analysis of this type of frame is covered by Chapter 5 of EN 1993-1-1 [2]. Sway imperfections of the frame are included, bow member imperfections are not modelled and are considered by individual stability checks of members according to 6.3 of [2]. Second order effects are small in this example but are accounted for by using an elastic second order analysis method.

The main frame members are composed of double C sections with lips. The effective cross-sectional properties of those sections are not determined in this example - the reference is made to Example L.

The frame is clad with purlins and sheeting on the roof, while the walls are clad with liner trays. The effect of this cladding arrangement is to restrain the rafters at purlin points (which provide both lateral and torsional restraint by means of stays to the compression flange of the rafter where required), while the restraining effect of the liner trays on the columns is neglected.

All connections are pinned, with the exception of the ridge which is assumed to be fully rigid. Arrangement of connections is shown in Fig. 2.24. Check of joints is not covered in this example.

In addition to the above assumptions, it is also assumed that the individual channel sections are connected together at sufficiently small centres as to prevent them buckling separately.

The design requires the following calculation steps:

- Definition of structural model, load cases including sway imperfections and relevant load case combinations.
- Effective section properties at the ultimate and serviceability limit states.
- Analysis of the frame for each load case combination together with a determination of deflections. The analysis used for this example was a second order analysis with sway imperfections introduced into the frame.
- Serviceability checks to ensure that neither the vertical nor the horizontal deflections are excessive.
- Axial resistance of each section using the effective section properties, considering buckling for members under compression.
- Bending resistance of the section using the effective section properties, considering lateral-torsional buckling where relevant.

- Design check for each member under a governing load case combination.

2.5.2 Frame appearance, loading

Details of the frame and loading are shown in Fig. 2.24. The figure illustrates the overall dimensions of the structure.

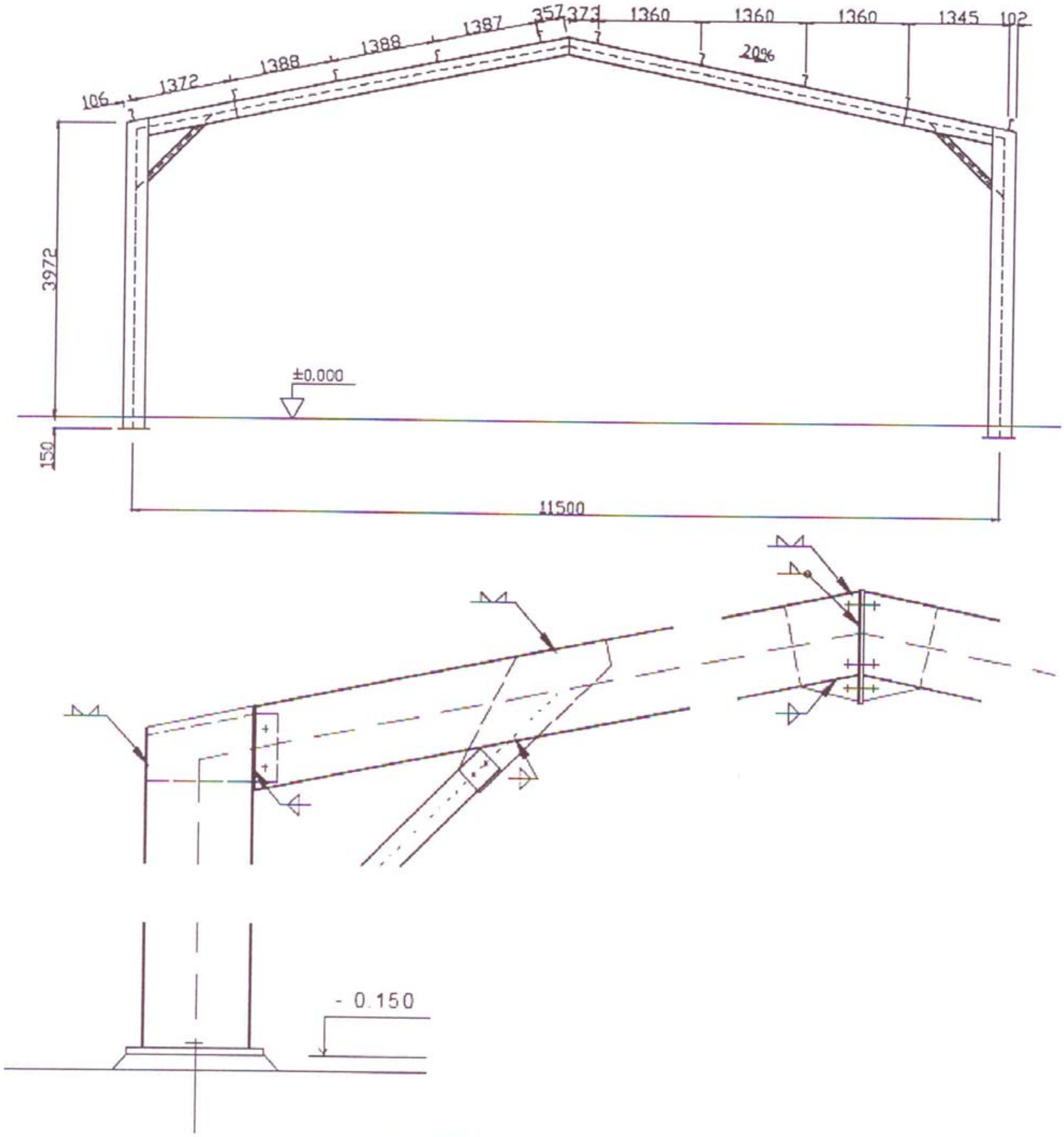

Fig. 2.24: Appearance of the frame

Note: If the ridge connection was designed without end plates the behaviour of the joint would be semi-rigid.

Frame spacing: 4.5 m centres

Dead load (both on roof and on walls)
- Upper bond value: $g_{k,sup} = 0.15$ kN/m^2
- Lower bond value: $g_{k,inf} = 0.10$ kN/m^2

Selfweight of frame members is applied by static computer program.

