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SOME ISSUES FOR COLUMN STABILITY CRITERIA

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***Abstract**. The strength and stability of columns have been the subjects of numerous studies since the original work of Euler in 1744 and 1759. Elastic buckling of perfectly straight columns was examined during the 19th century, with landmark theories developed by Engesser and Considère. Several series of column tests were conducted to establish the correlation between theory and physical behavior. Research in the 20th century examined the influence of material and member imperfections, including the tangent modulus study of Shanley. These efforts provided the resolution of the effects of material non-linearity, residual stress and out-of-straightness. The definitive solutions were developed in the 1970-s, when modeling and numerical procedures allowed all nonlinear effects to be included. Since that time reliability and probabilistic solutions have provided state-of-the-art limit states criteria for steel columns. Such approaches are now the bases for columns in all advanced design codes in the world.*

1 INTRODUCTION

The strength and behavior of columns is one of the longest lasting, continuous research endeavors in structural engineering. From the classical studies of Euler, Tredgold, Tetmajer, Engesser, Considère, von Kármán, Salmon, Shanley, Beedle and others until today [1], the level of knowledge has advanced from understanding the characteristics of the straight, elastic member to the current treatment of columns with all forms of nonlinearities. The columns are now analyzed and designed as structural members that interact with and are affected by the other members and the connections and indeed the entire structure. The advancement has closely paralleled the evolution of testing and computational tools.

The most significant developments have taken place since the late 1960-s, and this was only possible because of the advent of computers. For example, the solution of the general inelastic flexural buckling problem dates from that time, and studies of the influence of end restraint on inelastic columns were only finalized in the late 1990-s [2]. In brief, the solution of two- and three-dimensional inelastic stability problems with randomly variable column strength parameters was far too complex for the traditional closed-form techniques. Numerical solutions were the only option, and advances in testing equipment and measurement tools have allowed for close agreement between tests and theory.

A variety of column strength formulas have been developed over the years, being based on theoretical models or the results of column tests or combinations thereof. Some of these have appeared in design codes, but most have not survived the tests of time and usefulness. However, even the most advanced of some of these approaches have suffered from certain drawbacks, whether due to complexity in formulation, limited applicability or consistency for all types of members and steel types, or any number of other reasons.

At this time, however, the states-of-the-art of computation and testing have progressed so far that accuracy in modeling and realism in testing make it possible to expand the design criteria to take into account a number of additional features. For example, two- and three-dimensional response characteristics are incorporated into some of the advanced software that is available, as is improved correlation and interaction between the column and the surrounding structure. Thus, connection restraint is now built into some modeling schemes, and overall structural reliability has become a realistic feature of several international codes [3, 4, 5].

Technical advances have significantly shortened the time lag between research results availability and code adoption. In some respects this is a desirable development; it also has certain disadvantages. This is particularly because too rapid acceptance may cause some, if not all, practical needs and implications to be overlooked. Contradictory as this may sound, it is nevertheless a fact that design criteria need a certain period of thought and practical maturation before they are adopted by a design standard. As an example (although not a stability consideration), the early incorporation of the initial block shear provisions into the AISC Specification in 1978 led to a great number of difficulties. Since that time a number of researchers have advanced proposals for correct representation of limit states and improved design accuracy. However, changes continue to be developed, including in the AISC Specification that was recently adopted [3].

For the case of steel columns, the current level of knowledge reflects the fact that much effort has been devoted to evaluating practical applications of the strength and performance data that are available [2, 6, 7, 8, 9]. It is a matter of record that the present amount of data and other information significantly exceeds the size of the data base that was used to establish earlier column formulas. Improvements therefore can and have been made on the basis of factual results, with continuing improvements through earlier and current code editions [2, 3, 4, 5].

2 ELEMENTARY AND ADVANCED STABILITY CONCEPTS

The basic mechanistic column models can be categorized as follows, in ascending order of accuracy and complexity:

- (i) Individual pinned-end column, perfectly straight, elastic material
- (ii) Individual pinned-end column, initially curved, elastic or inelastic material, elastic or inelastic structural response, incorporating residual stress, for example.
- (iii) Individual column with restrained ends, initially curved, inelastic material and/or response, incorporating end restraint developed by members that frame into the column, for example. Random behavior of the column strength parameters can be included, for complete reliability analysis.
- (iv) The column is no longer an individual element, but part of a planar subassemblage. This consists of the member itself with all its material and geometric imperfections, and at least the immediately adjacent columns in the frame. End restraint effects may be taken into account by using the properties of the actual connections, although the original model used rigid joints [10].
- (v) The column is one of the elements of the original planar structure, including all of the individual performance parameters. The overall influence of framed and leaning columns is accounted for by this model.
- (vi) The column is part of a three-dimensional subassemblage which incorporates end restraint, initial curvature and inelastic response characteristics. The ultimate limit state may be flexural buckling about any axis, or flexural-torsional buckling.
- (vii) The column is one of the elements of a three-dimensional frame. The model incorporates all conceivable strength and stiffness parameters, including the influence of leaning columns.

