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PLATE BUCKLING ACCORDING TO EUROCODE 3. COMPARISON OF THE EFFECTIVE WIDTH METHOD AND THE REDUCED STRESS METHOD

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Abstract. *The Eurocode 3 section 1-5 (EC3 1-5) on plated structures presents two approaches to calculate the ultimate resistance of compressed plated elements affected by plate buckling (class 4): the effective width method and the reduced stressed method. Given its complexity, the latter of these methods has not been the subject of much research until now. Therefore, the scope of its applicability as well as its relative advantages and disadvantages with respect to the other methods remain to a large extent uncertain. The aim of the present paper is to contribute to a better understanding of the background and practical use of these methods. To that end, a brief introduction to the background of these methods, as well as their application to non-stiffened and stiffened plated structures is first included. Then, the article reproduces a comparative evaluation of these methods, based on the parametric study carried on the paper 'Estudio comparativo de los métodos de cálculo propuestos en los artículos 4 y 10 del EN1993-1.5' [1]. Finally, the main conclusions of this study are summarised at the end of the article.*

1 INTRODUCTION

Eurocodes are produced in the EU to harmonise different types of structures and building materials. Steel structures are included in Eurocode 3 (EN 1993). Section 1-5 of Eurocode 3 [2] on plated structures presents two approaches to calculate the ultimate resistance of compressed plated elements affected by plate buckling (class 4): the effective width method and the reduced stressed method. Given its complexity, the latter of these methods has not been the subject of much research until now [3], [4]. Therefore, the scope of its applicability as well as its relative advantages and disadvantages with respect the other methods remain to a large extent uncertain.

The paper 'Estudio comparativo de los métodos de cálculo propuestos en los artículos 4 y 10 del EN1993-1.5' [1], which can be downloaded from [A], contributes to a better understanding of the implications of using these alternative calculation methods and covers a comparative evaluation of the methods. This article summarizes the main points of the paper, in order to give a concise but clear introduction to the subjcet. To that end, the background of each method is first presented, in particular the fundamental concepts of the reduced stress method. This document covers the case of non-stiffened as well as stiffened plates. Then, the results of the application of each method to a particular case are reproduced. Finally, the comparative evaluation of the effective width and reduced stress methods is achieved by a parametric study. The parameters and conclusions of this study are summarized at the end of the article. The parametric study was performed using an EXCEL spreadsheet, which can be downloaded from [A].

2 BACKGROUND THEORY OF THE EC3 1-5 APPROACHES

2.1 The effective width method

a) Non-stiffened plates

This approach was first developed by Von Karman (1932) and subsequently modified by Winter (1947). It is included in section 4 of Eurocode 3, part 1-5.

The effective width approach is based on the fact that a plate subjected to in-axial compressive stresses will go through a post-buckling stage where higher stresses will be transferred to the stiffer areas of the plate due to the membrane effect (Figure 1). Therefore, the strength of a plate affected by buckling can be assumed to be that of an imaginary plate of reduced breadth, b_{eff}, not subjected to buckling. According to this, the plate will fail when compressive stresses reach the maximum strength (yield strength, f_y) of the reduced plate.

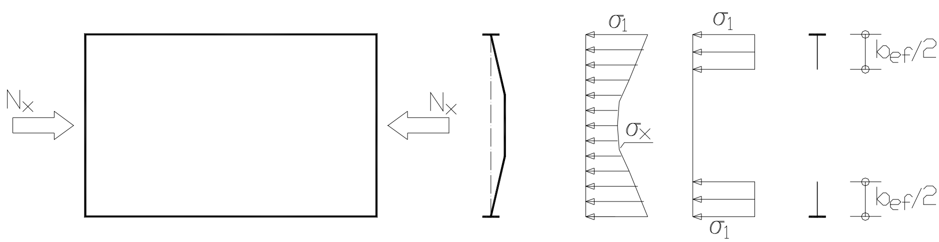

Figure 1: Stress transfer

The behavior of a plate under compression stresses depends to a large extent on the ratio of its principal dimensions. Considering a thin plate of length *a*, width *b* and thickness *t* mm (a,b<<t), if *a* is of the same magnitude as *b* (square shaped plate) its response will be similar to that of a compressed plate and will be therefore governed by the plate-buckling theory. However, if *a* is significantly smaller than *b* its response will be better described by the column buckling theory [5], [6]. These responses are known as the plate and the columns mechanisms respectively. The theory of this mechanisms is fully covered on section 4.1.2 a and 4.1.2 b of EN1993-1.5 (EC3 1-5) [2].