Imposed load on roof: 0.65 kN/m^2

Wind loading on structure: As shown in the Fig. 2.25

Two wind load cases will be considered:
- Wind load case 1: External side load + internal overpressure;
- Wind load case 2: External side load + internal underpressure

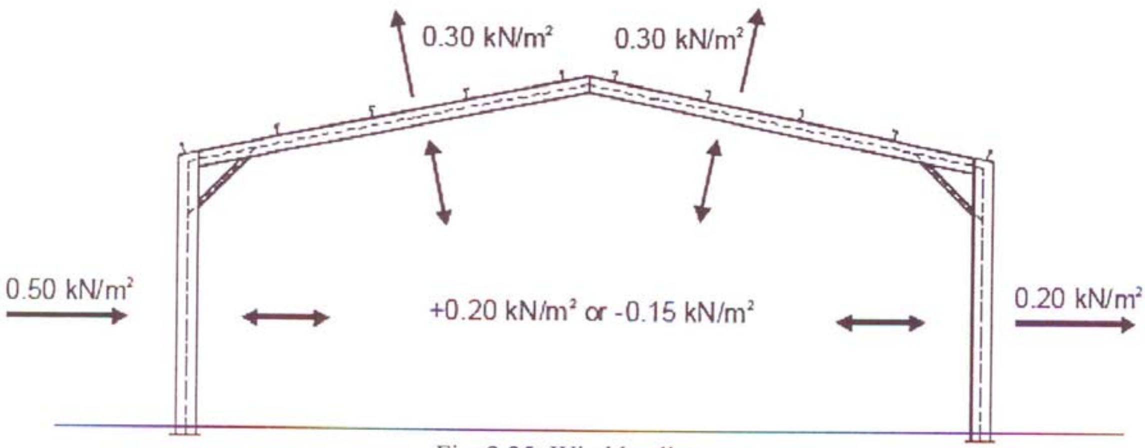

Fig. 2.25: Wind loading

Sway imperfections, see 5.3.2 of EN 1993-1-1 [2], are taken into account by equivalent lateral forces introduced for each column. Those forces are considered for load cases that develop compression in columns (Dead load and Imposed load) and belong to appropriate load case. This is simplified attitude: more precisely the sway imperfections should be determined separately for each load combination.

Initial inclination of columns ([2], Eq. (5.5)):

$$\phi = \phi_0 \cdot \alpha_h \cdot \alpha_m = 1/200 \cdot 1.0 \cdot 0.87 = 1/230$$

where $\phi_0 = 1/200$

$$\alpha_h = \frac{2}{\sqrt{h}} = \frac{2}{\sqrt{4}} = 1.0$$

$$\alpha_m = \sqrt{0.5\left(1+\frac{1}{m}\right)} = \sqrt{0.5\left(1+\frac{1}{2}\right)} = 0.87$$

The following load case combinations (LCC) were considered (in accordance with EN 1990):

- Ultimate limit state (ULS):

1) 1.35 Dead load + 1.5 Imposed load
2) 1.35 Dead load + 1.5 Imposed load + ψ_0 1.5 Wind load 2, where $\psi_0 = 0.6$
3) 1.0 Dead load + 1.5 Wind load 1
4) 1.0 Dead load + 1.5 Wind load 2

- Serviceability limit state (SLS):

5) 1.0 Dead load + 1.0 Imposed Load
6) 1.0 Wind load 1

2.5.3 Global analysis

2.5.3.1 General data

Partial Safety factors $\gamma_{M0} = \gamma_{M1} = 1.00$

Steel grade S350GD ... $f_{yb} = 350$ N/mm^2
$f_u = 420$ N/mm^2

Fig. 2.26 shows the line diagram (to the centre lines of steelwork) used for the analysis, together with the node and member numbering system.

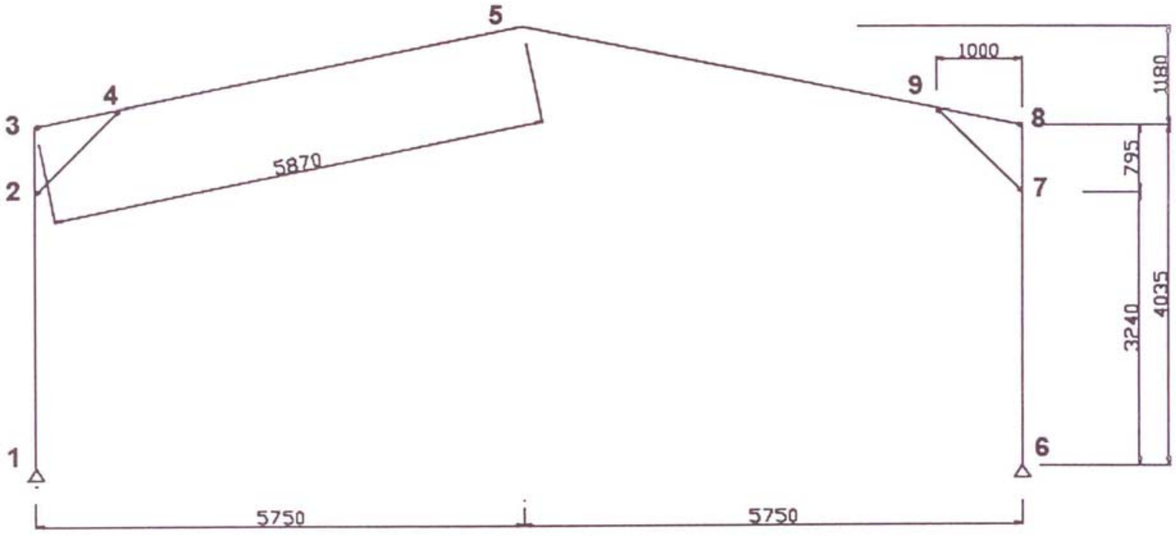

Fig. 2.26: Static scheme

2.5.3.2 *Design of profiles*

Because the structure is statically indeterminate the profiles shall be designed before the static analysis calculations. Generally, when profiles are changed after the analysis due to insufficient resistance or stiffness, the static analysis shall be repeated with redesigned profiles.

2.5.3.2.1 *Column*

Use double C section with lips.
Profile 300 x 160 x 25 x 3 (height x flange total width x lip x thickness)

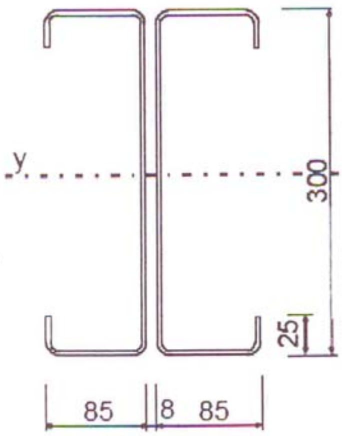

Fig. 2.27: Dimensions of the section

The specified nominal dimensions are the outer ones. Internal radius $r = 2 \cdot t$.
Nominal thickness $t_N = 3.00$ mm

Core thickness (design thickness, Clause 3.2.4) is equal to nominal thickness minus zinc coating:

$$t = 3.00 - 0.04 = 2.96 \text{ mm}$$

2.5.3.2.2 Rafter

Use double C section with lips.

Profile 276 x 125 x 20 x 3 (Millpac 276 x 3.0 section, tabulated e.g. in http://www.corusconstruction.com [4])

The specified nominal dimensions are the outer ones. Internal radius $r = 2\,t$.

Design thickness $t = 2.96$ mm (see Section 2.5.3.2.1.).