It is understood that any of the mechanistic models may treat the column strength parameters as deterministic or probabilistic. In the latter case, the evaluation of the column stability will also incorporate the reliability aspects of the member and at least the subassemblage; the complete frame may be used in lieu of the subassemblage. In a limit states code format, this is required.

Model (i) was used by Euler for his classical solution. It was also that of the Engesser and Considère tangent modulus (and double modulus) solution. Model (ii) was the state-of-the-art in the early 1970-s [1, 2, 7, 11]. Figure 1 illustrates the basic load-deflection characteristics of the inelastic column models, including the tangent modulus response of the perfectly straight column. Various forms of Model (iii) were developed through an intensive, international research effort in the 1980-s that led to realistic modeling and quantification of the restraint effects, including criteria for code implementation and design [12, 13, 14, 15, 16].

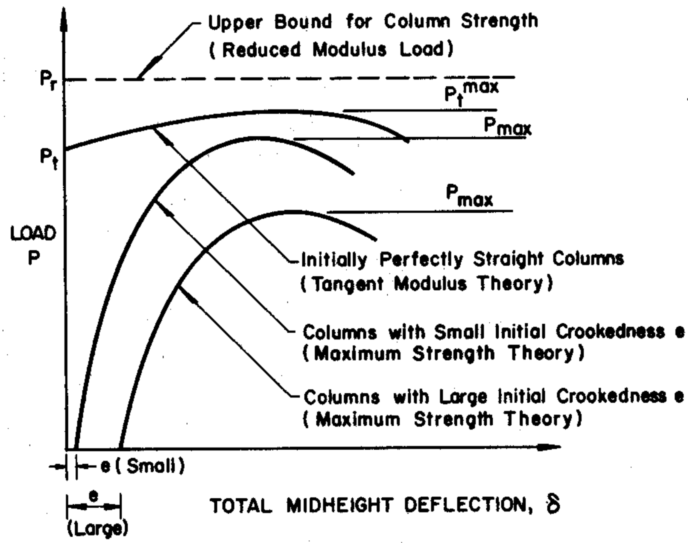

Figure 1: Load-Deflection Curves for Inelastic Columns

Common to Models (ii) through (iv) is the fact that they provide detailed theoretical solutions for the column strength, and the influence of the strength parameters is covered explicitly. Notwithstanding the limitations, the results are not subject to the random variations that are associated with physical tests. The sources, but not the magnitudes of the test variations are largely known, and emphasize the need for a very large body of test data if such are to serve as the sole basis for the design criteria [7, 17, 18]. This is further emphasized by the approximately 100 full scale column test results that are illustrated in Fig. 2. Only an accurate mechanistic model, whose performance can be verified by comparisons with the test results, is capable of giving the kind of rational design criteria which will not be at the mercy of new test data at any time.

A much better approach than the test-based criterion would have been to use multiple column curves, as proposed in the early 1970s [6, 7, 9, 19]. The curves developed for the United States are shown in Fig. 3, labeled as Curves 1 through 3 and 1P through 3P [7, 9], where the first set used an out-of-straightness of 1/1000 of the member length and the second set was based on the probabilistically-based out-of-straightness of 1/1500 (actually, 1/1470, which was the mathematical mean on the basis of the shape of the probability density function for the out-of-straightness [7]). Curve 2P is the single curve that has been used by AISC for its limit states code (LRFD) since 1986, specifically because of the maximum strength basis and the use of the mean crookedness, rounded off to 1/1500.

The Canadian steel design standard CSA S16-1 has used two curves for several years [5]. The current Eurocode 3 [4] has five curves, as originally proposed by Beer and Schulz [6]; these are shown in Fig. 4, where the current, single AISC column curve also is included for comparison purposes [3]. As should be expected, the curves are all fairly close, with the AISC very close to EC3 curves a and b for most of the slenderness ratio range. For very large slenderness ratios the EC3 a_0 curve and the AISC curve overlap, as they should, since elastic buckling governs for columns of this type. The yield stress of the steel does not play a role in this slenderness range.