In fact, the actual response of a compressed plate falls between these two mechanisms and the greater influence of one above the other depends purely on the plate's dimensions. The EC3 1-5 takes this fact into consideration and suggests an interpolation formula between the two mechanisms in order to obtain a more realistic value of the plate's strength:

$$\rho_c = (\rho - \chi_c)\xi(2-\xi) + \chi_c \tag{1}$$

where ρ and χ_c are the reduction factors for the plate and column mechanisms respectively, and ξ is a 'measure' between the critical strength of the plate mechanism and that of the column mechanism given by

$$\xi = \frac{\sigma_{cr,p}}{\sigma_{cr,c}} - 1 \tag{2}$$

where $\sigma_{cr,p}$ and $\sigma_{cr,c}$ are the critical stresses according to the plate mechanism and the column mechanism, respectively. Figure 2 represents the interpolation between the two mechanisms according to ξ.

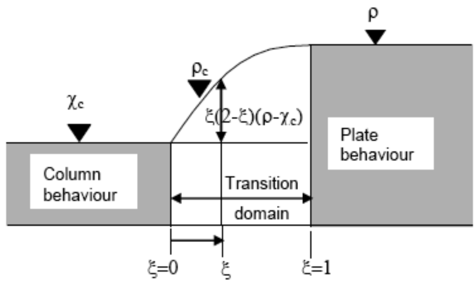

Figure 2: Interpolation between the column behaviour and the plate behaviour

b) Stiffened plates

Plate girder

The first step in the calculation of the critical strength of a compressed plate girder by applying the effective width approach is to study the buckling effects in every individual plate that conforms the plate girder and its stiffeners. This is known as local plate-buckling study. Subsequently, the study of the plate-buckling phenomenon of the whole plate takes place. To this end, the effective width method refers to the interpolation between the column mechanism and the plate mechanism [7], [8]. The column mechanism establishes imaginary ‘cuts’ in the plate between two stiffeners that eliminates its parts affected by buckling. Thus, the remaining parts of the plate along with each stiffener can be treated as a column under an axial compressive force. This mechanism assumes that columns do not receive any support from the adjacent one but are able to buckle in a direction normal to their plane (Figure 3a). The plate mechanism is based on the fact that these ‘cuts’ do not exist and therefore the column will be supported by the adjacent columns (Figure 3b).

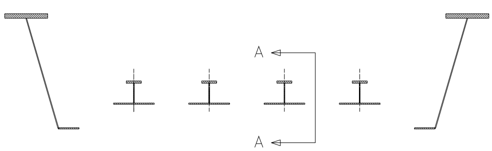

Figure 3a: Column mechanism

Figure 3b: Plate mechanism

The reduction factor for the plate mechanism is determined from:

$$\rho = \left[\bar{\lambda} - 0{,}055(3+\psi)\right]\frac{1}{\bar{\lambda}^2} \tag{3}$$

where $\bar{\lambda}$ is the slenderness of the plate expressed as:

$$\bar{\lambda} = \sqrt{\frac{\beta_{A,c} f_y}{\sigma_{cr,p}}} \tag{4}$$

$\beta_{A,c}$ being the ratio between the effective area and the gross area of the stiffened plate, in order to take into account the effects of plate buckling in the parts between the stiffeners. The term $\sigma_{cr,p}$ represents the critical stress according to the plate mechanism, which in Annex A of EN 1993. 1-5 is given by:

$$\sigma_{cr,p} = k_{\sigma,p}\sigma_E \quad (5)$$

where

$$k_{\sigma,cp} = \frac{2\left[(1+\alpha^2)^2 + \gamma - 1\right]}{\alpha^2(\psi+1)(1+\delta)} \qquad \text{when} \quad \alpha \le \sqrt{\gamma}$$

(6a)

$$k_{\sigma,cp} = \frac{4\left(1+\sqrt{\gamma}\right)}{(\psi+1)(1+\delta)} \qquad \text{when} \quad \alpha \ge \sqrt{\gamma} \quad (6b)$$

The column mechanism will apply the European curves for buckling, in which the coefficient α given by

$$\alpha = \alpha + \frac{0{,}09}{i/e} \quad (7)$$

takes the values α =0,49 or α=0,34 depending on whether the stiffeners are open or closed. In the column mechanisms calculations, each longitudinal stiffener will be treated as a separate 'column', since they are simply supported between two transversal stiffeners.

