DIRECT STRENGTH DESIGN OF COLD-FORMED SECTIONS FOR SHEAR AND COMBINED ACTIONS

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Abstract. *In order to extend the Direct Strength Method (DSM) of design of cold-formed sections to shear, and combined bending and shear, a research program has been performed recently at the University of Sydney. This includes evaluation and calibration of DSM design rules for shear and combined actions when applied to an extensive series of purlins tested at the University of Sydney, as well as shear only and combined bending and shear tests on channel sections. The paper summarises this research, as well as making proposals for shear, and combined actions. Two features researched are the effect of full section shear buckling (as opposed to web only shear buckling), and tension field action. Full section buckling is a feature of the DSM but requires software that can evaluate full sections for shear. Methods for doing this are summarised in the paper.*

1 INTRODUCTION

In both the Australian Standard and American Specification for the Design of Cold-Formed Steel Structures, which include the newly developed Direct Strength Method (DSM) of design, the method presented [Chapter 7 of AS/NZS 4600:2005 (Standards Australia [1]), Appendix 1 of the North American Specification (AISI [2])] is limited to pure compression and pure bending. The Direct Strength Method (Schafer and Peköz [3]) was formally adopted in the North American Design Specification in 2004 and in AS/NZS 4600:2005 as an alternative to the traditional Effective Width Method (EWM) in 2005. It uses elastic buckling solutions for the entire member cross section to give the direct strength rather than for elements in isolation.

The first advantage of the DSM is that it allows direct computation of the capacity of cold-formed thin walled members of complex section shape (eg. with intermediate stiffeners). Secondly, the interaction between local and overall modes, distortional and overall modes is easily taken into account. The DSM uses numerical solutions for elastic buckling and requires computer software such as THIN-WALL (CASE [4]) or CUFSM (Schafer and Ádány [5]) to evaluate elastic buckling stresses. There is no need to calculate cumbersome effective sections especially with intermediate stiffeners.

The development of the DSM for columns and beams, including the reliability of the method is well researched. In the review of the DSM of cold-formed steel member design, Schafer [6] noted that no formal provisions for shear currently exist for the DSM. However, as recommended in the AISI Direct Strength Design Guide [7], the existing provisions in the North American Design Specification and AS/NZS 4600:2005 could be suitably modified into the DSM format.

To investigate this proposition, vacuum rig tests on continuous lapped cold-formed purlins at the University of Sydney over a 10 year period, have been used to calibrate DSM design proposals for shear and combined bending and shear (Pham and Hancock [8]). The conclusions from this calibration are that the existing bending and shear equations in AS/NZS 4600:2005 in DSM format will provide reliable designs irrespective of whether the limiting design moment in the interaction equation is based on the lesser of the local buckling and distortional buckling moments (called Proposal 1) or the local buckling moment alone (called Proposal 2). To further investigate these proposals, additional tests on C-sections in predominantly shear, combined bending and shear, and bending alone have been performed at the University of Sydney (Pham and Hancock [9], [10])

The main objectives of this paper are:

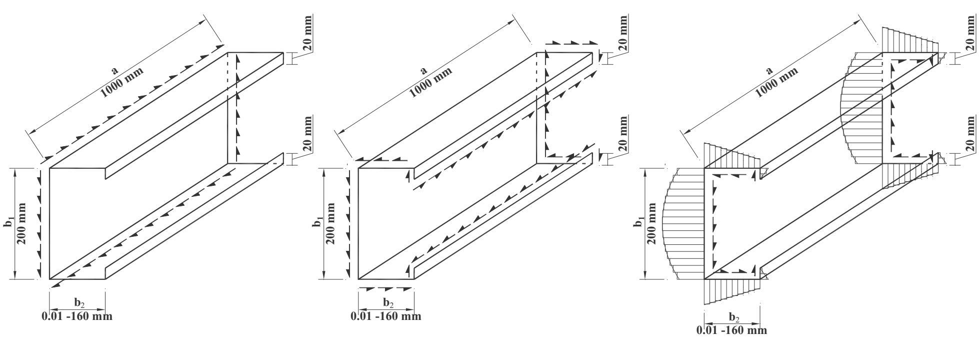
- To summarise analyses of full sections in shear, with and without intermediate stiffeners, with a view to providing elastic shear buckling loads V_{cr} which can be used as input to the Direct Strength Method of design of complete sections in shear.
- To summarise tests on high strength cold-formed lipped C-Sections (with and without intermediate stiffeners) in shear and combined bending and shear.
- To summarise proposals for extension of the Direct Strength Method to shear and combined bending and shear. The proposals are made both with and without Tension Field Action (TFA) and are compared with the test results on the lipped C-sections.