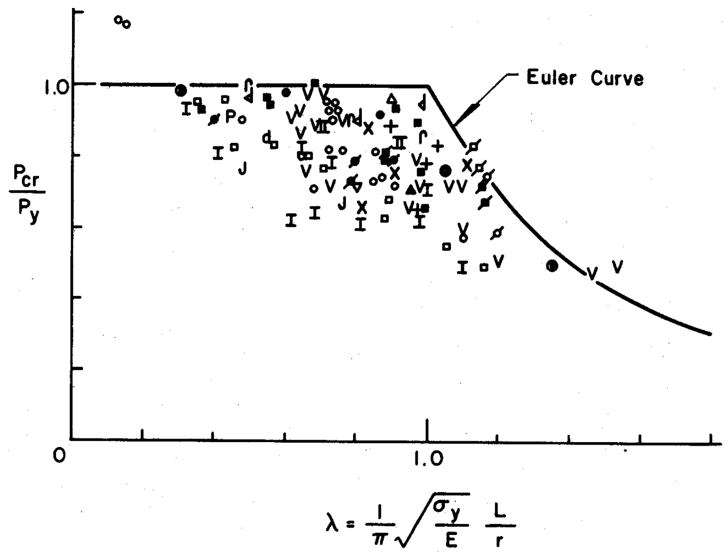

Figure 2: Approximately 100 Full-Scale Column Test Results for a Variety of Shapes, Steel Grades and Fabrication Methods

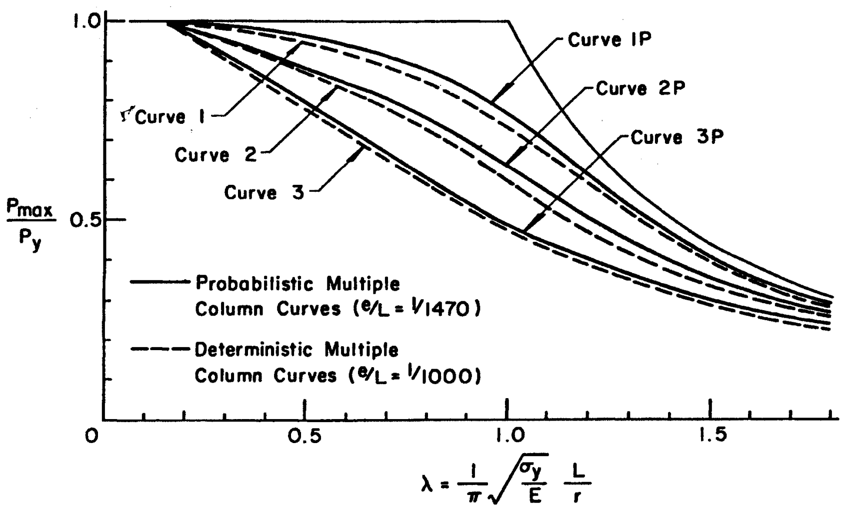

Figure 3: Proposed American Multiple Column Curves, Including the Current AISC Curve = Curve 2P

Models (iv) through (vii) reflect progressively more accurate, but also significantly more complex approaches to the evaluation of the stability of columns as parts of frameworks. In its original form, Model (iv) was developed as an idealized, elastic buckling solution for a three-story column, with beams framing into it with rigid connections [10, 20]. Using a slope-deflection analysis, the characteristic equation for the stability of the column as part of the subassemblage was obtained for the sway and non-sway cases. This led to the development of the well-known effective length (K-factor) alignment charts (= nomograms), which have been used extensively for years. At this time, however, due to advanced analysis techniques and much better computation software and facilities, elastic and inelastic second order procedures have made the use of the K-factor un-necessary, indeed undesirable. However, many engineers prefer the older method, since they are familiar with the approach and the solution is acceptable for many types of structures.

