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EFFECTIVE DESIGN OF COLD-FORMED THIN-WALLED CHANNEL BEAMS WITH BENT EDGES OF FLANGES

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Abstract. *An analytical and numerical elastic buckling study and an effective design of cold-formed thin-walled channel beams with various flange bend shapes under pure bending are presented. Flanges of these beams are with open or closed bends. Buckling problems of flanges or webs of the beams are considered. Mathematical models of critical states for these beams are formulated and critical loads determined. Moreover, critical loads are calculated with the use of the Finite Strip Method – FSM and compared to analytical solution. Results of the numerical investigations are presented in figures. Effective design problem of thin-walled beams is formulated. A criterion includes two problems: maximization of the load and minimization of the beam cross section area. Results of the solution of the effective design problem are presented in figures.*

1 INTRODUCTION

Strength and stability problems of thin-walled beams with open cross sections are intensively investigated since 1940's and described in many monographs of the 20^{th} century, for example in chronological order [1] - [7]. Numerical calculations of stresses and critical states of thin-walled beams are carried out with the use of the finite strip method (FSM) [8] or the finite element method (FEM). The fundamental numerical study of the local, distortional, and flexural-torsional buckling of I-beams with the use of the finite strip method is presented in [9]. Stability problems of thin-walled members are also at present investigated, selected papers are the following: [10]-[33]. Global and local/distortional buckling problems are described with consideration of the Generalized Beam Theory (GBT). Moreover, optimal design problems are also intensively developed. The first paper on optimal design of a thin-walled beam with open cross section concerned an I-beam under pure bending [34]. Results of later studies of these problems are presented in the following papers: [35]-[53]. Solutions of the optimization tasks of thin-walled beams take into account the strength and stability constraints.

Flanges and webs of cold-formed C-sections or I-sections are variously shaped in order to improve their stiffness. Evolution of section shapes of these beams is presented for example in the papers [15], [30] and [53]. The subject of the study includes thin-walled channel beams with open or closed bends of flanges under pure bending. Geometric properties are described with consideration of warping functions.

2 GEOMETRIC PROPERTIES OF TWO CROSS SECTIONS

Geometric properties of two cross sections of cold-formed channel beams are determined with respect to the principal axes *y*, *z*. The origin (the point O) and the shear centre (the point C) are located on the z-axis of symmetry. Scheme of two mono-symmetrical C-sections of channel beams with open or closed bends of flanges are shown in figure 1.

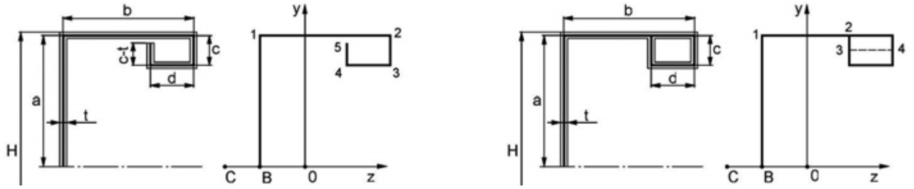

Figure 1: Cross sections of two channel beams with open or closed bends of flanges.

Geometric properties of two cross sections are defined by the following dimensionless parameters

$$x_1 = \frac{b}{a}, \; x_2 = \frac{c}{b}, \; x_3 = \frac{d}{b}, \; x_4 = \frac{t}{b}. \tag{1}$$

where: a and b – dimensions of the cross section, c and d – dimensions of the bend, t – thickness of the wall, $H = 2a + t = a(2 + x_1 x_4)$ - depth of the beam.

2.1 C-section with open bends of flanges

The total area and geometric stiffness for Saint-Venant torsion of the cross section

$$A = 2at \cdot f_0(x_i), \; J_t = \frac{2}{3} at^3 \cdot f_0(x_i), \text{ where } f_0(x_i) = 1 + x_1(1 + 2x_2 + x_3 - x_4). \tag{2}$$

The location of the centroid

$$z_B = -b \cdot \frac{f_1(x_i)}{f_0(x_i)}, \text{ where } f_1(x_i) = x_1\left[1 + x_2 - \frac{1}{2}(1 - x_3)(1 - 2x_2 - x_3 + 2x_4)\right]. \tag{3}$$

Moments of inertia of the plane area with respect to the y and z axes

$$J_y = 2a^3 t \cdot f_2(x_i), \; f_2(x_i) = x_1^2 \left\{ x_1 \left[\frac{2}{3} + x_2 - \frac{1}{3}(1 - x_3)^2 (1 - 3x_2 - x_3 + 3x_4) \right] - \frac{f_1^2(x_i)}{f_0(x_i)} \right\}, \tag{4}$$

$$J_z = 2a^3 t \cdot f_3(x_i), \; f_3(x_i) = \frac{2}{3} + x_1 - (1 - x_1 x_2)^2 \left[\frac{2}{3} - x_1 \left(\frac{2}{3} x_2 + x_3 \right) \right] + \frac{1}{3}(1 - x_1 x_4)^3. \tag{5}$$