2 SHEAR BUCKLING OF FULL SECTIONS

The Spline Finite Strip Method (SFSM) is a development of the semi-analytical finite strip method originally derived by Cheung [11]. It uses spline functions in the longitudinal direction in place of the single half sine wave over the length of the section, and has been proven to be an efficient tool for analyzing structures with constant geometric properties in a particular direction, generally the longitudinal one. The advantage of the spline finite strip analysis is that it allows more complex types of loading and boundary conditions other than simple supports to be easily investigated and buckling in shear is also easily accounted for. Initially, the spline finite strip method was fully developed for the linear elastic structural analysis of folded plate structures by Fan and Cheung [12].

The SFSM was then extended to buckling and nonlinear analyses of flat plates and folded-plate structures by Lau and Hancock [13] and Kwon and Hancock [14, 15]. The spline finite strip method involves subdividing a thin-walled member into longitudinal strips where each strip is assumed to be free to deform both in its plane (membrane displacements) and out of its plane (flexural displacements). The functions used in the longitudinal direction are normally B3 splines. The ends of the section under study are normally free to deform longitudinally but are prevented from deforming in a cross-sectional plane.

For the shear buckling analyses described by Pham and Hancock [16], three different methods, which represent different ways of incorporating the shear stresses in the thin-walled section, are used in this analysis as shown in Fig. 1.



(a) <u>Case A & Case B</u> *(b) <u>Case C</u>* *(c) <u>Case D</u>*

Figure 1. Shear Flow Distributions in Lipped Channels

These include pure shear in the web only (Cases A and B), pure shear in the web and the flanges (Case C) , and a shear distribution similar to that which occurs in practice allowing for section shear flow (Case D). For combined bending and shear, each method of the incorporation of the shear stress is combined with pure bending to produce the interaction relations. The stress states studied are not in equilibrium as shear can only be generated in a section by moment gradient. However, the studies allow the shear buckling, and combined bending and shear buckling to be isolated and investigated. Buckling modes from the SFSM described in Pham and Hancock [17] are shown in Fig. 2 for combined bending and shear where the shear is Case D in Fig. 1. They demonstrate a range of buckling modes including section twisting for sections with narrow flanges, flange distortional buckling and web shear buckling depending upon the width of the flanges.

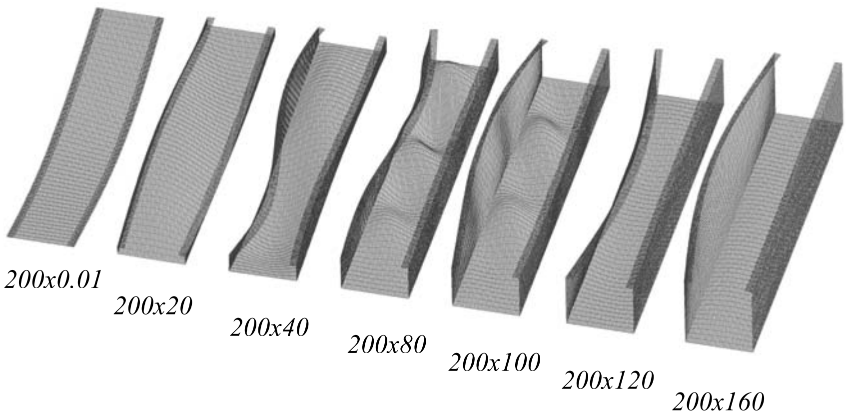


Figure 2. Buckling Mode Shapes of Lipped Channel Section under Combined Bending and Shear for *Case D*

The geometry of the lipped channel with an intermediate stiffener studied is shown in Fig. 3. The purpose of this study (Pham and Hancock [18]) was to determine the effect of an intermediate stiffener in the web on the shear buckling load of the section.