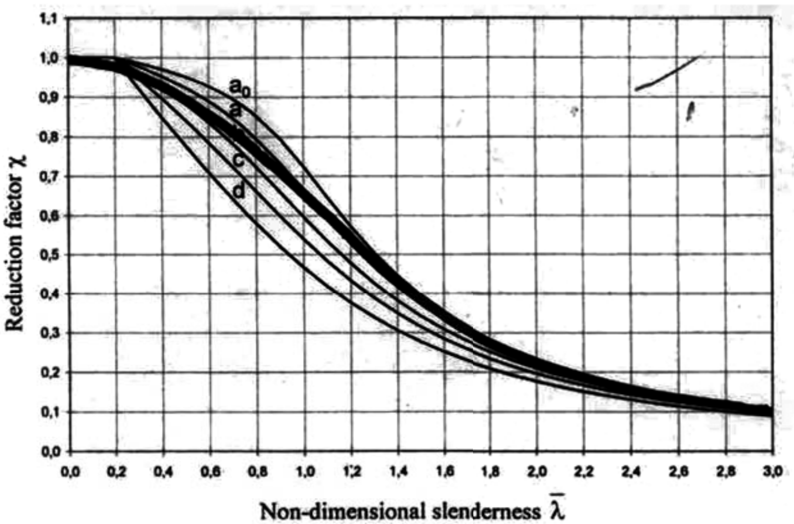

Figure 4: The Five Column Curves of Eurocode 3 and the AISC Column Curve

However, major assumptions were made for the effective length solutions, which is one of the reasons researchers and advanced designers have been questioning their use in practice. For example, Yura [21] modified the elastic treatment to take into account inelastic buckling; this led to the concept of the inelastic K-factor. Combined with the restraining effects of columns and the appropriate stiffness of beams and the far end support conditions, Johnston [22] formulated a much-improved solution for columns in frames, and Yura expanded on this approach. This was subsequently enhanced very significantly by Hellesland and Bjorhovde [23, 24].

Last, but by no means least, Yura [21] defined the concept of leaning columns, and provided a solution that considered their influence on the overall stability of the frame. This has become increasingly important as a result of refinements of structural analysis techniques. The advent of limit states design (LRFD) has also played a major role. Contemporary codes take these influences explicitly into account through the so-called Direct Analysis Method [3].

In the last several years, a number of studies have addressed the issue of using realistic connection restraint characteristics in the assessment of column stability. Some of this has been reflected in the criteria for the strength of individual columns, as indicated for Models (iv) through (vii). However, a major body of work has been devoted to incorporating such properties into the stability of columns in frame subassemblages. Numerous studies have offered improved understanding of the buckling mechanisms as well as how the end restraint concept may be implemented in practice. The advantage of this approach is that it ties directly into what has been common practice for years, with well-defined physical modifications of known and accepted methods.

Models (vi) and (vii) essentially expand the principles of Model (v), to incorporate the entire planar frame into the stability analysis. Beam-to-column connections are treated as rigid or pinned, or the actual moment-rotation response is used. The latter reflects the characteristics of semi-rigid (Type PR) connections, emphasizing the fact that realistic joints may exhibit elastic initial response, but the moment-rotation curve will sooner or later become non-linear. Having developed the mechanistic concepts through Models (ii) through (v), arriving at a complete frame solution basically requires only additional computational capacity. However, the word "only" can be misleading. This is especially true if PR connections are used, and/or if three-dimensional response is to be included.

For almost all structural types, and notably those with substantial gravity and/or lateral load from wind or seismic action, it is essential to account for second order effects. This applies whether the frame is braced or unbraced, although the latter tends to be much more affected. The original research work and the solution for these P-Delta effects were provided by P. F. Adams while he was a Ph.D. student at Lehigh University. Details of the historical development have been detailed by Galambos [11].

Significant advances have been made over the last few years towards practical analysis and design methods for PR frames. All major international design codes give provisions for such designs, but actual use continues to be limited. This is mostly because commercial PR analysis and design software is missing, but connection modeling is also a major issue. Data bases for certain types of connections have been developed [25, 26], but the real problem lies with the fact that it is nearly impossible to use actual test data in practical designs. The approach that appears to offer the most useful procedure is based on a classification system for the connections, focusing on the key characteristics of the moment-rotation behavior [4, 27].

Many designers still prefer to use elastic methods of analysis and design, and there are a number of software packages for such work. However, researchers and advanced designers now have transitioned to complete inelastic analysis. State-of-the-art standards have adopted such procedures [3, 4, 5], especially when seismic response has to be taken into account [28].

In a major development towards inelastic design, Appendix 1 of the new AISC Specification [3] contains a comprehensive section on this approach. Several studies have already examined the proposed procedure, and although the method is complex and likely will only be used by very advanced designers, the method offers a clear view of where design is headed [29, 30, 31, 32]. Most importantly, the 2010 AISC stability provisions represent a major improvement that will make for an all-inclusive approach [3].

The preceding has given an overview of the history and status of the stability treatment of individual columns and of columns as parts of frameworks. The contemporary approach is the evaluation of the complete structure, but by necessity most of the strength criteria have been and will continue to be based on individual members. Therefore, the better the data base for structural elements, the better will be the design procedure and the easier will be the eventual transition to the full structure.

3 MECHANISTIC VERSUS EMPIRICAL MODELING

The preceding has demonstrated that much has been done to develop mechanistic column response models. Further, over the years a substantial number of column tests have been conducted, in some cases only to verify theoretical solutions. In other projects, however, series of tests have been carefully planned and executed, to cover all column strength variables, with the specific aim of arriving at experimentally based design criteria. Well-known historical examples of the latter are the column tests of Tetmajer and von Kármán [1]. Among the more recent tests are the series of the European Convention for Constructional Steelwork (ECCS) [33, 34]. These were carried out to verify and supplement the theoretical studies of Beer and Schulz [6] in arriving at the ECCS, now the Eurocode 3 column curves, as shown in Fig. 3 [4].