To conclude, the effective width approach is based on an interpolation between the two mechanisms using equation (8), yielding the final reduction factor, ρ_c. This factor is applied to the effective area of the plate girder (i.e. the resultant area of the plate girder after the local plate-buckling study) to obtain the total effective area:

$$A_{eff,T,p} = \rho_c A_{c,eff,loc} + 2\left(\frac{b_{c,eff}}{2}\right)t \quad (8)$$

<u>Webs</u>

The application of the effective width method to stiffened webs remains analogous to that of the plate girder except for a number of adjustments arising from its linearly varying stress distribution: in the local plate-buckling analysis, the reduction factor applied to each subplate will depend on its particular stress distribution, ψ. Therefore the dimensions of the subplates parts adjacent to each stiffener will vary along the web. The EN-1993-1.5 [2] presents the effective width of the subplates as a function of the stress distribution in Table 5.3.2.

2.2.- Reduced stress method

<u>a) Non-stiffened plates</u>

This method is briefly covered in section 10 of EC3 1-5 as an alternative to the effective width method.

The reduced stress method states that the strength of a plate affected by buckling can be obtained by applying a reduction factor to the entire section with no reduction due to plate buckling. In this way, the plate could be treated as a 'class 3' element without carrying any reduction in its dimensions. The reduction factor is a function of the coefficient $\sigma_{ult,k}$, relating the applied stress to the yield stress of the plate. The latter may be obtained by one of the established stresses criteria, like the Von Mises criterion, which is the criterion adopted in the EC3 1-5.

In contrast with the effective width method, the reduced stress method depends on both the normal and the shear stress. This increases the complexity of its application to compressed plates, specially in the case of webs, since they concentrate the majority of the shear stresses. This explains its limited of use in common engineering practice.

The coefficient $\alpha_{ult,k}$ which when applied to the design stress, σ_{Ed}, leads to yielding of the most compressed point of the plate, can be obtained from:

$$\frac{1}{\alpha_{ult,k}^2} = \left(\frac{\sigma_{x,Ed}}{f_y}\right)^2 + \left(\frac{\sigma_{z,Ed}}{f_y}\right)^2 - \left(\frac{\sigma_{x,Ed}}{f_y}\right)\left(\frac{\sigma_{z,Ed}}{f_y}\right) + 3\left(\frac{\tau_{Ed}}{f_y}\right)^2 \tag{9}$$

In an analogous manner, the coefficient α_{crit}, given as the scaling factor that when applied to the design stress, σ_{Ed}, yields the critical stress, is determined from:

$$\frac{1}{\alpha_{crit}} = \frac{1+\psi_x}{4\alpha_{crit,x}} + \frac{1+\psi_z}{4\alpha_{crit,z}} + \left[\left(\frac{1+\psi_x}{4\alpha_{crit,x}} + \frac{1+\psi_z}{4\alpha_{crit,z}}\right)^2 + \frac{1-\psi_x}{2\alpha_{crit,x}^2} + \frac{1-\psi_z}{2\alpha_{crit,z}^2} + \frac{1}{\alpha_{crit,\tau}^2}\right]^{1/2} \tag{10}$$

The slenderness of the plate will be given by the ratio of these coefficients:

$$\bar{\lambda} = \sqrt{\frac{\sigma_{\max}}{\sigma_{crit}}} = \sqrt{\frac{\alpha_{ult,k}}{\alpha_{crit}}} \tag{11}$$