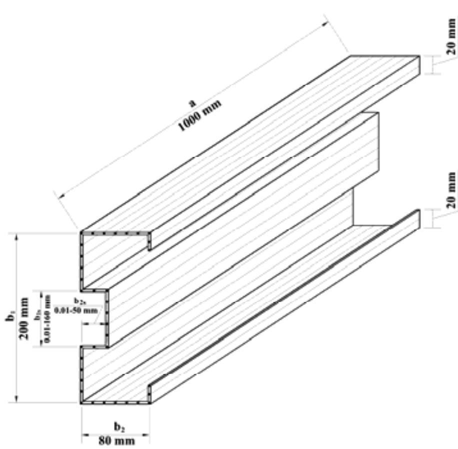


Figure 3. Lipped Channel Geometry with an Intermediate Stiffener

The channel section consists of a web of width 200 mm (b_1), a flange of width 80 mm (b_2), a lip size of 20 mm, all with thickness of 2 mm. The stiffener is positioned at the longitudinal center line of the web and the main variables are the dimensions of the stiffener. The depth of stiffener (b_{1s}) increases from 0.01 mm to 160 mm whereas the width of the stiffener (b_{2s}) varies from 0.05 mm to 50 mm. The length of the member studied is 1000 mm. The aspect ratio of the web rectangular plate is therefore $a/b_1 = 5$. The buckling modes for the section in shear according to Case D in Fig. 1 with an intermediate stiffener of depth 50mm are shown in Fig. 4 as the height of the stiffener increases. Modes ranging from flange distortional buckling to local buckling in shear in the intermediate stiffener can be observed.

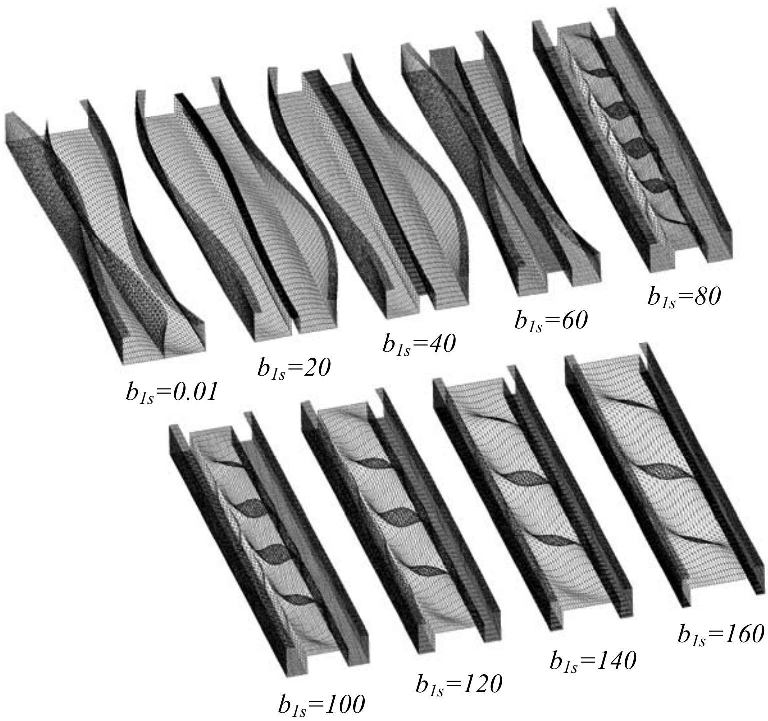


Figure 4. Buckling Mode Shapes of Lipped Channel Sections with an Intermediate Stiffener

3 EXPERIMENTS ON PLAIN C- LIPPED SECTIONS AND SUPACEE® SECTIONS IN SHEAR, AND COMBINED BENDING AND SHEAR

3.1 Experimental Rig and Tests Specimens

LaBoube and Yu [19] conducted a series of tests including a total of forty three beam specimens subjected primarily to shear stress. They found that, for shear, the exact critical buckling load for a beam webs is difficult to determine experimentally and the post-buckling strength of web elements due to tension field action increases as the h/t ratio of the web, the aspect ratio of the web, and the yielding point of the material increase. Moreover, the arrangement of connections has a significant effect on the ultimate shear capacity of the unreinforced webs. The current Effective Width Method for shear was calibrated against these tests.