2.5.3.2.3 Brace

Use double plain channel section (U section without lips).

Profile 80 x 96 x 2.4 (Metsec section, tabulated e.g. in http://www.corusconstruction.com [4])

2.5.3.2.4 Cross section properties

Both gross cross-section and effective cross-section properties are listed in Tab. 2.2.

Calculation of effective cross-section properties for C sections with lips is not presented here, it is illustrated in Example L. Calculation of effective cross-section properties for plain channel section is shown below the table.

Table 2.2: Cross-section properties

			Column	Rafter	Brace
Gross section	A	mm²	2872	2463	784
	I_y	mm⁴	$36.45 \cdot 10^6$	$25.28 \cdot 10^6$	$811 \cdot 10^6$
	I_z	mm⁴	$3.36 \cdot 10^6$	$1.568 \cdot 10^6$	$431.4 \cdot 10^6$
Effective section uniform compression	A_{eff}	mm²	1691	1498	648
Effective section bending along y-y	$I_{eff,y}$	mm⁴	$34.80 \cdot 10^6$	$24.652 \cdot 10^6$	

Single plain channel section 80 x 48 x 2.4 effective properties for uniform compression.

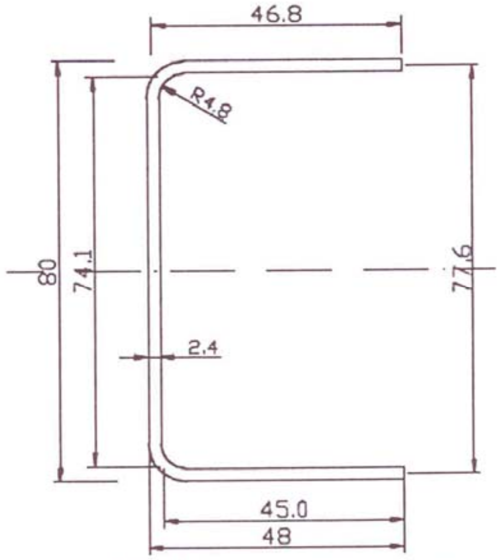

Fig. 2.28: Dimensions of the section

Design thickness $t = 2.40 - 0.04 = 2.36$ mm
Internal radius $r_{in} = 2 \cdot t_{nom} = 2 \cdot 2.4 = 4.8$ mm

Centreline dimensions:

$h_c = 80 - 2.4 = = 77.6$ mm
$b_c = 48 - 2.4/2 = 46.8$ mm
$r = 4.8 + 2.4/2 = 6.0$ mm

Notional flat widths:

$h_p = 77.6 - 2 \cdot 0.293 \cdot 6 = 74.1$ mm
$b_p = 46.8 - 0.293 \cdot 6 = 45.0$ mm

Flange effective width (using [3], Clause 4.4 for outstand element and constant stress diagram):

The relative slenderness is

$$\overline{\lambda}_p = 1.052 \frac{b_p}{t} \sqrt{\frac{f_y}{E \cdot k_\sigma}} = 1.052 \frac{45.0}{2.36} \sqrt{\frac{350}{210000 \cdot 0.43}} = 1.25$$

where k_σ is the buckling factor, $k_\sigma = 0.43$ (see [3], Tab. 4.2).

For $\overline{\lambda}_p > 0.673$, the reduction factor is

$$\rho = \frac{1.0 - 0.188 / \overline{\lambda}_p}{\overline{\lambda}_p} = \frac{1 - 0.188/1.25}{1.25} = 0.68 < 1.0$$

Effective width:

$$b_{eff} = \rho \cdot b_p = 0.68 \cdot 45.0 = 30.6 \text{ mm}$$

Portion of flange to deduct:

$$b_p - b_{eff} = 45.0 - 30.6 = 14.4 \text{ mm}$$

Web effective width (using [3], Clause 4.4 for doubly supported element and constant stress diagram):

The relative slenderness is

$$\bar{\lambda}_p = 1.052 \frac{b_p}{t} \sqrt{\frac{f_y}{E \cdot k_\sigma}} = 1.052 \frac{74.1}{2.36} \sqrt{\frac{350}{210000 \cdot 4}} = 0.674$$

where k_σ is the buckling factor, $k_\sigma = 4$ (see [3], Tab. 4.1).

For $\bar{\lambda}_p \cong 0.673$, the reduction factor $\rho = 1.0$ and
$h_{eff} = h_p = 74.1$ mm.

Effective cross-section area - double section:

$$A_{eff} = 784 - 2 \cdot 2.36 \cdot (2 \cdot 14.4) = 648 \text{ mm}^2$$

Effective cross-section is shown on Fig. 2.29. Effective properties for bending need not to be determined.

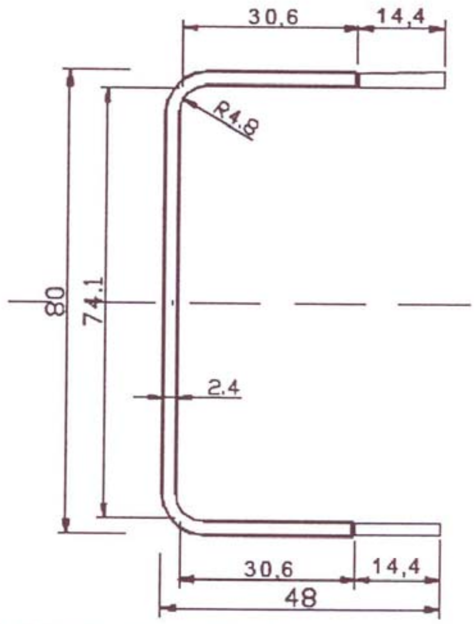

Fig. 2.29: Effective cross-section of brace under compression

2.5.3.3 Serviceability limit state (SLS) check

The deflections should be determined using effective section properties for SLS in global analysis. Those properties can be either constant along the member length (based on the maximum absolute stress on member due to serviceability loading), or may be taken as variable along the member length according to most severe location.

In the example the maximum stresses in SLS are much smaller than yield strength and vary considerably along member lengths. Therefore, as simplification, the deflections are determined using gross section properties in global analysis.

Deflection limits:

- Vertical deflection: Span/250 = 11800/250 = 47.2 mm
- Horizontal side-sway: Height/150 = 4035/150 = 26.9 mm

Note: The deflection limits are not specified in EN 1993. If the limits are not specified in National Annexes they should be specified for each project and agreed with the client.

Calculated deflections:

- Vertical deflection (LCC 5): 30 mm < 47.2 mm
- Horizontal side-sway (LCC 6): 27 mm ≈ 26.9 mm

The limits are satisfied.

Note: The horizontal deflection was limiting for the design of profiles.