Subsequently, the reduced stress method accounts for the effects of plate buckling by introducing a reduction factor, ρ, which can be either the minimum of the reduction factor obtained by the interpolation formula, ρ_c, and the reduction factor due to shear stresses, χ_v, or the value obtained by interpolating between these factors. The final verification of this approach is expressed as follows:

$$\frac{\sigma_{Ed}}{\sigma_{Rd}} = \frac{\sigma_{Ed}}{\rho \dfrac{\alpha_{ult,k}\sigma_{Ed}}{\gamma_{M1}}} \leq 1 \tag{12}$$

<u>b) Stiffened plates</u>

<u>Plate girder</u>

The application of the reduced stress method to a stiffened plate girder proves to be relatively simple due to the fact that it is only subjected to normal compressive stresses. Therefore, the coefficient $\alpha_{ult,k}$ will only be a function of the applied normal stress $\sigma_{x,Ed}$,

$$\frac{1}{\alpha_{ult,k}^2} = \left(\frac{\sigma_{x,Ed}}{f_y}\right)^2 \tag{13}$$

And the coefficient α_{crit} is given by

$$\frac{1}{\alpha_{crit}} = \frac{1+\psi_x}{4\alpha_{crit,x}} + \left[\left(\frac{1+\psi_x}{4\alpha_{crit,x}}\right)^2 + \frac{1-\psi_x}{2\alpha_{crit,x}^2} + \frac{1}{\alpha_{crit,\tau}^2}\right]^{1/2} \tag{14}$$

<u>Webs</u>

Webs concentrate the majority of the shear stresses of a section. Therefore, in addition to the complexity of the variant distribution of normal stress, the incorporation of the shear stresses in calculations appears as a new difficulty. The coefficient $\alpha_{ult,k}$ will in this case be expressed as a function of the design normal stress, $\sigma_{x,Ed,}$ and the shear stress, τ_{Ed}:

$$\frac{1}{\alpha_{ult,k}^2} = \left(\frac{\sigma_{x,Ed}}{f_y}\right)^2 + 3\left(\frac{\tau_{Ed}}{f_y}\right)^2 \quad (15)$$

and the coefficient α_{crit} remains

$$\frac{1}{\alpha_{crit}} = \frac{1+\psi_x}{4\alpha_{crit,x}} + \left[\left(\frac{1+\psi_x}{4\alpha_{crit,x}}\right)^2 + \frac{1-\psi_x}{2\alpha_{crit,x}^2} + \frac{1}{\alpha_{crit,\tau}^2}\right]^{1/2} \quad (16)$$

3.- APPLICATION TO A PARTICULAR CASE (REFERENCE CASE)

In order to illustrate all the steps involved in the effective width method and in the reduced stress method, they have been applied to a particular case of plate girder and webs issued from a common bridge cross-section. The dimensions of these elements are represented in figures 4 and 5, respectively.

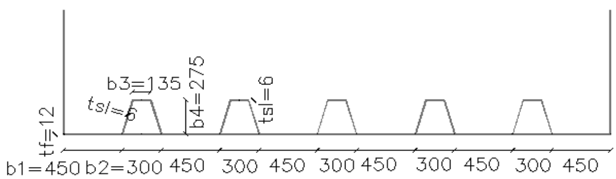

Figure 4: Plate girder dimensions (mm)

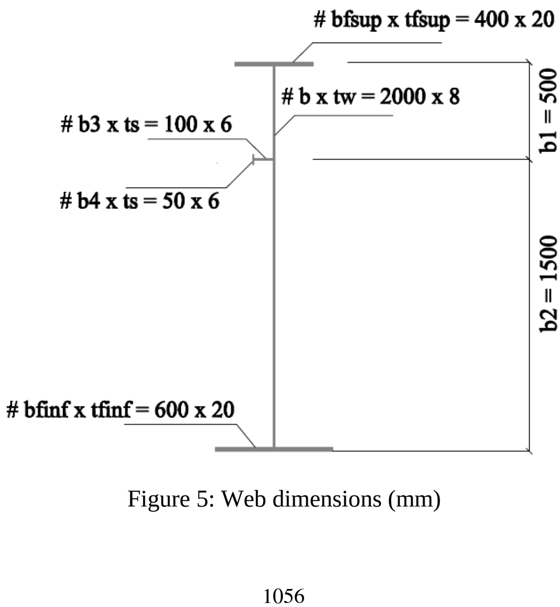

Figure 5: Web dimensions (mm)

The detailed calculations as well as the EXCEL spreadsheet used to perform these calculations [1] can be downloaded from [A]. The results issued from this particular study are summarized in Table 1.