Another testing program was also performed at the University of Missouri-Rolla for the structural strength of beam webs subjected to combined bending and shear by LaBoube and Yu [20]. The results of twenty five beam tests indicated that the circular formula, originally developed for a separate individual sheet, would be conservative for beam webs with adequate transverse stiffeners, for which a diagonal tension field action can be developed. Based on these test results, the linear interaction equation was developed for beams webs with transverse stiffeners for combined bending and shear in the current Effective Width Method.

This paper summarises two series of tests on both high strength cold formed steel plain C- lipped sections and SupaCee® sections (Lysaght [21]) for the extension to the Direct Strength Method for shear, and combined bending and shear. The first experimental program comprised a total of sixty tests which included three test series conducted in the J. W. Roderick Laboratory for Materials and Structures at the University of Sydney. All tests were performed in the 2000 kN capacity DARTEC testing machine, using a servo-controlled hydraulic ram. Two different commercially available plain C- lipped channel sections of 150 mm and 200 mm depths as shown in Fig. 5 were chosen with three different thicknesses of 1.5 mm, 1.9 mm and 2.4 mm. The first series (V) is predominantly shear; and the second series (MV) is combined bending and shear. These series each consisted of twenty four tests and used the same test rig configuration. The third series is bending only (M) which used the common four point loading configuration. A total of twelve tests of this series were conducted. Although the tests described in LaBoube and Yu [20] contained straps at the loading points as described later, tests both with and without straps are included in the test program.

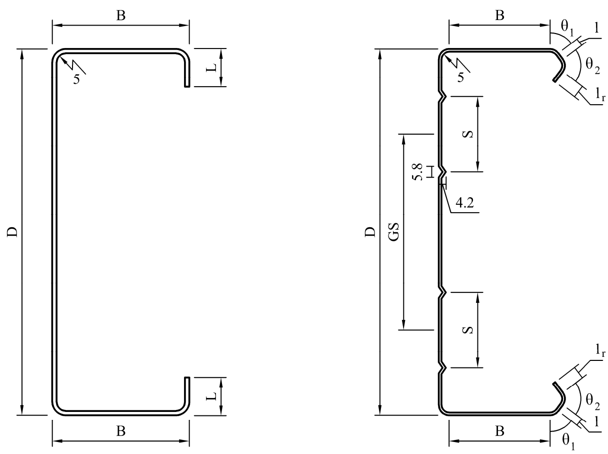


Figure 5. Dimensions of Plain and SupaCee® Channel Sections

The second series of test specimens was also performed at the University of Sydney including a total of thirty six tests of commercially available SupaCee® sections as shown in Fig. 5. Two different depths of 150 mm and 200 mm were chosen with three different thicknesses of 1.2 mm, 1.5 mm and 2.4 mm. Three test series, which were conducted identically to the first test program, also consisted of predominantly shear test (V), combined bending and shear test (MV) and bending only test (M). These test series each included twelve tests each and used the same test rig configurations with the first test program.

The basic design of the test rig was developed by LaBoube and Yu [19, 20]. A diagram of the test set-up is shown in Fig. 6 for both the V and MV series. The only difference between the V and MV tests is the ratio of shear span to depth where the shear span is the distance between the lines of bolts at the loading and support points. With the V series, the ratio of span to depth is 1:1 whereas that of MV series is 2:1.