2.5.3.4 Internal forces and moments

Table 2.3: Axial forces and moments for load case combinations

Member	LCC	1		2		3		4	
	Node No.	$M_{y.Ed}$ [kNm]	N_{Ed} [kN]	$M_{y.Ed}$ [kNm]	N_{Ed} [kN]	$M_{y.Ed}$ [kNm]	N_{Ed} [kN]	$M_{y.Ed}$ [kNm]	N_{Ed} [kN]
Left column - bottom	1	0	-39.69	0	15.63	0	-26.03	0	-33.96
	2	43.28	-35.65	-37.76	17.82	16.33	-21.92	23.71	-29.89
Left column - top	2	43.28	31.77	-37.76	-37.43	16.33	6.16	23.71	12.52
	3	0	32.87	0	-36.79	0	7.18	0	13.54
Right column – bottom	6	0	-39.94	0	8.88	0	-30.31	0	-38.60
	7	44.48	-35.90	-1.38	11.14	40.71	-26.31	**50.20**	**-34.61**
Right column – top	7	44.48	33.38	-1.38	3.62	40.71	33.93	50.20	42.70
	8	0	34.49	0	4.18	0	35.02	0	43.83
Rafter – left corner	3	0	60.79	0.02	-54.68	0.01	21.96	0	31.77
	4	-27.24	60.80	**27.50**	**-54.68**	-7.83	22.03	-12.92	31.83
Rafter – right corner	8	0	62.94	0.02	0.32	0.01	59.07	0	72.05
	9	-28.48	62.95	-3.63	0.36	-29.12	59.08	**-36.11**	**72.02**
Rafter – span	4	0	0	0	0	0	0	0	0
	9	-28.48	-18.74	-3.63	9.37	-29.12	-11.91	-36.11	-19.15
	max	**25.38**	**-17.29**	23.10	11.36	18.24	-10.43	22.43	-17.66
Brace –left	4	0	-97.57	0	88.90	0	-37.02	0	-61.66
	2	0	-97.75	0	88.76	0	-37.21	0	-61.84
Brace – right	7	0	-100.25	0	0.30	0	-92.51	**0**	**-111.27**
	9	0	-100.06	0	0.44	0	-92.33	0	-111.08

Governing forces and moments are boltprinted.

The results of ultimate limit state analyses are given in Tab.2.3 for each member. In addition, the moment diagram for LCC 4 that is governing for most members is shown on Fig. 2.30.

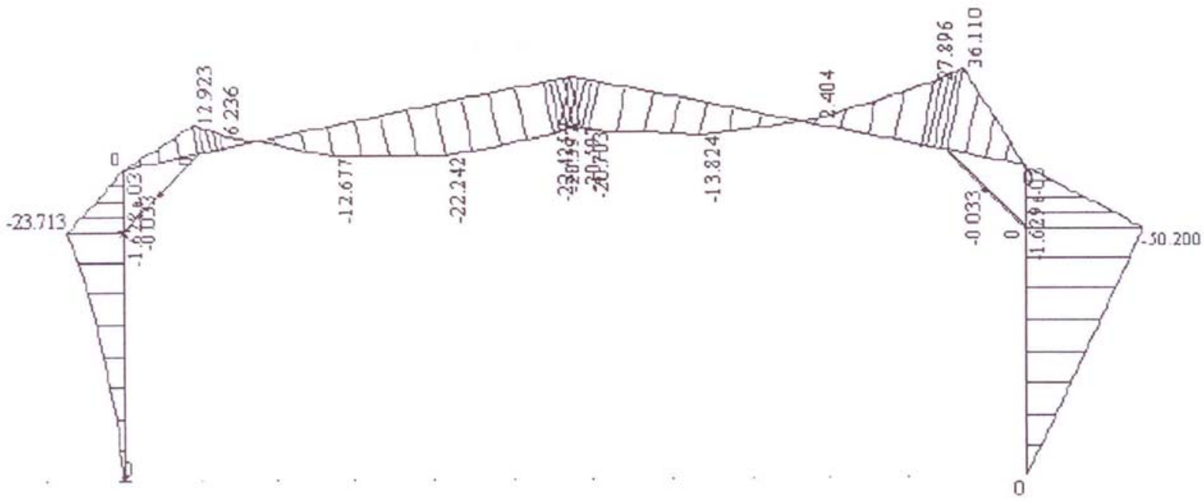

Fig. 2.30: Bending moments for LCC 4

2.5.4 Column check

2.5.4.1 Design of profile

Section 300 x 160 x 25 x 3

Gross section properties (see Tab. 2.2):

$A_g = 2872\ \text{mm}^2$
$I_y = 36.45 \cdot 10^6\ \text{mm}^4$
$I_z = 3.36 \cdot 10^6\ \text{mm}^4$
$i_y = 112.7\ \text{mm}$
$i_z = 34.2\ \text{mm}$
$I_t = 8390\ \text{mm}^4$
$I_w = 71.5 \cdot 10^9\ \text{mm}^6$

Effective cross-section properties (see Tab. 2.2):

- For uniform compression: $A_{eff} = 1691\ \text{mm}^2$

- For bending along *y-y*: $I_{eff.y} = 34.80 \cdot 10^6\ \text{mm}^3$
  $W_{eff.y} = 227.7 \cdot 10^3\ \text{mm}^3$

2.5.4.2 Axial compression buckling resistance

Member length: 4.035 m

Effective lengths for flexural buckling

- $L_{cr,y}$: Because of sway behaviour, the buckling length in plane of the frame was determined using linear stability computer calculation. The results and procedure of determination of $L_{cr,y}$ is shown in Tab. 2.4 for governing load combinations.
- $L_{cr,z}$ = member length = 4.035 m (brace connection does not restrain the column out of frame plane)

Table 2.4: Buckling length $L_{cr,y}$ of right column for governing load combinations

LCC	N_{Ed}	α_{cr}	N_{cr}	I_y	$L_{cr,y}$
	[kN]		[kN]	[mm^4]	[mm]
1	(35.9+39.94)/2 = 37.92	12.00	455	$34.80 \cdot 10^6$	
4	(34.61+38.6)/2 = 36.60	13.11	480	$34.80 \cdot 10^6$	12260

Formulas and variables used in Table 2.4:

α_{cr}: ratio of critical to applied load, determined for appropriate load combination by linear stability computer calculation taken the effective cross-section properties into account;

$$N_{cr} = \alpha_{cr} \cdot N_{Ed}$$

$$L_{cr,y} = \pi \sqrt{\frac{EI_{eff,y}}{N_{cr}}}$$

Note: Because α_{cr} > 10 the internal forces need not to be determined by second order global analysis according to 5.2.1 of EN 1993-1-1 [2].