The location of the shear centre (the point C)

$$z_B - z_C = \frac{1}{J_z} \int_A \omega_B y dA = b \cdot \tilde{z}_{BC}, \text{ where } \tilde{z}_{BC} = \frac{1}{2} x_1 \frac{f_4(x_i)}{f_3(x_i)}, \; f_4(x_i) \text{ - composite function.} \tag{6}$$

The warping moment of inertia

$$J_\omega = \int_A \omega^2 dA = 2a^5 t \cdot f_5(x_i), \text{ where } f_5(x_i) = \frac{1}{3} x_1^2 \left[\tilde{\omega}_1^2 + x_1 \left(\tilde{\omega}_1^2 - \tilde{\omega}_1 \tilde{\omega}_2 + \tilde{\omega}_2^2 \right) + \ldots \right], \tag{7}$$

and dimensionless warping functions: $\tilde{\omega}_1 = \tilde{z}_{BC}$, $\tilde{\omega}_2 = 1 - \tilde{z}_{BC}$, $\tilde{\omega}_3 = 1 + x_1 x_2 - \tilde{z}_{BC}(1 - x_1 x_2)$,
$\tilde{\omega}_4 = 1 + x_1 x_2 - (x_3 + \tilde{z}_{BC})(1 - x_1 x_2)$, $\tilde{\omega}_5 = 2x_1 x_2 x_3 + (1 + x_1 x_4)(1 - x_3) - \tilde{z}_{BC}(1 - x_1(2x_2 + x_4))$.

2.2 C-section with closed bends of flanges

The formulas for geometric properties of the C-section are analogous to the ones of the C-section with open bends of flanges. The geometric stiffness for torsion and the warping moment of inertia are differently defined.

The geometric stiffness for torsion

$$J_t = \frac{2}{3} a t^3 \cdot f_t(x_i), \text{ where } f_t(x_i) = 1 + x_1(1 - x_3) + 6 \frac{x_1}{x_2 + x_3} \left(\frac{x_2 x_3}{x_4} \right)^2 . \tag{8}$$

The location of the shear centre (the point C)

$$z_B - z_C = \frac{1}{J_z} \int_A \omega_B y dA = b \cdot \tilde{z}_{BC} \text{ , where } \tilde{z}_{BC} = \frac{1}{2} x_1 \frac{f_4(x_i)}{f_3(x_i)}, \tag{9}$$

and $f_4(x_i) = (1 - x_3)\tilde{\omega}_{B2} + x_3(2 - x_1 x_2)(\tilde{\omega}_{B3} + \tilde{\omega}_{B4})$, $\tilde{\omega}_{B2} = 1 - x_3$, $\tilde{\omega}_{B3} = (1 - x_3)\left(1 + \frac{1}{2} x_1 x_2\right)$.

The warping moment of inertia

$$J_\omega = \int_A \omega^2 dA = 2a^5 t \cdot f_5(x_i) \text{ , where } f_5(x_i) = \frac{1}{3} x_1^2 \left[\tilde{\omega}_1^2 + x_1(1 - x_3)\left(\tilde{\omega}_1^2 - \tilde{\omega}_1 \tilde{\omega}_2 + \tilde{\omega}_2^2\right) + 2 x_1 x_3 \left(\tilde{\omega}_3^2 + \ldots\right)\right], \tag{10}$$

and dimensionless warping functions: $\tilde{\omega}_1 = \tilde{z}_{BC}$, $\tilde{\omega}_2 = 1 - x_3 - \tilde{z}_{BC}$,
$\tilde{\omega}_3 = (1 - x_3)(1 + 0.5 x_1 x_2) - \tilde{z}_{BC}(1 - 0.5 x_1 x_2)$, $\tilde{\omega}_4 = 1 + 0.5 x_1 x_2 (1 - 2x_3) - \tilde{z}_{BC}(1 - 0.5 x_1 x_2)$.

3 GLOBAL BUCKLING

The global buckling problem of thin-walled beams is described for example in [3], [5] and [6]. The lateral buckling moment of thin-walled beam under pure bending

$$M_{CR}^{(Glob)} = \frac{\pi E}{L} \sqrt{\frac{J_y J_t}{2(1+\nu)} \left[1 + 2(1+\nu) \frac{\pi^2}{L^2} \frac{J_\omega}{J_t} \right]} , \tag{11}$$

where: E – Young's modulus, ν – Poisson ratio, L – length of the beam.