Both of the fundamental approaches that have been described in the preceding have been used to develop column design curves and other criteria. In addition, a number of studies have aimed at devising mathematically simple equations and curves that approximate those with an engineering theory background but otherwise offer no physical interpretation. The work of Rondal and Maquoi [35] and Rotter [36] are the most comprehensive of these efforts. Loov [37] provided an elegant simplification into a single equation for the two Canadian column curves which were SSRC Curves 1 and 2 [5, 7, 9, 19].

The most significant benefit of using mechanics-based column curves is their predictability and their ability to replicate test results. In addition, a model that offers a clear physical illustration and solid grounding in mechanics has a definite advantage. In other words, given the material properties and other relevant member data, such column curves can be used to predict the outcome of physical tests. The more column strength parameters that are incorporated into the model, the closer will be the magnitudes of the theoretical and physical strengths. Thus, column models (ii) through (iv) are generally capable of producing results that are within 5 percent of the test results [2, 7, 9, 11, 36].

Many such column curves have been developed over the years. For example, the formula of the Column Research Council (CRC), the CRC Curve [22], was used for many years as the basis for the AISC allowable stress design specification. It reflected the tangent modulus solution for a perfectly straight column with residual stress. The two sets of the three SSRC Curves (Curves 1 to 3 and Curves 1P to 3P), as shown in Fig. 3, were maximum strength solutions, taking into account residual stress as well as out-of-straightness [7]. SSRC 1P to 3P also incorporated the random nature of the column strength parameters; these were a perfect fit for a limit states design code. This was accomplished for the US in the 1970s and early 1980s [2, 7, 9, 11, 19, 30] and for Europe in the 1970s to 1990s [4, 6, 33, 34]. The Canadian steel design standard adopted SSRC Curve 2 in 1974; subsequently SSRC Curve 1 was added, such that this standard has used two column curves since 1989 [5]. SSRC Curve 2P has been the column curve for the AISC Specification since 1986 [3]. Finally, Fig. 5 shows the column curve selection table that was developed for use with the SSRC multiple column curves [7, 9]; this would have been part of the code if AISC had chosen to use several curves in lieu of a single one. For reasons of simplicity, AISC chose to use the one curve.

Fabrication Details		Axis	Specified Minimum Yield Stress of Steel (ksi)				
			≤ 36	37 to 49	50 to 59	60 to 89	≥ 90**
Hot-rolled W-shapes	Light and medium W-shapes	Major	2	2	1	1	1
		Minor	2	2	2	1	1
	Heavy W-shapes (flange over 2 in.)	Major	3	2	2	2	2
		Minor	3	3	2	2	2
Welded Built-up H-shapes	Flame-cut plates	Major	2	2	2	1	1
		Minor	2	2	2	2	1
	Universal mill plates	Major	3	3	2	2	2
		Minor	3	3	3	2	2
Welded Box Shapes	Flame-cut and universal mill plates	Major	2	2	2	1	1
		Minor	2	2	2	1	1
Square and Rectangular Tubes	Cold-formed	Major	N/A	2	2	2	2
		Minor	N/A	2	2	2	2
	Hot-formed and cold-formed heat-treated	Major	1	1	1	1	1
		Minor	1	1	1	1	1
Circular Tubes	Cold-formed	N/A	2	2	2	2	2
	Hot-formed	N/A	1	1	1	1	1
All Stress-relieved Shapes		Major and Minor	1	1	1	1	1

Figure 5: The SSRC Column Curve Selection Table [2, 7, 9, 38]

The major drawback of the test-based approach is that without a mechanistic base, any new test results would have to be incorporated into the data base for the design curve. This would require a full analysis of the expanded data base, to determine whether resistance factor changes would be needed. This might also lead to changes in the curve itself. Most importantly, such changes would have to be made any time new steel grades or products were developed by the industry. Apart from the impractical aspect of having a basic design curve change every so often, and then only due to certain test data, the real problem is rooted in the fact that test results are highly subject to interpretation. In other words, due to the many factors that influence the column strength, the outcome of an experiment is not always clearly understood, much less properly interpreted. Some of the parameters are not easily quantified, there is a certain amount of interaction between others, and yet others may not have been recognized as playing a role.

On this background, it is clear that a proper and reliable column strength criterion is one that gives excellent correlation between tests and theory, and that takes all major strength parameters explicitly into account. Such can only be achieved with an accurate and robust mechanistic model.