Table 1: results for the application of the effective width method and the reduced stress method to a particular example of plate girder and web (reference case)

	Effective width method	Reduced stress method
Plate girder	$\frac{A_{c,eff}}{A_{bruta}} = \frac{A_{c,eff}}{A_c + \sum b_{extremos} t} = \frac{54.487,34}{65.550 + 450 \cdot 12} = 0,768$	$\frac{\sigma_{Rd}}{f_y} = \frac{262,89}{355} = 0,740$
Web	$\frac{W_{eff}}{W_{bruto}} = \frac{19.631.758,6}{22.927.995,2} = 0.856$	$\frac{\sigma_{Rd}}{f_y} = \frac{\sigma_{Rd,c}}{f_y} = \frac{141,46}{355} = 0,398$

4.- PARAMETRIC STUDY

In order to evaluate the accuracy of both methods, a parametric study has been performed. In this study [1], each method is applied to a number of different symmetries and dimensions of stiffened plate girders and webs. As it has been said, all these calculations have been performed using an EXCEL spreadsheet, which can be downloaded from [A].

4.1.- Case studies

The parametric study includes 12 cases studies based on 4 examples of plate girders and 8 examples of webs derived by varying the thickness of the reference case. Every case study is divided in groups of analysis: 3 for the plate girder and 1 for the webs. Each group covers the application of each method when varying a single dimension parameter from the reference case: the width, b, length, a, and number of stiffeners, N, will be the varying parameters for the plate girder, and the height, h, for the web's case. For each group, the effective area, $A_{c,eff,}$ (or effective modulus $W_{c,eff}$ in the case of webs) and the reduced strength, $\sigma_{ult,d}$, will be obtained**.** Finally the comparison of the two methods will follow from

the analysis of a number of graphs showing the variation of the results ratio $\frac{A_{c,eff} / A_{bruta}}{\sigma_{ult,d} /(f_y / \gamma_{M1})}$ when varying each dimension.

Plate girder

The 4 study cases are based in a plate girder of thickness 12, 16, 20 and 25 mm respectively. For all the study cases, the values for the varying parameters of each group are:

- width b: from b=4000mm to b=5000 at 200mm intervals, and from b=5000mm to b=10000 at 500mm intervals
- length a: from a=1000mm to a=10000 at 1000mm intervals
- Number of stiffeners N: N=4, 5, 6, 7

Webs

The parametric study includes 8 case studies considering webs of thickness t_w= 6, 8, 10, 12, 16, 20, 25 and 30mm. Two types of flanges were considered: flanges of different dimensions (using the dimensions of the reference case) and flanges of equal dimensions. The study will involve only one group of analyses, with the height of the web h as the varying parameter. This parameter will take values from h=1500 to h=4000mm at 250mm intervals. It is important to note that the stiffeners dimensions also vary as the height of the web increases, as can be seen in the results.

4.2.- Analysis of the study's results

In the parametric study the 'loss of strength' of a compressed element has been quantified, in the case of the effective width method and the reduced stress method respectively, by the following ratios:

$$\frac{A_{c,eff}}{A_{bruta}} \qquad \frac{\sigma_{ult,d}}{f_y/\gamma_{M1}} \qquad (17a) \quad (17b)$$

In order to compare the results of each method the ratio $\frac{A_{c,eff}/A_{bruta}}{\sigma_{ult,d}/(f_y/\gamma_{M1})}$ has been computed for each group of analysis and plotted in a graph against its corresponding varying parameter. Consequently, a ratio>1 implies that the effective width method yields higher estimations of post-buckling strength than the reduced stress method.

<u>Plate girder</u>

The following graphs show the ratio of the reduced strengths obtained with the effective width and the reduced stress methods, $\frac{A_{c,eff}/A_{bruta}}{\sigma_{ult,d}/(f_y/\gamma_{M1})}$, when varying the width, *b*, and the length, *a*, of the plate.