4 DISTORTIONAL BUCKLING

The local and distortional buckling problems are described in the papers [11], [14], [15], [17], [20] and [26]. This problem is also studied for thin-walled channel beams with open or closed bends of flanges. Scheme of displacements for distortional buckling is shown in figure 2.

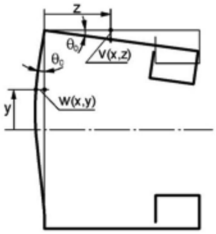

Figure 2: Theoretical shape of distortional buckling mode.

Functions of deflections of the flange and the web

$$v(x,z)=\theta_0\frac{b}{r\pi}\sin\frac{r\pi z}{b}\sin\frac{m\pi x}{L},\quad w(y,z)=-\theta_0\frac{a}{2\pi}\left(2\cos\frac{\pi y}{2a}+\sin\frac{\pi y}{a}\right)\sin\frac{m\pi x}{L},\tag{12}$$

where: θ_0 – angle of rotation (figure 2), r – real number, m – natural number, $b_1=b-d/2$, and coordinates $0\le z\le b_1$, $-a\le y\le a$.

The elastic strain energy U_ε and the work W of the load for the beam under pure bending are described with the functions (12). Taking into account the principle of minimum of the total potential energy $\delta(U_\varepsilon-W)=0$ takes the elastic distortional buckling stress in the following form

$$\sigma_{x,CR}^{(Dist)}=\min_{m,r}\left\{\frac{f_\varepsilon^{(fp)}+f_\varepsilon^{(fb)}+f_\varepsilon^{(w)}}{f_\sigma^{(f)}+f_\sigma^{(w)}}\right\}E,\tag{13}$$

where:

$$f_\varepsilon^{(fp)}=\frac{\pi^2}{12(1-\nu^2)}\left(\frac{t}{b_1}\right)^2\frac{r^2}{2}\left\{\left(X_1+\frac{1}{X_1}\right)^2-\frac{\sin(2\pi r)}{2\pi r}\left[\left(X_1-\frac{1}{X_1}\right)^2+4\nu\right]\right\},\quad X_1=\frac{m}{r}\frac{b_1}{L},$$

$$f_\varepsilon^{(fb)}=(\pi r)^2\left[\frac{1}{2(1+\nu)}\frac{J_t^{(b)}}{b_1^3t}\cos^2(\pi r)+\frac{J_z^{(b)}}{b_1^3t}X_1^2\sin^2(\pi r)\right],$$

$$f_\varepsilon^{(w)}=\frac{\pi^2}{12(1-\nu^2)}\left(\frac{t}{a}\right)^2\left(\frac{r}{2}\right)^2\left(5X_2^2+4+\frac{5}{4X_2^2}\right)\left(\frac{a}{b_1}\right)^3,\quad X_2=m\frac{a}{L},$$

$$f_\sigma^{(f)}=\frac{1}{2}\left(1-\frac{\sin(2\pi r)}{2\pi r}\right)+\left(1-\frac{1}{2}x_1x_2\right)\frac{A_b}{b_1t}\sin^2(\pi r),\quad f_\sigma^{(w)}=\frac{128}{9\pi^2}\left(\frac{a}{b_1}\right)^3\left(\frac{r}{2}\right)^2,$$

$$J_t^{(b)}=J_t^{(b-open)}=\frac{2}{3}t^3(c+d),\ J_t^{(b)}=J_t^{(b-closed)}=2t\frac{c^2d^2}{c+d},\ J_z^{(b)}=\frac{1}{2}c^2t\left(\frac{1}{3}c+d\right),\ A_b=2t(c+d).$$

Numerical calculation is performed for the example steel channel beams with open or closed bends of flanges: $E=205\,GPa$, $\nu=0.3$, $a=100\,mm$, $b=100\,mm$, $c=15\,mm$, $10\,mm\le d=25\,mm$, $t=1.4\,mm$. Values of critical stresses of distortional buckling are calculated from the formula (13) and with the use of the finite strip method (FSM-Cufsm – B. Schafer). The comparison of results of both methods is shown in figure 3. Differences in critical stresses values are below five percent.

