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CROSS-SECTIONAL STABILITY OF STRUCTURAL STEEL

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***Abstract**. The objective of this paper is to more fully understand local cross-section stability of hot-rolled steel members and more accurately account for web-flange interaction to create a more robust method for the design of high yield stress structural steel cross-sections that are locally slender. First, analytical expressions for the elastic cross-section local buckling stress, including element interaction, of hot-rolled steel structural shapes are provided. The expressions are based on plate buckling coefficients (k's) determined by finite strip analysis (FSA). The k's from FSA are then compared to the values inherently assumed in the U.S. (AISC) Specification, and significant differences are observed. Finally, a series of nonlinear finite element analyses are conducted to compare three commonly used design methods for locally slender steel beams and columns for the purpose of understanding and highlighting the parameters that lead to divergence between the capacity predictions of the different design methods.*

1 INTRODUCTION

Cross-section stability of structural steel and the local slenderness limits for a section to remain compact are function of the yield stress. As new steels are introduced and yield stress increases the potential for cross-section stability to control the strength also increases. Today, with the availability of high and ultra-high yield strength steels, it is becoming uneconomical to continue avoiding the use of locally slender cross-sections, which essentially ignores the beneficial post-buckling reserve that exists in the local buckling modes. The objective of this ongoing effort towards a fuller understanding of hot-rolled steel cross-sectional local stability, and a more accurate accounting of web-flange interaction, is to create a more robust design method for high yield stress structural steel cross-sections that are locally slender.

2 SLENDERNESS LIMITS OF STRUCTURAL STEEL

2.1 Overview

Finite strip analysis (FSA) is used to study and evaluate the slenderness limits that are currently defined by design codes. These codes use a single slenderness limit for each type of element, indicating a single value of elastic local buckling coefficient. Contrary to this, FSA results show plate buckling coefficients fall in a wide range. Based on the FSA results a series of simple empirical equations were developed to provide an approximate means of predicting the local plate buckling coefficients for all of the section types under different loading conditions. The equations developed were used to construct a proposed alternative to Table B4.1 in the AISC manual for analyzing local stability (2005 AISC manual of steel construction [1]).

2.2 AISC local buckling criteria

In this section the local buckling width-to-thickness limits of the AISC Specification are examined, and a method provided for determining the assumed local plate buckling coefficients (k's) inherent in the AISC Specification. Currently, the AISC Specification defines, in Table B4.1, the local buckling criteria in terms of width-to-thickness ratios, i.e., for an element of width b, and thickness, t:

$$\lambda = \frac{b}{t} \tag{1}$$

The elastic critical local buckling stress of this element is:

$$f_{cr} = k\frac{\pi^2 E}{12\left(1-\nu^2\right)}\left(\frac{t}{b}\right)^2 = k\frac{\pi^2 E}{12\left(1-\nu^2\right)}\left(\frac{1}{\lambda}\right)^2 \tag{2}$$

where E is Young's modulus, ν is Poisson's ratio, and k is the local plate buckling coefficient which accounts for the boundary conditions and loading. Plate slenderness, (e.g., see [2]) is expressed as:

$$\alpha = \sqrt{f_y / f_{cr}} \tag{3}$$

where f_y is the yield stress. Note the usual notation for plate slenderness is λ, but AISC uses this symbol for b/t, so the symbol α has been adopted here for plate slenderness. The AISC compactness limits, λ_r (associated with α_r) define the non-slender/slender element limits for columns or the non-compact/slender element limit for beams as:

$$\lambda_r = \left(\frac{b}{t}\right)_r = \beta_r\sqrt{E / f_y} \tag{4}$$

where β_r is given for different element types and loading conditions in AISC Table B4.1. The inter-relationship between the plate buckling coefficient, k, plate slenderness limit, α_r, and width-to-thickness ratio coefficient β_r may be found by substituting f_{cr} from (2) into (3) and solving for the $(b/t)_r$ limit of (4):

$$\sqrt{k\frac{\pi^2}{12\left(1-\nu^2\right)}\alpha_r^{\,2}}\sqrt{E / f_y} = \left(\frac{b}{t}\right)_r \tag{5}$$

Using (4) and solving for k results in:

$$k = \frac{\beta_r^{\,2}}{\alpha_r^{\,2}}\frac{12\left(1-\nu^2\right)}{\pi^2} \tag{6}$$

Implying that, if the plate slenderness limit α_r is known, and given β_r is provided (from AISC Table B4.1) then the k value assumed by the AISC Specification may be back-calculated.