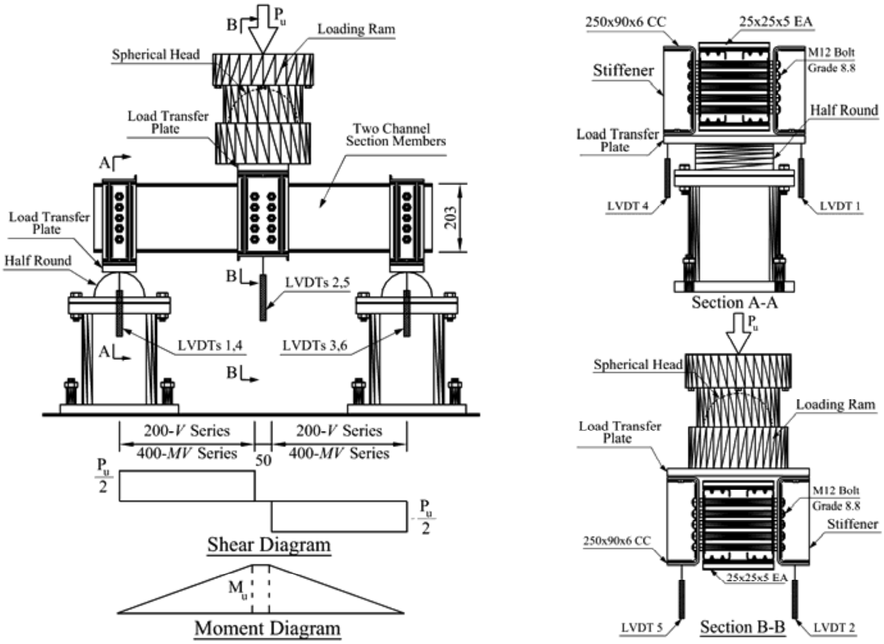


Figure 6. *V* and *MV* Test Series Configuration (Dimensions for 200 mm Deep Section)

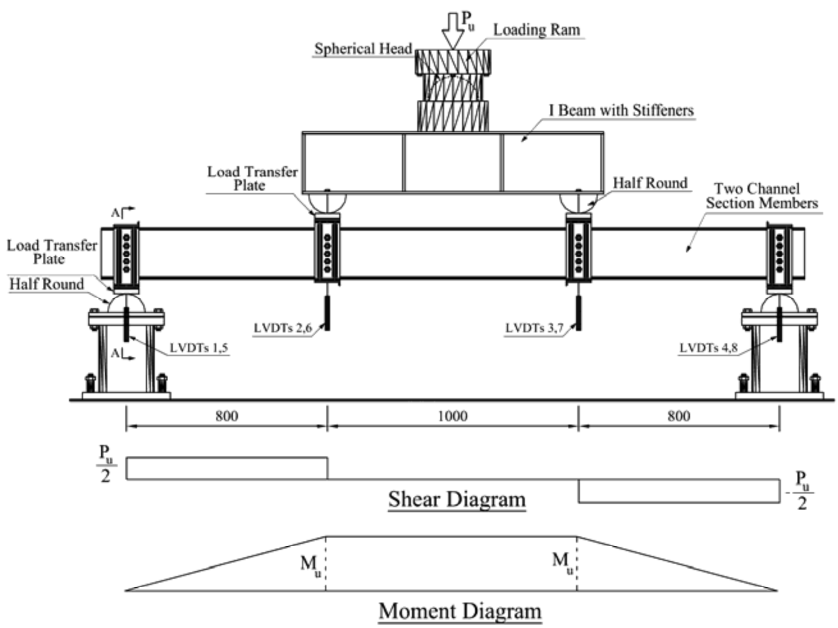


Figure 7. *M* Test Series Configuration (Dimensions for 200 mm Deep Section)

The detailed test configuration of the bending only series is shown in Fig. 7. The four point bending arrangement provided a central region of uniform bending moment and zero shear force. At the two supports, the rig assembly is exactly the same as that of the predominantly shear test set-up. The difference is at the loading points which have a similar configuration to the support points. The channel section members were loaded symmetrically at two points via a centrally loaded spreader I beam with stiffeners. The distance between the two half rounds bolted to the I beam at the loading points was 1000 mm. These two half rounds bore upon two 20 mm thick load transfer plates. The half round ensured that the applied loads were vertical. The distance between the support and the adjacent loading point was 800 mm.