Relative slenderness

$$\lambda_y = \frac{L_{cr,y}}{i_y} = \frac{12260}{112.7} = 108.8 \qquad \bar{\lambda}_y = \frac{\lambda_y}{\lambda_1}\sqrt{\frac{A_{eff}}{A}} = \frac{108.8}{76.9}\sqrt{\frac{1691}{2872}} = 1.085$$

$$\lambda_z = \frac{L_{cr,z}}{i_z} = \frac{4035}{34.2} = 117.9 \qquad \bar{\lambda}_z = \frac{\lambda_z}{\lambda_1}\sqrt{\frac{A_{eff}}{A}} = \frac{117.9}{76.9}\sqrt{\frac{1691}{2872}} = 1.176$$

where $\lambda_1 = 93.9 \cdot \sqrt{235/f_y} = 93.9 \cdot \sqrt{235/350} = 76.9$

Reduction buckling factors

$\chi_y = 0.61$ for buckling curve *a* (see [1], Tab. 6.3)
$\chi_z = 0.49$ for buckling curve *b* (see [1], Tab. 6.3)

The section is doubly symmetric and torsional buckling length $L_{cr,T} = L_{cr,z}$. Therefore torsional and torsional-flexural buckling according to 6.2.2.3 is not critical and will not be checked.

2.5.4.3 *Lateral-torsional buckling moment resistance*

Effective length $L = L_{cr,z} = 4.035$ m

Procedure to calculate the elastic critical buckling moment M_{cr} is given neither in EN 1993-1-1 nor in EN 1993-1-3; any appropriate calculation method or numerical solution can be used. Here the procedure of EN 1999-1-1, Annex I [5] will be applied.

End support conditions:

$k_z = 1.0$
$k_w = 1.0$

Moment pattern:

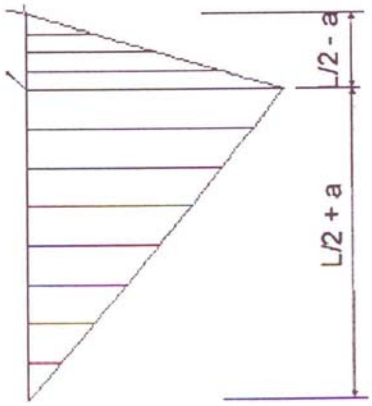

Fig. 2.31: Moment pattern

Shown moment pattern is not covered by formulas of [5]. Safely the case of force in the midspan point can be applied, then $C_1 = 1.35$. Alternatively, the formula derived by Vrany [6] for general position of force on simply supported beam can be used (for notation see Fig. 2.31):

$$C_1 = 1.35 + 0.49\left(\frac{2 \cdot 0.8}{4.035}\right)^{2.5} = 1.49$$

Governing load is represented by lateral force from brace that is considered as acting into shear centre of the section. Therefore

$z_g = 0$

For doubly symmetric cross-section with $z_g = 0$ the critical moment M_{cr} is

$$M_{cr} = C_1 \frac{\pi^2 \cdot E \cdot I_z}{(k_z \cdot L)^2} \left[\frac{(k_z \cdot L)^2 G \cdot I_t}{\pi^2 \cdot E \cdot I_z} + \frac{I_w}{I_z} \left(\frac{k_z}{k_w} \right)^2 \right]^{1/2} = 96.4\,\text{kNm}$$

Note: For doubly symmetrical sections, $k_z = k_w = 1$ and loading applied in shear center ($z_g = 0$) the presented formula is identical with formula according to Annex F of ENV 1993-1-1.

Relative slenderness

$$\bar{\lambda}_{LT} = \sqrt{\frac{W_{eff,y} \cdot f_y}{M_r}} = \sqrt{\frac{227.7 \cdot 10^3 \cdot 350}{96.4 \cdot 10^6}} = 0.909$$

Reduction buckling factor using 6.3.2.2 of EN 1993-1-1 for buckling curve *a* (with reference from 6.2.4 of EN 1993-1-3):

$$\chi_{LT} = 0.73$$

2.5.4.4 Check of combination of bending and axial compression

After calculating the capacity of the member subject to axial compression force and bending moment, the combined action is covered by clause 6.2.5. Load combination 4 governs:

$N_{Ed} = 34.6$ kN
$M_{y,Ed} = 50.2$ kNm

Interaction formula (6.38) of EN 1993-1-3 [1] will be used:

$$\left(\frac{N_{Ed}}{N_{b,Rd}} \right)^{0.8} + \left(\frac{M_{y,Ed}}{M_{b,y,Rd}} \right)^{0.8} \leq 1$$

$$\left(\frac{N_{Ed}}{\chi_{\min} \cdot A_{eff} \cdot f_y / \gamma_{M1}} \right)^{0.8} + \left(\frac{M_{y,Ed}}{\chi_{LT} \cdot W_{eff,y} \cdot f_y / \gamma_{M1}} \right)^{0.8} \leq 1$$

$$\left(\frac{34600}{0.49 \cdot 1691 \cdot 350 / 1.0} \right)^{0.8} + \left(\frac{50.2 \cdot 10^6}{0.73 \cdot 227700 \cdot 350 / 1.0} \right)^{0.8} =$$

$$= 0.182 + 0.891 = 1.07 > 1$$

The condition is not fulfilled.

Alternatively, it is also possible to check the member using general and more complicated procedure to 6.3.3(4) of EN 1993-1-1 [2], with the interaction factors k_{yy}, k_{zy} obtained from Annex B of [2]:

$C_{my} = 0.9$ for sway buckling mode
$C_{m,LT} = 0.6 + 0.4\psi = 0.6$ for end moment ratio $\psi = 0$

Interaction factors from Tab. B.2 of Annex B (for members susceptible to torsional deformations):

$$k_{yy} = C_{my}\left(1 + 0.6\cdot\overline{\lambda}_y \frac{N_{Ed}}{\chi_y \cdot A_{eff}\cdot f_y/\gamma_{M1}}\right) =$$

$$= 0.9\left(1 + 0.6\cdot 1.085\frac{34600}{0.61\cdot 1691\cdot 350/1.0}\right) = 0.95$$

$$k_{zy} = \left(1 - \frac{0.55\cdot\overline{\lambda}_y}{(C_{mLT} - 0.25)}\frac{N_{Ed}}{\chi_z\cdot A_{eff}\cdot f_y/\gamma_{M1}}\right) =$$

$$= 1 - \frac{0.05\cdot 1.176}{(0.6 - 0.25)}\frac{34600}{0.49\cdot 1691\cdot 350/1.0} = 0.98$$

Interaction buckling check for combined bending and axial compression:

Because the shift of neutral axis for the effective cross-section subjected to uniform compression is zero the additional moment $\Delta M_{y,Ed} = 0$.