The plate slenderness limits α_r are generally determined from testing. For compression AISC employs an α_r=0.7 (which itself implies $f_{cr}=2f_y$) (e.g., see [3]). This agrees well with Winter's equation, employed extensively in cold-formed steel design (2007 AISI manual [4]), where α_r=0.673. The k value assumed in AISC may now be found for any section, for example, consider the case of compression in webs of doubly symmetric I-shaped sections, β_r is 1.49 (from AISC's Table B4.1), which will yield a k value of 5.0 when using α_r of 0.7; which is about one-third of the way between simply supported (k = 4.0) and fixed (k = 6.97) boundary conditions.

For flexural members, much less is provided in the literature about the assumed plate slenderness limit, α_r. The best discussion the authors have been able to find is provided in relation to plate girders where the criteria α_r=1.0 is clearly employed (which implies f_{cr}=f_y) [Salmon et al. 2009]. For doubly symmetric I-shaped sections in flexure Table B4.1 provides a β_r of 5.7, assuming α_r=1.0 results in a k of 36, which is about 80% between the range of simply supported (k = 23.9) and fixed (k = 39.6) boundary conditions, and this k is consistent with the discussions in White [5] and Salmon et al. [3]. It is worth noting that cold-formed steel design [4] uses α_r=0.673 for elements in flexure as well as in compression.

2.3 Local buckling finite strip analysis

FSA was performed on all sections in the AISC shape database (v3) from the Manual of Steel Construction (excluding pipe sections) (AISC 2005 [1]). The analysis was completed using CUFSM version 3.12 [6]. Sections were simplified to their centerline geometry (the increased width in the k-zone was thus ignored) and analyzed under different loading conditions: axial compression, positive and negative major-axis bending, and positive and negative minor-axis bending.

The cross-section elastic local buckling stress, $f_{cr\ell}$, is found from the finite strip analysis. The local buckling stress is converted into local plate buckling coefficients (k's) for comparison to existing design provisions and for the development of the new approximate design expressions as follows. The plate buckling solution for the flange is:

$$f_{crb} = k_f \frac{\pi^2 E}{12\left(1-\nu^2\right)}\left(\frac{t_f}{b}\right)^2 \tag{7}$$

where k_f is the flange (horizontal element) local plate buckling coefficient, b is the unsupported flange width (i.e., ½ of b_f for a W-section, b_f is the total flange width), t_f is the flange thickness, E is the Modulus of elasticity, and ν is Poisson's ratio. Setting $f_{crb} = f_{cr\ell}$ (from the FSA) and solving for k_f:

$$k_f = f_{cr\ell} \frac{12\left(1-\nu^2\right)}{\pi^2 E}\left(\frac{b}{t_f}\right)^2 \tag{8}$$

Similarly, the web buckling coefficient, k_w, can be found, where:

$$f_{crh} = k_w \frac{\pi^2 E}{12\left(1-\nu^2\right)}\left(\frac{t_w}{h}\right)^2 \tag{9}$$

and setting $f_{crh} = f_{cr\ell}$, we can solve for k_w as:

$$k_w = f_{cr\ell} \frac{12\left(1-\nu^2\right)}{\pi^2 E}\left(\frac{h}{t_w}\right)^2 \tag{10}$$

where k_w is the web (vertical element) local plate buckling coefficient, h is the distance between the centerline of the flanges less the fillet, and t_w is the web thickness. Using the full cross-section elastic local buckling stress, $f_{cr\ell}$, the plate buckling coefficients resulting from (8) and (10) will thus include web-flange interaction.

2.4 Finite strip analysis results

Consider the AISC W-sections as an example; the flange plate buckling coefficient, k_f, including web-flange interaction can be calculated from each finite strip analysis from (8). For the AISC W-sections in pure compression, the resulting k_f's are provided in Figures 1(a) and (b). Figure 1(a) highlights that the flange plate buckling coefficient is not independent of the web slenderness h/t_w, i.e., web-flange interaction is real and unavoidable. Figure 1(b) shows that if both web and flange slenderness are considered, relatively simple functional relationships may exist for predicting when local buckling occurs. The web plate buckling coefficients are provided for the W-sections in pure compression in Figures 1(c) and 1(d). The web plate buckling coefficient is dependent on the flange slenderness, but again a simple combination of slenderness may adequately describe the plate buckling coefficient, as shown in Figure 1(d). The same observations are true for the different loading cases for all types of sections.