3.2 Plain Lipped C-Section Test Results

The full set of test results for the plain lipped C-Sections is given in the research report by Pham and Hancock [9]. The tests results are compared with existing design methods in AS/NZS 4600:2005 and with the tension field action included using the rules of AS 4100:1998 (Standards Australia, [22]). All slender section specimens in the *V* and *MV* Series were found to develop tension field action as the connection bolts at the loading and support points extended over the full depth of the section whether 150 mm (4 bolts) or 200 mm (5 bolts). The inclusion of straps attached to the flanges to prevent distortion at the loading and support points further enhanced the tension field action. They are compared with DSM design proposals in Section 4 following

3.3 SupaCee® Section Test Results

The full set of test results for the SupaCee® sections is given in the research report by Pham and Hancock [10]. The tests results are compared with existing design methods in AS/NZS 4600:2005 and with the tension field action included using the rules of AS 4100:1998 (Standards Australia, [22]). As for the plain lipped C-Sections, all slender section specimens in the *V* and *MV* Series were found to develop tension field action. The inclusion of straps attached to the flanges to prevent distortion at the loading and support points further enhanced the tension field action.

4 DIRECT STRENGTH METHOD (DSM) OF DESIGN

4.1 Introduction

In the paper "Direct Strength Design of Cold-Formed Purlins" (Pham and Hancock [8]), test results from eight different test series, which were implemented at the University of Sydney by using the vacuum testing rig or more than 10 years, have been utilized to extend DSM to purlins systems. These tests have covered a wide range of parameters including: single, double and triple spans; inwards and outwards loading; zero, one and two rows of bridging per span; screw fastened and concealed fixed sheeting systems; and cleat and flange bolting of purlins to rafters. The paper (Pham and Hancock [8]) has outlined two current approaches to the design of purlin systems using an extension to the Direct Strength Method (DSM) in Section 7 of AS/NZS 4600:2005 which are referred to herein as the C_b-factor and FELB approaches. The results are compared with the Effective Width Method (EWM) as well as the ones from purlins test results.

The paper has also made two proposals for the bending and shear failure mode for use in the DSM. Proposal 1 uses the lesser of the local buckling and distortional buckling section strengths in the combined bending and shear interaction equation, and Proposal 2 uses only the local buckling section strength in the interaction equation. All methods were calibrated and produced acceptable safety indices including those test cases where combined bending and shear dominated such as the triple spans under uplift loading with one and two rows of bridging. It therefore appears that Proposal 2 is an acceptable method for safe design even though it ignores the distortional buckling strength.

The main purpose of this section is to further refine the proposals for DSM based on tests which concentrated on shear, and combined bending and shear.

4.2 DSM Design Rules for Flexure

The nominal section moment capacity at local buckling *(M_{sl})* is determined from Section 7.2.2.3 of AS/NZS 4600:2005 {Appendix 1, Section 1.2.2.2 of NAS [2]} as follows:

$$\text{For } \lambda_l \le 0.776 \ : \ M_{sl} = M_y \tag{1}$$

$$\text{For } \lambda_l > 0.776 \ : \ M_{sl} = \left[1 - 0.15\left(\frac{M_{ol}}{M_y}\right)^{0.4}\right]\left(\frac{M_{ol}}{M_y}\right)^{0.4} M_y \tag{2}$$

where λ_l is non-dimensional slenderness used to determine M_{sl} ; $\lambda_l = \sqrt{M_y / M_{ol}}$; $M_y = Z_f f_y$,

M_{ol} is elastic local buckling moment of the section; $M_{ol} = Z_f f_{ol}$,

Z_f is section modulus about a horizontal axis of the full section,

f_{ol} is elastic local buckling stress of the section in bending.