$$\frac{N_{Ed}}{\frac{\chi_y \cdot N_{Rk}}{\gamma_{M1}}} + k_{yy}\frac{M_{y,Ed} + \Delta M_{y,Ed}}{\chi_{LT}\frac{M_{y,Rk}}{\gamma_{M1}}} \le 1$$ EN 1993-1-1, eq. (6.61)

$$\frac{34600}{0.61\cdot 1691\cdot 350/1.0} + 0.95\frac{50.2\cdot 10^6}{0.73\cdot 227700\cdot 350/1.0} =$$

$$= 0.096 + 0.824 = 0.92 \le 1$$

$$\frac{N_{Ed}}{\frac{\chi_z \cdot N_{Rk}}{\gamma_{M1}}} + k_{zy}\frac{M_{y,Ed} + \Delta M_{y,Ed}}{\chi_{LT}\frac{M_{y,Rk}}{\gamma_{M1}}} \le 1$$ EN 1993-1-1, eq. (6.62)

$$\frac{34600}{0.49 \cdot 1691 \cdot 350 / 1.0} + 0.98 \frac{50.2 \cdot 10^6}{0.73 \cdot 227700 \cdot 350 / 1.0} =$$

$$= 0.119 + 0.851 = 0.97 \leq 1$$

The conditions are fulfilled. The column is satisfactory.

2.5.5 Rafter – corner area

Procedures and comments used for column check apply here.

2.5.5.1 Design of profile

Section 276 x 125 x 20 x 3 (Millpac 276 x 3.0 section)

Gross section properties:

$A_g = 2463$ mm^2
$I_y = 25.28 \cdot 10^6$ mm^4
$I_z = 1.568 \cdot 10^6$ mm^4
$i_y = 101.3$ mm
$i_z = 25.2$ mm
$I_t = 7200$ mm^4
$I_w = 27.434 \cdot 10^9$ mm^6

Effective cross-section properties (see Tab. 2.2):

- For uniform compression:

$A_{eff} = 1498$ mm^2

- For bending along *y-y*:

$I_{eff.y} = 24.65 \cdot 10^6$ mm^3
$W_{eff.y.com} = 178.1 \cdot 10^3$ mm^3
$W_{eff.y.ten} = 183.2 \cdot 10^3$ mm^3

2.5.5.2 Axial tension resistance

The strain hardening by the cold forming is introduced by the mean yield strength:

$$f_{ya} = f_{yb} + \left(f_u + f_{yb}\right) \frac{k \cdot n \cdot t^2}{A_g} = 350 + \left(420 - 350\right) \frac{7 \cdot 8 \cdot 2.96^2}{2463} = 364 \text{ N/mm}^2$$

where f_{yb}, f_u are the yield strength and the ultimate strength of material before cold-forming,
$k = 7$ for roll-forming,
$n = 8$ number of 90° bends in the cross-section with an internal radius $r \leq 5t$

The mean yield strength is limited by

$$f_{ya} \leq \frac{f_u + f_{yb}}{2} = \frac{420+350}{2} = 385 \text{ N/mm}^2, \qquad \textit{which is satisfied.}$$

The design tension resistance:

$$N_{t,Rd} = A_{net} \cdot f_{ya}/\gamma_{M0} = 2463 \cdot 364/1.00 = 896.5 \cdot 10^3 \text{ N} = 896.5 \text{ kN}$$

2.5.5.3 Axial compression buckling resistance

Effective lengths for flexural buckling:

- $L_{cr,y} = 0.87$ m (distance from column face to the brace connection)
- $L_{cr,z} = 1.387$ m (distance of purlins)

Relative slenderness:

$$\lambda_y = \frac{L_{cr,y}}{i_y} = \frac{870}{101.3} = 8.6 \qquad \bar{\lambda}_y = \frac{\lambda_y}{\lambda_1}\sqrt{\frac{A_{eff}}{A}} = \frac{8.6}{76.9}\sqrt{\frac{1498}{2463}} = 0.09 < 0.2$$

$$\lambda_z = \frac{L_{cr,z}}{i_z} = \frac{1387}{25.2} = 55.0 \qquad \bar{\lambda}_z = \frac{\lambda_z}{\lambda_1}\sqrt{\frac{A_{eff}}{A}} = \frac{55.0}{76.9}\sqrt{\frac{1498}{2463}} = 0.557$$

where $\lambda_1 = 93.9\sqrt{235/f_y} = 93.9\sqrt{235/350} = 76.9$

Reduction buckling factors:

$\chi_y = 1.00$ for $\bar{\lambda}_y < 0.2$
$\chi_z = 0.86$ for buckling curve b (see Tab. 6.3 of [1])

2.5.5.4 Lateral-torsional buckling moment resistance – frame corner area

The bottom flange of the rafter is laterally restrained by means of stays at the first internal purlin position. Therefore both flanges are equally restrained and the following calculation applies both for hogging and sagging moment.

Effective length $L = L_{cr,z}$ = 1.387 m (distance of purlins)

As for column, the procedure of ENV 1993-1-1, Annex F [2] will be used.

End support conditions:

$k = 1.0$ $k_w = 1.0$

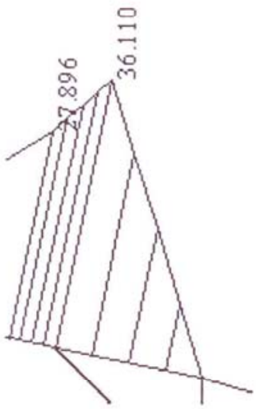

Fig. 2.32: Moment pattern

Shown moment pattern is not covered by formulas of [5]. Using value for single force applied in the mid-span of simply supported beam is on safe side, therefore

$C_1 = 1.35$

As for column, governing load is represented by lateral force from brace that is considered as acting into shear centre of the section. Therefore

$z_g = 0$

For doubly symmetric cross-section with $z_g = 0$ the critical moment M_{cr} is

$$M_{cr} = C_1 \frac{\pi^2 \cdot E \cdot I_z}{(k_z \cdot L)^2} \left[\frac{(k_z \cdot L)^2 G \cdot I_t}{\pi^2 \cdot E \cdot I_z} + \frac{I_w}{I_z} \left(\frac{k_z}{k_w} \right)^2 \right]^{1/2} = 304.6\,\text{kNm}$$

Relative slenderness

$$\bar{\lambda}_{LT} = \sqrt{\frac{W_{eff,y} \cdot f_y}{M_r}} = \sqrt{\frac{178.1 \cdot 10^3 \cdot 350}{304.6 \cdot 10^6}} = 0.452$$

Reduction buckling factor using 6.3.2.2 of EN 1993-1-1 for buckling curve *a* (with reference from 6.2.4 of EN 1993-1-3):

$\chi_{LT} = 0.94$

2.5.5.5 *Check for bending and axial tension*

The member should be checked for combined bending and axial tension, considering lateral-torsional buckling. Load combination 4 governs:

$N_{Ed} = 72.0$ kN
$M_{y,Ed} = 36.11$ kNm

The member shall fulfil the following conditions. Resistance of cross-section in tension fibres is covered by Clause 6.1.8 of [1]:

$$\frac{N_{Ed}}{A_{net} \cdot f_y / \gamma_{M0}} + \frac{M_{y,Ed} + N_{Ed} \cdot e_{Ny}}{W_{eff,y,ten} \cdot f_y / \gamma_{M0}} \leq 1$$

$$\frac{72000}{2463 \cdot 350 / 1.0} + \frac{36.11 \cdot 10^6}{183.2 \cdot 10^3 \cdot 350 / 1.0} = 0.084 + 0.563 = 0.647 < 1$$

Resistance for combined bending and axial tension considering lateral-torsional buckling is not directly covered by any clause of EN 1993-1-1 or EN 1993-1-3. Though, the interaction is considered by following way:

$$\frac{1}{\chi_{LT}}\left(\frac{M_{y,Ed} + N_{Ed} \cdot e_{Ny}}{W_{eff,y,com} \cdot f_y / \gamma_{M1}} - \frac{N_{Ed}}{A_{net} \cdot f_y / \gamma_{M1}}\right) \leq 1$$

$$\frac{1}{0.94}\left(\frac{36.11 \cdot 10^6}{178.1 \cdot 10^3 \cdot 350 / 1.0} - \frac{72000}{2463 \cdot 350 / 1.0}\right) =$$

$$= \frac{1}{0.94}(0.579 - 0.084) = 0.53 < 1$$

Both conditions are fulfilled.

2.5.5.6 *Check for bending and compression*

Combined bending and axial compression takes place at left corner area due to load combination 2:

$N_{Ed} = 54.7$ kN
$M_{y,Ed} = 27.5$ kNm

The interaction check will be made according to 6.3.3(4) of EN 1993-1-1 [2], with the interaction factors k_{yy}, k_{zy} obtained from Annex B of [2].

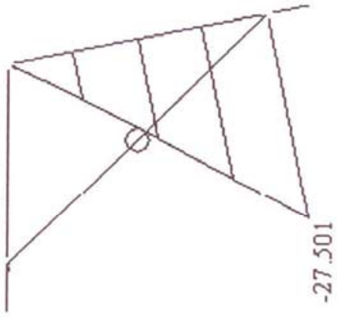

Fig. 2.33: Moment diagram

C_{my} (from Table B.3 of [2]): moment diagram as for $L_{cr,y}$ (see Fig. 2.33) therefore end moments ratio $\psi = 0$

C_{my} = 0.6 + 0.4ψ = 0.6 for non-sway buckling mode (because buckling of the rafter is not governed by global sway buckling mode)

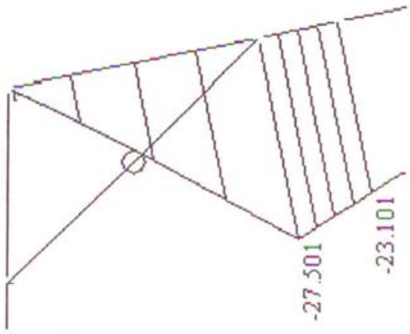

Fig. 2.34: Moment diagram

C_{mLT} (from Table B.3 of [2]): moment diagram as for $L_{cr,z}$ (see Fig. 2.34) therefore $\psi = 0$

$$\alpha_h = \frac{M_h}{M_s} = \frac{23.1}{27.5} = 0.84 \quad \text{for} \left|\frac{M_h}{M_s}\right| < 1$$

$$C_{mLT} = 0.9 + 0.1 \cdot \alpha_h = 0.98$$

Interaction factors from Tab. B.2 of Annex B (for members susceptible to torsional deformations):

$$k_{yy} = C_{my}\left(1 + 0.6 \cdot \bar{\lambda}_y \frac{N_{Ed}}{\chi_y \cdot A_{eff} \cdot f_y / \gamma_{M1}}\right) =$$

$$=0.6\left(1+0.6\cdot 0.09\frac{54700}{1.0\cdot 1498\cdot 350/1.0}\right)=0.60$$

$$k_{zy}=\left(1-\frac{0.05\cdot\overline{\lambda}_z}{(C_{mLT}-0.25)}\frac{N_{Ed}}{\chi_z\cdot A_{eff}\cdot f_y/\gamma_{M1}}\right)=$$

$$=1-\frac{0.05\cdot 0.557}{(0.98-0.25)}\frac{54700}{0.858\cdot 1498\cdot 350/1.0}=0.995$$

Interaction buckling check for combined bending and axial compression:

Because the shift of neutral axis for the effective cross-section subjected to uniform compression is zero the additional moment $\Delta_{My,Ed}=0$.

Equation (6.61) of EN 1993-1-1 could not govern because there is not any effect of buckling along *y-y*.

$$\frac{N_{Ed}}{\dfrac{\chi_z\cdot N_{Rk}}{\gamma_{M1}}}+k_{zy}\frac{M_{y,Ed}+\Delta M_{y,Ed}}{\chi_{LT}\dfrac{M_{y,Rk}}{\gamma_{M1}}}\le 1 \qquad \text{EN 1993-1-1, eq. (6.62)}$$

$$\frac{54700}{0.86\cdot 1498\cdot 350/1.0}+0.995\frac{27.5\cdot 10^6}{0.94\cdot 178100\cdot 350/1.0}=$$

$$=0.122+0.468=0.590\le 1$$

The condition is fulfilled.

2.5.6 Rafter span area

Procedures and comments used for column check apply here.

2.5.6.1 Design of profile

Section 276 x 125 x 20 x 3 see 2.5.5.1.

2.5.6.2 Axial compression buckling resistance

Effective lengths for flexural buckling:

$L_{cr,y}\cong 4.84$ m (distance from the brace connection to the ridge)
$L_{cr,z}=1.387$ m (distance of purlins)

Relative slenderness:

$$\lambda_y = \frac{L_{cr,y}}{i_y} = \frac{4840}{101.3} = 47.8 \qquad \bar{\lambda}_y = \frac{\lambda_y}{\lambda_1}\sqrt{\frac{A_{eff}}{A}} = \frac{47.8}{76.9}\sqrt{\frac{1498}{2463}} = 0.484$$

$$\lambda_z = \frac{L_{cr,z}}{i_z} = \frac{1387}{25.2} = 55.0 \qquad \bar{\lambda}_z = \frac{\lambda_z}{\lambda_1}\sqrt{\frac{A_{eff}}{A}} = \frac{55.0}{76.9}\sqrt{\frac{1498}{2463}} = 0.557$$

where $\lambda_1 = 93.9\sqrt{235/f_y} = 93.9\sqrt{235/350} = 76.9$

Reduction buckling factors:

$\chi_y = 0.93$ for buckling curve *a* (see [1],Tab. 6.3)
$\chi_z = 0.86$ for buckling curve *b* (see [1],Tab. 6.3)