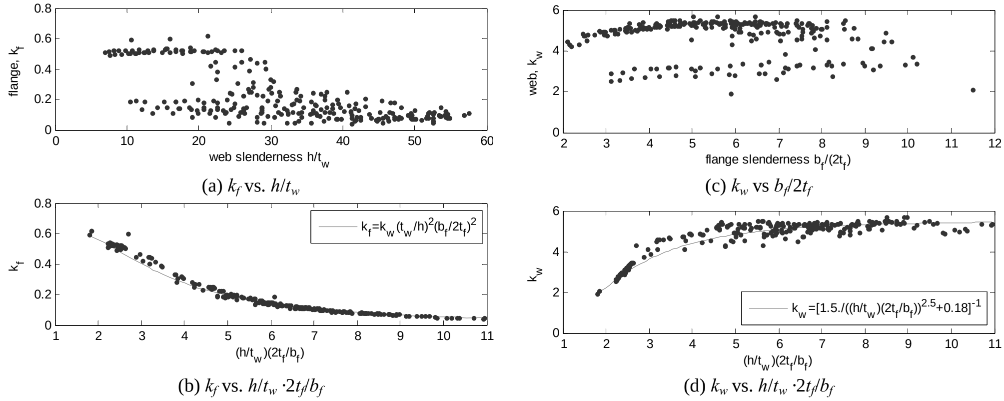

Figure 1: Flange and web local buckling coefficients for w-sections under axial loading.

2.5 Comparison to AISC specification limits

Comparison of the AISC assumed k values with those from the FSA indicates (a) the k values fall in a wide range and use of a single k value is bound to be quite approximate, and (b) in some cases AISC falls near the mean k value predicted from finite strip analysis in other cases it may be significantly higher or lower than the mean. For example, for tees the AISC values are quite near the mean k values, while for W-sections the cases of the flange in flexure and the web in compression are near the mean k values, while the cases of the web in flexure and the flange in compression are significantly higher (unconservative) compared with the mean finite strip k values. To judge the actual impact of the selected k values they must be taken in the context of the AISC Specification, for instance, a high k value for an unstiffened element may have little impact given that post-buckling of slender unstiffened elements is essentially ignored in the AISC Specification. Nonetheless, the lack of a consistent rational basis for the assumed k values employed in the AISC Specification would seem to be an impediment to advancing prediction of local buckling phenomenon. For a complete comparison and histograms of the *k values* see [7].

2.6 Development plate buckling coefficients expressions

As shown in Figure 1 simple functional relations exist such that the local plate buckling coefficients can be expressed as a function of section geometry. Further, note that using the same cross-section elastic local buckling stress, f_{cr}, instead of the individual f_{crb} and f_{crh}, implies that (7) and (9) must be equal, thus the flange and web local buckling coefficients are related by:

$$k_f = k_w \left(\frac{t_w}{h}\right)^2 \left(\frac{b}{t_f}\right)^2 \quad \text{or} \quad k_w = k_f \left(\frac{t_f}{b}\right)^2 \left(\frac{h}{t_w}\right)^2 \tag{11}$$

Due to (11) only one local plate buckling coefficient needs to be determined for a cross-section. Therefore, for each loading case, either k_f or k_w was selected and a series of simple empirical equations were developed to provide an approximate means of predicting the local plate buckling coefficients. These equations represent a potential beginning for the evolution of Table B4.1 in the AISC Specification for analyzing local stability. Note that the k expressions were developed to best match the results obtained from the FSA of the sections in the AISC shape database. Applying these expressions for sections with dimensions falling outside the range of the current database will need further assessment for accuracy. Finite strip analysis may still be used for sections or loading not covered herein. For a complete list of the developed equations see [7].

3 COMPARISON OF DESIGN METHODS FOR LOCALLY SLENDER MEMBERS

3.1 Overview

A series of nonlinear finite element analyses are used to compare three commonly used design methods for locally slender steel beams and columns. To aid the comparison of the available methods the design strength formulas, for locally slender W-section beams and columns, are provided in a common notation. The resulting design expressions highlight the prominent role of elastic cross-section stability as the key parameter for strength prediction. A nonlinear finite element analysis parameter study, using ABAQUS, is performed for the purpose of understanding and highlighting the parameters that lead to the divergence between the capacity predictions of the different design methods.