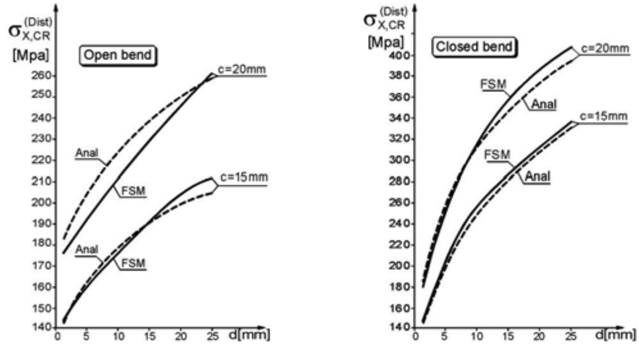

Figure 3: Critical stresses of distortional buckling of channel beams

5 EFFECTIVE SHAPING OF C-SECTIONS

Minimal manufacturing cost, minimal mass or maximal safe load are usually criterions for effective constructions design. The optimization criterion with regard to the papers [42], [45], [46], [47] and [53] is formulated in the following form

$$\max_{x_i}\{\Phi_1(x_i),\Phi_2(x_i),\ldots,\Phi_5(x_i)\}=\Phi_{\max}\text{, and the objective function }\Phi_j(x_i)=\frac{M_j}{EA^{3/2}}, \tag{14}$$

where:

$M_1 = 2\dfrac{J_z}{H}\sigma_{all}$ - the allowable moment from the strength condition (*j=1*),

$M_2 = \dfrac{M_{CR}^{(Glob)}}{c_{s1}}$ - the allowable moment from the lateral buckling condition (*j=2*),

$M_3 = \dfrac{J_z}{c_{s2}a}\sigma_{x,CR}^{(Dist)}$ - the allowable moment from the distortional buckling condition (*j=3*),

$M_4 = \dfrac{J_z}{c_{s3}a}\sigma_{x,CR}^{(Flange)}$ - the allowable moment from the local buckling condition for the flange (*j=4*) ,

$M_5 = \dfrac{J_z}{c_{s4}a}\sigma_{x,CR}^{(Web)}$ - the allowable moment from the local buckling condition for the web (*j=5*),

$\sigma_{x,CR}^{(Flange)} = \dfrac{\pi^2 E}{12(1-\nu^2)}k_f\left(\dfrac{t}{b_p}\right)^2$, $k_f = 4$ - the critical stress of the flange plate [3],

$\sigma_{x,CR}^{(Web)} = \dfrac{\pi^2 E}{12(1-\nu^2)}k_w\left(\dfrac{t}{2a}\right)^2$, $k_w = \dfrac{81}{32}\pi^2$ - the critical stress of the web (obtained from (13)),

σ_{all} - allowable stress, $c_{s1},\ldots,c_{s4}$ - safety coefficients, $b_p = b$ - open bend, $b_p = b - d$ - closed bend.

Strength and buckling conditions ($M_0 \le M_1$, where M_0 is the moment-load) are formulated for the simply supported beam under pure bending.

Effective shaping of cold-formed thin-walled channel beams with bent edges of flanges is realized for the family of beams: $\sigma_{all}/E = 0.0012$, $\nu = 0.3$, $c_{s1} = 1.2$, $c_{s2} = c_{s3} = c_{s4} = 1.3$, and relative length $\lambda = L/H = 7.5, 10.0, 15.0, 20.0, 25.0$. Results of the numerical calculations of dimensionless functions $\Phi_{\max}$ are shown in figure 4.

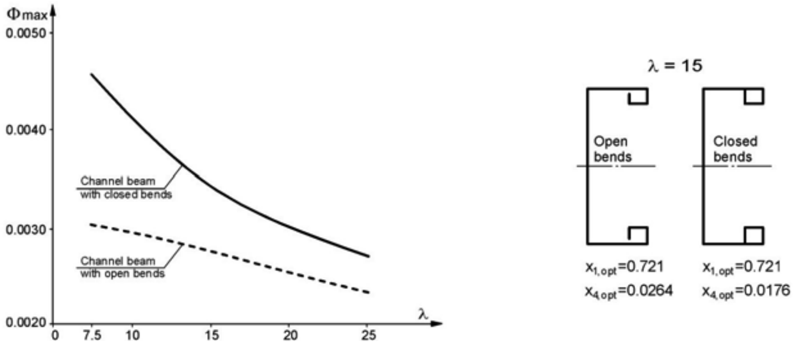

Figure 4: The comparison of effective channel beams with open or closed bends of flanges

6 CONCLUSION

The shapes of cross sections of cold-formed thin-walled beams are rather complicated. Strength and buckling resistance are strictly related with the shapes of cross sections. Effective design of beams with respect to the criterion and the dimensionless objective function (14) enables improving the structures. This criterion is a quality measure for beams. Values of objective function Φ_{max} for the beam with closed bends of flanges are greater than the values for the beam with open bends of flanges (fig.4). Thin-walled channel beams with closed bends of flanges are decidedly better than the ones with open bends of flanges.

The formula (13) described the elastic distortional buckling stress of the channel beams with open or closed bends of flanges. Values of stresses calculated with the use of (13) approximate the values calculated with the use of finite strip method (FSM-Cufsm – B. Schafer).

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