4 SOME COLUMN STRENGTH CONSIDERATIONS

Due to the complexity of the inelastic flexural buckling problem, all of the primary strength parameters were not included in the analyses until adequate computational tools were available. Previously, closed-form solutions were always sought. As a result, it was feasible to take into account the presence of residual stresses, but initial crookedness could only be considered through stress-based analyses. For example, the secant formula did this by an elastic flexural analysis which limited the maximum stress in the cross section to the yield level [1]. Similar attempts were made to include restraint effects.

The following gives a condensed review of the evolution of column modeling through the stages of parameter incorporation.

4.1 Influence of residual stress

Very well known and documented, through extensive tests and analyses, residual stress is one of the main column strength parameters. Residual stress data are available for a variety of shapes, plates, sizes, grades of steel and manufacturing practices, including the influence of welding, flame cutting and cold straightening [2, 11, 19, 38]. Some limitations still exist for the very heavy shapes, although additional measurement and computation results are becoming available [9].

Column formulas around the world have incorporated the influence of residual stress since the 1950s. The initial applications were all based on tangent modulus formulations [1, 22], and some were still using such criteria until a few years ago, like cold-formed members.

4.2 Out-of-straightness effects

Crookedness was early recognized as a major factor in the column strength equation, but the difficulty of obtaining closed form solutions prevented it from being adopted into design codes. The most common approach was to use a variable factor of safety to account for the effects of the crookedness. This solution was chosen by a number of design standards. For example, the old AISC allowable stress design criteria used a factor of safety that varied from 1.67 to 1.92, covering the combined effects of out-of-straightness, load eccentricity and so on.

Current limit states design criteria [3, 4, 5] cover the crookedness effects explicitly by using the maximum strength as the basic criterion. This was made feasible when computer technology allowed for numerical solutions of the inelastic load-deflection column problem. Some standards have focused on the maximum value of the out-of-straightness; depending on the type of cross section and manufacturing method, this is commonly around 1/1000 of the member length. The Canadian [5] and the European [4] codes have chosen this approach. The LRFD criteria of AISC use the mean value of 1/1500 of the length, on the premise that in the first order, second moment approach to limit states design, all parameters use the mean as the key central tendency. The value of 1/1500 is based on probabilistic evaluations of the out-of-straightness effects [7].

4.3 Straightening effects

Much of the residual stress data that has been mentioned represent those of hot-rolled and welded built-up shapes, mostly of the wide-flange or H- or I-type. However, it has been common practice for years in steel mills to straighten the shapes to meet tolerance requirements. The straightening is done at room (ambient) temperature, and is referred to as cold-straightening. The process is either continuous (rotary straightening) or point-applied (gag straightening). Small to medium size shapes are usually rotary straightened; heavier shapes are gag-straightened. But it is known that rotary cold-straightening has a significant influence on the magnitudes and distribution of the residual stresses in a shape; this is not so for gag-straightened shapes [2].

Tests results are limited, as are theoretical evaluations, but it is known that rotary straightening will lower the peak values of the residual stresses within a shape. Most importantly, the compressive residual stress maxima will be reduced. As a result, the axial compressive strength of a rotary straightened shape will be higher than that of an unstraightened shape. This effect is more pronounced for small to medium size shapes. Gag straightening does not produce such an effect, since the residual stress distribution is only affected in a areas close to the gag load application point [39, 40].

Although current column models are capable of incorporating cold-straightening effects, the lack of a large body of cohesive, carefully developed residual stress data has prevented design standards from taking advantage of the benefit. Further research is clearly needed.

4.4 Some other considerations

Overall frame stability and methods of taking actual connection characteristics into account continue to be studied today. This is reflected in part by the six international steel connection workshops that have been arranged since 1987. The first such workshop addressed the great variety of connections and how their properties could be built into frame analysis and stability considerations [15]. The use and design of frames with semi-rigid (PR) connections continue to be the subject of several international research projects, and a large number of papers continue to address the subject [for example: 27, 31, 32, 41, 42, 43].

5 STRUCTURAL SAFETY, STRENGTH AND ECONOMY

In engineering terms, safety is the issue of overriding concern, but strength is generally much easier to address. Design criteria have therefore tended to focus on achieving adequate safety by setting the strength requirements sufficiently high. Albeit an admirable goal, the lack of attention to the variability of the strength parameters have typically led to highly variable factors of safety in allowable stress design. The old AISC ASD criteria offer a good example, where the theoretical column safety for a range of member types and sizes would be from 1.67 to 1.92, but the actual safety margins varied from 1.4 to 2.5 [8].