3.2 Design methods

The design of locally slender steel cross-sections may be completed by a variety of methods, three of which are examined in this study: (1) The AISC method, as embodied in the 2005 AISC Specification, labeled **AISC** herein, (2) The AISI Effective Width Method from the main body of the 2007 AISI Specification for cold-formed steel, labeled **AISI** herein, and, (3) The Direct Strength Method as given in Appendix 1 of the 2007 AISI Specification, labeled **DSM** herein.

The AISC method uses the Q-factor approach to adjust the global slenderness in the inelastic regime of the column curve to account for local-global interaction, and further uses a mixture of effective width (for stiffened elements) and average stress (for unstiffened elements) to determine the final reduced strength. The AISI method uses the effective width approach. In the AISI method the global column curve is unmodified but the column area is reduced to account for local buckling in both stiffened and unstiffened elements via the same effective width equation. Finally, the DSM uses a new approach where the global column strength is determined and then reduced to account for local buckling based on the local buckling cross-section slenderness.

To provide a more definitive comparison between these three methods the formulas are presented in a common set of notation in Table 1. The format of presentation is modified from that used directly in the respective Specifications so that (i) the methods may be most readily compared to one another and (ii) the key input parameters are brought to light. It is noted that if the cross-section local buckling (f_{cr}) is used in place of isolated plate buckling solutions (f_{crb} and f_{crh}) equations become even simpler.

The number of free parameters in slender column design is actually significantly less than one might typically think. Based on Table 1, and performing a simple non-dimensional analysis, the parameters for determining the column strength of an idealized W-section are:

AISC: $P_n/P_y = \mathrm{f}\,(f_e/f_y, f_{crb}/f_y, f_{crh}/f_y, ht_w/A_g)$

AISI: $P_n/P_y = \mathrm{f}\,(f_e/f_y, f_{crb}/f_y, f_{crh}/f_y, ht_w/A_g \text{ or } 2b_ft_f/A_g)$

DSM: $P_n/P_y = \mathrm{f}\,(f_e/f_y, f_{cr}/f_y,)$

The central role of elastic buckling prediction both globally (f_e) and locally (f_{crb}, f_{crh} or f_{cr}) in determining the strength of the column is clear. Further, the "direct" nature of the DSM approach is highlighted as DSM only uses ratios of critical buckling values to determine the strength; where AISC and AISI still involve cross-section parameters beyond determination of gross area and critical stress.

Table 1 Comparison of column design equations for a slender W-section in a common notation