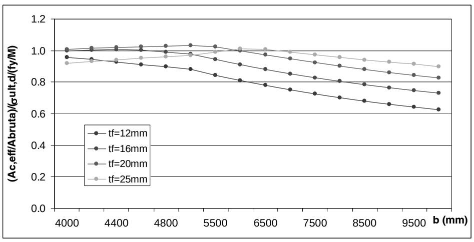

Figure 6a: Plate girder. Results depending on the width (*b*)

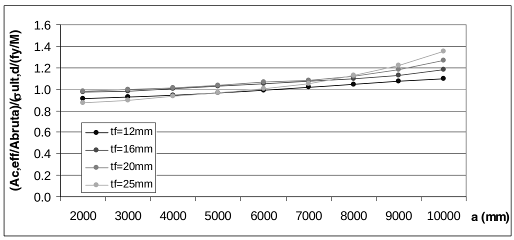

Figure 6b: Plate girder. Results depending on the length (*a*)

<u>Web</u>

The following graphs show the ratio of the reduced strengths obtained with the effective width and the reduced stress methods, $\frac{W_{c,eff}/W_{bruta}}{\sigma_{ult,d}/(f_y/\gamma_{M1})}$, when varying the height, *h*, of the webs.

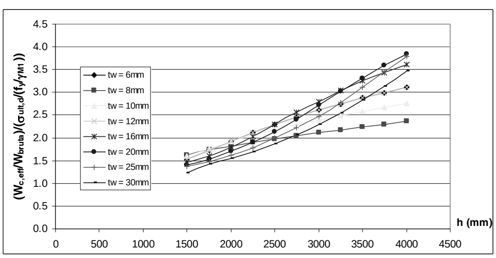

Figure 7a: Web with equal flanges. Results depending on the height (*h*)

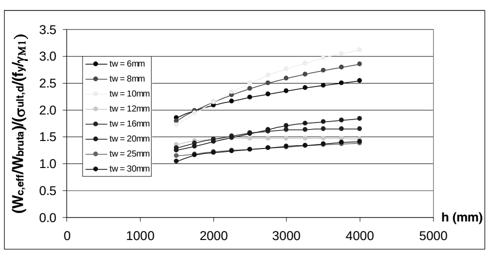

Figure 7b: Web with different flanges. Results depending on the height (*h*)

5.- CONCLUSIONS

5.1.-Conclusions relatives to the plate girder case

- As the width, b, of the plate girder increases (fig. 7a), the reduction factors obtained by each method converges (i.e. $\frac{A_{c,eff} / A_{bruta}}{\sigma_{ult,d} /(f_y / \gamma_{M1})} \approx 1{,}00$) until b=6000mm. For higher values of b, the slenderness of the plate becomes excessive and the reduced stress method proves to be less conservative than the effective width method. This can be most clearly observed in the case of the slenderest plate girder, of t_f=12mm and b=10000mm, for which the reduction factor yielded by the effective width method is 0,625 times that yielded by the reduced stress method.

- When varying the length of the plate girder the opposite conclusion is derived: the ratio between the reduction factor remains constant up to a value of a=6000mm and then, for higher values, the effective width method gives less conservative values that the reduced stress method. In particular, the reduction factor for the effective width method is 1,345 times that of the reduced stress method for the case of a plate girder of t_f=25mm and b=10000mm.

- Varying the number of stiffeners implies varying the width of the subplates. In this case, the outcomes of each method are very similar since the slenderness of the subplates is never excessive, remaining within the range $10{,}5 \le b/t_f \le 50$.However, for significantly small slenderness ($b/t_f \approx 10$), the effective width method becomes too conservative, yielding a reduction factor 0,529 times the reduction factor obtained by the reduced stress method

5.2.- Conclusions relatives to the web with different flange dimensions

- The slenderest webs, with thickness t_w between 6 and 10mm, are highly unstable against normal stresses and therefore their strength is significantly reduced when applying the reduced strength method. However, the effective width method allows for the redistribution of the centre of gravity, so that the strength is less reduced. Therefore, this method leads to resistances up to 3 times that obtained with the reduced stress method for the case of h=4000mm.

- For thicker elements, of thickness between t_w=12 and 30mm, the same conclusion is obtained. However, the difference between the two methods is less pronounced, the reduction factor for the effective width case been only 1,5 times that of the reduced strength case for h=4000mm

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