Economy, on the other hand, is an elusive concept. It is possible to estimate structural costs by considering the amount of steel and other materials that are used. However, this does not address fabrication and construction costs, for instance, and these can vary substantially from one area to another. For the designer, therefore, the key issue is to concentrate on providing as accurate calculations as possible, and this is helped by accurate design requirements. In other words, the more of the strength and stiffness parameters that are taken into account in the code, the better will be the resulting structure. On the other hand, selecting member and connection sizes and details based strictly on minute numerical differences is likely to prove costly. A balance has to be struck between the numerical and the practical demands, to ensure improved economy of construction.

In the evolution of the column stability criteria, it is a fact that more and more of the important strength parameters have been incorporated into the code equations. The column curves of today reflect residual stress and out-of-straightness through the maximum strength concept, and the limit state format allows for the random variability of these and other factors of influence. Safety is therefore achieved in the best possible fashion. Future refinements of the column criteria therefore should only be regarded in conjunction with the improvements that are being contemplated for the overall design of the frame. That is clearly the direction that must be pursued for the future.

6 SUMMARY

A comprehensive review has been provided of the development of the column stability criteria as they have appeared in research studies and design codes for the past 50 years. It has been demonstrated that through improved analytical and experimental techniques, all of the major strength parameters are now explicitly accounted for in the primary design codes of the world. The advantages of mechanics-based formulations of the design criteria are explained in detail, offering the rationale for code development that is not subject to short term practice and market variations.

REFERENCES

[1] Johnston, B.G., "Column Buckling Theory: Historical Highlights", *Journal of the Structural Division*, ASCE, **107**(ST4), 649-670, 1981.

[2] Ziemian, R.D. (ed.), *Guide to Stability Design Criteria for Metal Structures*, 6th Ed., John Wiley & Sons, New York, NY, 2010.

[3] American Institute of Steel Construction (AISC), *Specification for Structural Steel Buildings,* ANSI/AISC Standard No. 360-10, AISC, Chicago, IL, 2010.

[4] Comité Européen de Normalisation (CEN), *Eurocode 3 – Design of Steel Structures,* Standard No. 1993-1, CEN, Brussels, Belgium, 2005.

[5] Canadian Standards Association (CSA), *Steel Structures for Buildings,* CSA Standard No. S16-01, CSA, Mississauga, Ontario, Canada, 2009.

[6] Beer, H. and Schulz, G., "Theoretical Bases for the European Buckling Curves", *Construction Métallique,* **7**(3), 37-55, 1970 (in French).

[7] Bjorhovde, R.,"Deterministic and Probabilistic Approaches to the Strength of Steel Columns", Ph.D. Dissertation, Lehigh University, Bethlehem, PA, 1972.

[8] Bjorhovde, R., "The Safety of Steel Columns", *Journal of the Structural Division,* ASCE, **104**(ST3), 463-477, 1978.

[9] Bjorhovde, R., "Columns: From Theory to Practice", *Engineering Journal,* AISC, **25**(1), 21-34, 1988.

[10] Galambos, T.V. and Surovek, A.E., *Structural Stability of Steel – Concepts and Applications for Structural Engineers,* John Wiley & Sons, New York, NY, 2008.

[11] Galambos, T.V. (ed.), *Guide to Stability Design Criteria for Metal Structures,* 4th Ed., John Wiley & Sons, New York, NY, 1988.

[12] Jones, S.W., Kirby, P.A. and Nethercot, D.A., "Effect of Semi-Rigid Connections on Steel Column Strength", *Journal of Constructional Steel Research,* **1**(1), 35-46, 1980.

[13] Sugimoto, H. and Chen, W.-F., "Small End Restraint Effects on Strength of H-Columns", *Journal of the Structural Division,* ASCE, **108**(ST3), 661-681, 1982.

[14] Bjorhovde, R., "Effect of End Restraint on Column Strength – Practical Applications", *Engineering Journal,* AISC, **20**(1), 1-13, 1984.

[15] Bjorhovde, R., Brozzetti, J. and Colson, A. (eds.), *Connections in Steel Structures: Behaviour, Strength and Design,* Elsevier Applied Science, London, England, 1988.

[16] Bjorhovde, R., Colson, A., Haaijer, G. and Stark, J.W.B. (eds.), *Connections in Steel Structures II,* AISC, Chicago, IL, 1992.

[17] Fukumoto, Y. and Itoh, Y., "Evaluation of Multiple Column Curves from the Experimental Data Base Approach", *Journal of Constructional Steel Research,* **3**(3), 2-19, 1983.

[18] Hall, D.H., "Proposed Steel Column Design Criteria", *Journal of Structural Engineering*, ASCE, **109**(9), 2086-2096, 1983.

[19] Johnston, B.G. (ed.), *Guide to Stability Design Criteria for Metal Structures*, 3rd Ed., John Wiley & Sons, New York, NY, 1976.