AISC Inputs to find P_n: A_g = gross area f_e = global buckling stress f_y = yield stress f_{crb} = flange local buckling stress f_{crh} = web local buckling stress ht_w/A_g = web/gross area Comments: shifts the slenderness in the global column curve in the inelastic range only, assumes that unstiffened elements (flange) should be referenced to f_y, only applies an effective width style reduction to stiffened elements (the web), includes an iteration for web stress f.	$P_n = A_g\hat{f}_n$ $\hat{f}_n = \begin{cases} Q_sQ_a(0.658)^{Q_sQ_a(f_e/f_y)}f_y & \text{if } f_e \geq 0.44Q_sQ_af_y \\ 0.877f_e & \text{if } f_e < 0.44Q_sQ_af_y \end{cases}$ $Q_s = \begin{cases} 1.0 & \text{if } f_{crb} \geq 2f_y \\ 1.415 - 0.59\sqrt{\dfrac{f_y}{f_{crb}}} & \text{if } \dfrac{3}{5}f_y < f_{crb} < 2f_y \\ 1.1\dfrac{f_{crb}}{f_y} & \text{if } f_{crb} \leq \dfrac{3}{5}f_y \end{cases}$ $Q_a = \begin{cases} 1.0 & \text{if } f_{crh} > 2f \\ 1 - \left(1 - 0.9\sqrt{\dfrac{f_{crh}}{f}}\left(1 - 0.16\sqrt{\dfrac{f_{crh}}{f}}\right)\right)\dfrac{ht_w}{A_g} & \text{if } f_{crh} \leq 2f \end{cases}$ $f = \dfrac{P_n}{Q_aA_g} \sim \hat{f}_n$ determined with $Q_s = Q_a = 1$
AISI - Effective Width Inputs to find P_n: A_g = gross area f_e = global buckling stress f_y = yield stress f_{crb} = flange local buckling f_{crh} = web local buckling bt_f = flange area ht_w = web area Comments: no shift in global column curve, effective width used for stiffened and unstiffened elements.	$P_n = A_{eff}f_n$ $f_n = \begin{cases} (0.658)^{(f_e/f_y)}f_y & \text{if } f_e \geq 0.44f_y \\ 0.877f_e & \text{if } f_e < 0.44f_y \end{cases}$ $A_{eff} = 4\rho_b bt_f + \rho_h ht_w$ $b_e = \rho_b b$ where $\rho_b = \begin{cases} 1 & \text{if } f_{crb} \geq 2.2f_n \\ \left(1 - 0.22\sqrt{\dfrac{f_{crb}}{f_n}}\right)\sqrt{\dfrac{f_{crb}}{f_n}} & \text{if } f_{crb} < 2.2f_n \end{cases}$ $h_e = \rho_h h$ where $\rho_h = \begin{cases} 1 & \text{if } f_{crh} \geq 2.2f_n \\ \left(1 - 0.22\sqrt{\dfrac{f_{crh}}{f_n}}\right)\sqrt{\dfrac{f_{crh}}{f_n}} & \text{if } f_{crh} < 2.2f_n \end{cases}$
AISI - DSM Inputs to find P_n: A_g = gross area f_e = global buckling stress f_y = yield stress $f_{cr\ell}$ = local buckling stress Comments: similar to AISI but reductions on whole section and "effective width" equation modified.	$P_n = A_{eff}f_n$ $f_n = \begin{cases} (0.658)^{(f_e/f_y)}f_y & \text{if } f_e \geq 0.44f_y \\ 0.877f_e & \text{if } f_e < 0.44f_y \end{cases}$ $A_{eff} = \rho A_g$ $\rho = \begin{cases} 1 & \text{if } f_{cr\ell} \geq 1.66f_n \\ \left(1 - 0.15\left(\dfrac{f_{cr\ell}}{f_n}\right)^{0.4}\right)\left(\dfrac{f_{cr\ell}}{f_n}\right)^{0.4} & \text{if } f_{cr\ell} < 1.66f_n \end{cases}$

AISC and AISI/DSM use different formats for the global (lateral-torsional buckling) provisions of beams. However, for no moment gradient (C_b = 1) the resulting expressions are actually quite similar with the exception that AISI only provides capacities up to first yield (M_y) for sections subject to lateral-torsional buckling. For AISI/DSM local-global interaction in beams is treated in the same conceptual manner as for columns; not so for AISC, which uses nothing like the Q-factor approach, and instead provides direct reductions based on the flange and web plate slenderness (see [5]). A result of AISC's approach (i.e., not adopting one consistent philosophy for local-global interaction in beams) some unusual changes and discontinuities in strength prediction occur as local slenderness is varied.

3.3 FE parameter study

A nonlinear finite element (FE) analysis parameter study was carried out for the purpose of understanding and highlighting the parameters that lead to the divergence between the capacity predictions of the different design methods under axial and bending loads. The FE analysis was performed on both short members where only local buckling modes exist, and long members, where the locally slender cross-sections may interact with global (flexural, lateral-torsional, etc.) buckling modes. Based on the authors' judgment, AISC W14 and W36 sections were selected for the study as representing "common" sections for columns and beams in high-rise buildings. The W14x233 section is approximately the average dimensions for the W14 group and the W36x330 for the W36 group.

Geometric variation: To examine the impact of slenderness in the local buckling mode, and the impact of web-flange interaction in I-sections, four series of parametric studies are performed under axial and bending loading: **W14FI**: a **W14**x233 section with a modified **F**lange thickness, that varies **I**ndependently from all other dimensions, **W14FR**: a **W14**x233 section with variable **F**lange thickness, but the web thickness set so that the **R**atio of the flange-to-web thickness remains the same as the original W14x233, **W36FR**: a **W36**x330 section with variable **W**eb thickness, but the flange thickness set so that the **R**atio of the flange-to-web thickness remains the same as the original W36x330, and **W36WI**: a **W36**x330 section with a variable **W**eb thickness, that varies **I**ndependently from all other dimensions.