2.5.6.3 Lateral-torsional buckling moment resistance

Effective length $L = L_{cr,z} = 1.387$ m (distance of purlins)

Compared to previous paragraph, the only difference is the moment distribution pattern. To cover all members between purlins (acting as restraints), the factor C_1 must be taken as 1.0.

$$M_{cr} = 304.6\frac{1.00}{1.35} = 225.6\,\text{kNm}$$

Relative slenderness

$$\bar{\lambda}_{LT} = \sqrt{\frac{W_{eff,y}\cdot f_y}{M_r}} = \sqrt{\frac{178.1\cdot 10^3\cdot 350}{225.6\cdot 10^6}} = 0.526$$

Reduction buckling factor using 6.3.2.2 of EN 1993-1-1 for buckling curve *a*:

$\chi_{LT} = 0.92$

2.5.6.4 Check of span area for bending and compression

The interaction check will be made according to 6.3.3(4) of EN 1993-1-1, with the interaction factors k_{yy}, k_{zy} obtained from Annex B of EN 1993-1-1. Load case combination 1 governs:

$N_{Ed} = 17.29$ kN

$M_{y,Ed}$ = 25.38 kNm

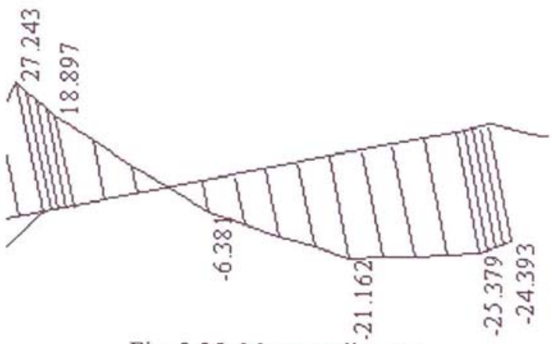

Fig. 2.35: Moment diagram

C_{my}: moment diagram as for $L_{cr,y}$ (see Fig. 2.35), therefore

$$\psi = -\frac{24.39}{27.24} = -0.90$$

$$\alpha_s = \frac{M_s}{M_h} = -\frac{25.38}{27.24} = -0.93 \qquad \text{for } \left|\frac{M_s}{M_h}\right| < 1$$

$C_{my} = 0.1 \cdot (1 - \psi) - 0.8 \cdot \alpha_s = 0.1 \cdot (1 + 0.90) - 0.8 \cdot (-0.93) = 0.93$ for non-sway buckling mode

C_{mLT} (from [2], Table B.3): let us safely consider moment diagram between two purlins as constant, therefore $\psi = 1$

$$C_{m,LT} = 0.6 + 0.4 \cdot 1 = 1.00$$

Interaction factors from Tab. B.2 of Annex B (for members susceptible to torsional deformations):

$$k_{yy} = C_{my}\left(1 + 0.6 \cdot \bar{\lambda}_y \frac{N_{Ed}}{\chi_y \cdot A_{eff} \cdot f_y / \gamma_{M1}}\right) =$$

$$= 0.93\left(1 + 0.6 \cdot 0.484 \frac{17290}{0.93 \cdot 1498 \cdot 350 / 1.0}\right) = 0.94$$

$$k_{zy} = \left(1 - \frac{0.05 \cdot \bar{\lambda}_z}{(C_{mLT} - 0.25)} \frac{N_{Ed}}{\chi_z \cdot A_{eff} \cdot f_y / \gamma_{M1}}\right) =$$

$$= 1 - \frac{0.05 \cdot 0.557}{(1.00 - 0.25)} \frac{17290}{0.858 \cdot 1498 \cdot 350 / 1.0} = 1.00$$

Interaction buckling check for combined bending and axial compression:

$$\Delta_{My,Ed} = 0$$

$$\frac{N_{Ed}}{\dfrac{\chi_y \cdot N_{Rk}}{\gamma_{M1}}} + k_{yy} \frac{M_{y,Ed} + \Delta M_{y,Ed}}{\chi_{LT} \dfrac{M_{y,Rk}}{\gamma_{M1}}} \leq 1$$ EN 1993-1-1, eq. (6.61)

$$\frac{17290}{0.93 \cdot 1498 \cdot 350/1.0} + 0.94 \frac{25.38 \cdot 10^6}{0.92 \cdot 178100 \cdot 350/1.0} = 0.036 + 0.416 = 0.45 \leq 1$$

$$\frac{N_{Ed}}{\dfrac{\chi_z \cdot N_{Rk}}{\gamma_{M1}}} + k_{zy} \frac{M_{y,Ed} + \Delta M_{y,Ed}}{\chi_{LT} \dfrac{M_{y,Rk}}{\gamma_{M1}}} \leq 1$$ EN 1993-1-1, eq. (6.62)

$$\frac{17290}{0.86 \cdot 1498 \cdot 350/1.0} + 1.00 \frac{25.38 \cdot 10^6}{0.92 \cdot 178100 \cdot 350/1.0} = 0.038 + 0.444 = 0.48 \leq 1$$

The conditions are fulfilled.

2.5.7 Brace

2.5.7.1 Design of profile

Profile 2U 80 x 96 x 2.4 (Metsec section)

Gross section properties:

$A_g = 784 \text{ mm}^2$
$I_y = 811 \cdot 10^3 \text{ mm}^4$
$I_z = 431.4 \cdot 10^3 \text{ mm}^4$
$i_y = 32.2 \text{ mm}$
$i_z = 23.5 \text{ mm}$

Effective cross-section properties for uniform compression (see Table 2.2):

$A_{eff} = 648 \text{ mm}^2$

The brace member should be checked for axial compression.

2.5.7.2 *Axial compression buckling resistance*

$N_{Ed} = 111.3$ kN (LCC 4, right brace)

Member length: 1.414 m

Effective lengths for flexural buckling:

$L_{cr,y} = L_{cr,z} = L = 1.414$ m

Slenderness λ_z governs.

Relative slenderness:

$$\lambda_z = \frac{L_{cr,z}}{i_z} = \frac{1414}{23.5} = 60.2 \qquad \bar{\lambda}_z = \frac{\lambda_z}{\lambda_1}\sqrt{\frac{A_{eff}}{A}} = \frac{60.2}{76.9}\sqrt{\frac{648}{784}} = 0.712$$

Reduction buckling factor:

$\chi_z = 0.78$ for buckling curve b (see [1], Tab. 6.3)

Design buckling resistance

$$N_{b,Rd} = \chi_{min} \cdot A_{eff} \cdot f_y \cdot \gamma_{M1} = 0.78 \cdot 648 \cdot 350/1.0 =$$

$$= 176900 \text{ N} = 176.9 \text{ kN} > 111.3 \text{ kN}$$

Satisfactory.