[20] Julian, O.G. and Lawrence, L.S., "Notes on S & L Nomograms for Determination of Effective Lengths", Unpublished Report, Jackson & Moreland Engineers, Boston, MA, 1959.

[21] Yura, J.A., "The Effective Length of Columns in Unbraced Frames", *Engineering Journal,* AISC, **8**(2), 37-42, 1971.

[22] Johnston, B.G. (ed.), *Guide to Stability Design Criteria for Metal Structures*, 2nd Ed., John Wiley & Sons, New York, NY, 1966.

[23] Hellesland, J. and Bjorhovde, R., "Restraint Demand Factors and Effective Lengths of Braced Columns", *Journal of Structural Engineering*, ASCE, **122**(10), 1216-1224, 1996.

[24] Hellesland, J. and Bjorhovde, R., "Improved Frame Stability Analysis with Effective Lengths", *Journal of Structural Engineering*, ASCE, **122**(11), 1275-1283, 1996.

[25] Nethercot, D.A., "Steel Beam-to-Column Connections – A Review of Test Data and Their Applicability to the Evaluation of the Joint Behaviour of the Performance of Steel Frames", CIRIA, London, England, 1985.

[26] Kishi, N. and Chen, W.-F., "Data Base of Steel Beam-to-Column Connections", Vols. I and II, Structural Engineering Report No. CE-STR-86-26, Purdue University, West Lafayette, IN, 1986.

[27] Bjorhovde, R., Colson, A. and Brozzetti, J., "Classification System for Beam-to-Column Connections", *Journal of Structural Engineering,* ASCE, **116**(11), 3063-3080, 1990.

[28] American Institute of Steel Construction (AISC), *Seismic Provisions for Structural Steel Buildings*, ANSI/AISC Standard No. 341-10, AISC, Chicago, IL.

[29] White, D.W. and Hajjar, J.F., "Buckling Models and Stability Design of Steel Frames: A Unified Approach", *Journal of Constructional Steel Research*, **42**(3), 171-207, 1997.

[30] White, D.W. and Hajjar, J.F., "Design of Steel Frames without Consideration of Effective Lengths", *Engineering Structures,* **19**(10), 797-810, 1997.

[31] Surovek, A.E., White, D.W. and Leon, R.T., "Direct Analysis for Design Evaluation and Design of Partially Restrained Steel Framing Systems", *Journal of Structural Engineering*, ASCE, **131**(9), 1376-1389, 2005.

[32] White, D.W. and Goverdhan, A.V., "Design of PR Frames Using the AISC Direct Analysis Method", in [41], *Bjorhovde et al. (2008)*, 255-264, 2008.

[33] Sfintesco, D., "Experimental Basis for the European Column Curves", *Construction Métallique*, **7**(3), 1970 (in French).

[34] Jacquet, J., "Column Buckling Tests and Their Statistical Evaluation", *Construction Métallique*, **7**(3), 1970 (in French).

[35] Rondal, J. and Maquoi, R., "Single Equation for SSRC Column Strength Curves", *Journal of the Structural Division,* ASCE, **105**(ST1), 247-250, 1979.

[36] Rotter, J.M., "Multiple Column Curves by Modifying Factors", *Journal of the Structural Division,* ASCE, **108**(ST7), 1665-1669, 1982.

[37] Loov, R., "A Simple Equation for Axially Loaded Steel Column Design Curves", *Canadian Journal of Civil Engineering,* **23**(1), 272-276, 1996.

[38] Galambos, T.V. (ed.), *Guide to Stability Design Criteria for Metal Structures*, 5th Ed., John Wiley & Sons, New York, NY, 1998.

[39] Brozzetti, J., Alpsten, G.A. and Tall, L., "Residual Stresses in a Heavy Rolled Shape 14WF730", Fritz Engineering Laboratory Report No. 337.1, Lehigh University, Bethlehem, PA, 1970.

[40] Aschendorff, K.K., Bernard, A., Bucak, Ö., Mang, F. and Plumier, A., "Buckling Tests on Heavy Rolled I-Shapes in St 37 and St 52, and in St E 460 with Standard Sizes", *Der Bauingenieur*, **58**, 261-268, 1983 (in German).

[41] Bjorhovde, R., Bijlaard, F.S.K. and Geschwindner, L.F. (eds.), *Connections in Steel Structures VI*, AISC, Chicago, IL, 2008.

[42] Deierlein, G.G., "An Inelastic Analysis and Design System for Steel Frames with Partially Restrained Connections", in [16], *Bjorhovde et al. (1992),* 408-417, 1992.

[43] Christopher, J.E. and Bjorhovde, R., "Semi-Rigid Frame Design Methods for Practicing Engineers", *Engineering Journal*, AISC, **36**(1), 12-28, 1999.