Modeling: ABAQUS was used to perform the analysis. Members were modeled using S4 shell elements. The choice of element type and density are based on a comparison study for different FE elements reported in [8]. All sections are modeled with globally pinned, warping fixed boundary conditions, and loaded via incremental displacements. The material model follows classical metal plasticity. The classic residual stress distribution of Galambos and Ketter [9] is employed. Initial geometric imperfections are added through linearly superposing a scaled local and a scaled global eigenmode solution from a FSA performed on each section, using CUFSM. The local buckling mode is scaled so that the maximum nodal displacement is equal to the greater of $b_f/150$ or $d/150$, while the global buckling mode is scaled so that the maximum nodal displacement is $L/1000$.

3.4 Results

The parametric study focuses on W14 and W36 sections, where through modification of element thicknesses, the flange slenderness, and/or web slenderness are systematically varied (from compact, to noncompact, to slender in the parlance of AISC). Due to limited space, as a sample, the results of the parametric study are presented for each group, including comparisons to the AISC, AISI, and DSM design methods for the stub columns in Figure 2. Results are plotted as a function of elastic local slenderness of the cross-section: $\sqrt{f_y/f_{crl}}$, determined by finite strip analysis. See [8] and [10] for full results and discussion. Generally, results indicate that AISC is overly conservative when the flange is slender; AISC's assumption of little to no post-buckling reserve in unstiffened elements is not borne out by the analysis. AISI's effective width method is a reliable predictor; only for the beam studies does AISI provide overly conservative solutions when the web is compact but the flange slender. DSM provides reliable predictions when both flange and web slenderness vary together, but is overly conservative when one element is significantly more slender than another.

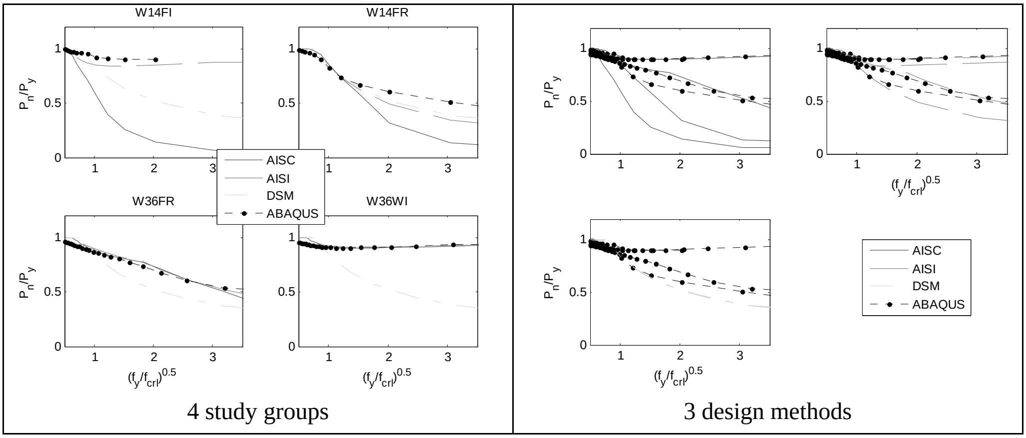

Figure 2: Results of stub column parametric study

4 CONCLUSIONS

Consideration of local buckling is an important part of the design of structural steel shapes. The primary means for consideration of local buckling in the AISC Specification is the use of width-to-thickness limits for each element of a cross-section. It is shown herein that this method assumes that a unique plate buckling coefficient, or k value, exists for each element of a section, and that the web-flange interaction is thus fixed. However, as demonstrated with finite strip analysis, the local plate buckling coefficients vary widely for a given section and loading. Nonetheless, the variation in k may be expressed as a function of the member geometry and loading and simple relations are provided for such k, which include web-flange interaction. The developed expressions provide a potential first step towards rationalizing the AISC Specification approach to local buckling limit states across the different sections. The design of locally slender steel cross-sections may be completed by a variety of methods, yet the key parameters are the elastic local (element, or member) buckling stress and the material yield stress. A parametric study conducted with nonlinear finite element analysis is used to examine the performance of available design methods as a function of local cross-section slenderness. The results, presented only in brief here, demonstrate that the AISC methodology may be overly conservative, and provide a basis for improving the Direct Strength Method and its application to structural steel sections.

5 ACKNOWLEDGEMENTS

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