The nominal section moment capacity at distortional buckling *(M_{sd})* is determined from Section 7.2.2.4 of AS/NZS 4600:2005 {Appendix 1, Section 1.2.2.3 of NAS [2]} as follows:

$$\text{For } \lambda_d \le 0.673 \ : \ M_{sd} = M_y \tag{3}$$

$$\text{For } \lambda_d > 0.673 \ : \ M_{sd} = \left[1 - 0.22\left(\frac{M_{od}}{M_y}\right)^{0.5}\right]\left(\frac{M_{od}}{M_y}\right)^{0.5} M_y \tag{4}$$

where λ_d is non-dimensional slenderness used to determine M_{sd} ; $\lambda_l = \sqrt{M_y / M_{od}}$; $M_y = Z_f f_y$,

M_{od} is elastic distortional buckling moment of the section; $M_{od} = Z_f f_{od}$,

Z_f is section modulus about a horizontal axis of the full section,

f_{od} is elastic distortional buckling stress of the section in bending.

4.3 Proposed DSM Design Rules for Shear

The equations in Section 3.2.1 of the North American Specification [2] which are expressed in terms of a nominal shear stress F_v have been changed to DSM format by replacing stresses by loads as follows:

$$\text{For } \lambda_v \le 0.815 \ : \ V_v = V_y \tag{5}$$

$$\text{For } 0.815 < \lambda_v \le 1.231 \ : \ V_v = 0.815\sqrt{V_{cr} V_y} \tag{6}$$

$$\text{For } \lambda_v > 1.231 \ : \ V_v = V_{cr} \tag{7}$$

where $\lambda_v = \sqrt{V_y / V_{cr}}$, (8)

V_y is yield load of web $V_y = 0.6 A_w f_y$, (9)

V_{cr} is elastic shear buckling force of the web $V_{cr} = \dfrac{k_v \pi^2 E A_w}{12\left(1-\nu^2\right)\left(\dfrac{d_1}{t_w}\right)^2}$ (10)

d_1 is depth of the flat portion of the web measured along the plane of the web,

t_w is thickness of web, A_w is area of web $A_w = d_1 \times t_w$,

k_v is the shear buckling coefficient.

To account for the shear buckling of the whole section rather than simply the web, the shear buckling coefficient k_v can be back-calculated from the shear buckling load V_{cr} of the whole section as computed in Section 2 using Eq. 10. In this way, the DSM philosophy of section rather than element buckling can now be incorporated in the nominal shear capacity. The value of the shear buckling coefficients (k_v) for the whole plain C- lipped sections and SupaCee® sections are now based on the elastic shear buckling studies as described in Section 2. Both lipped plain C- and SupaCee® channel sections with the thickness of 2 mm as shown in Fig. 5 are investigated. Two ratios of shear span *(s)* to depth (d_1) of 1:1 and 2:1 respectively are also considered. A shear distribution similar to that which occurs in practice allowing for section shear flow as shown in Fig. 1 Case D is used for modeling sections in pure shear resulting from a shear force parallel with the web. All edges of the end cross-section are simply supported.

The computed values of the shear buckling coefficients k_v for the plain channels increase from the theoretical value of a simply supported rectangular plate in shear of 9.34 for a Span:Panel Depth of 1:1 to 9.926 and 10.006 for the 150mm and 200 mm depth sections respectively. For the SupaCee® sections, the corresponding values are 12.204 and 11.709 as a result of the longitudinal intermediate stiffeners in the web.

The DSM nominal shear capacity (V_v) including Tension Field Action is proposed based on the local buckling (M_{sl}) equation (4) where M_{sl}, M_{ol} and M_y are replaced by V_v, V_{cr} and V_y respectively as follows:

$$V_v = \left[1 - 0.15\left(\frac{V_{cr}}{V_y}\right)^{0.4}\right]\left(\frac{V_{cr}}{V_y}\right)^{0.4} V_y \tag{11}$$

Fig. 8 shows that all of the plain C- lipped and SupaCee® *V*-tests lie close to the proposed DSM nominal shear capacity with Tension Field Action given by Eq. 11. They lie well above the AISI in DSM format equations (see Eqs. 5 – 10) presumably because significant tension field action was developed. This may be a result of the bolts connecting the webs of the channels spanning the full depth of the section for both 150 mm and 200 mm tests. In tests where full tension field action is not developed, the results may lie below Eq. 11. Investigation of other test results such as those from LaBoube and Yu [19] will be required to confirm these design curves for all situations.

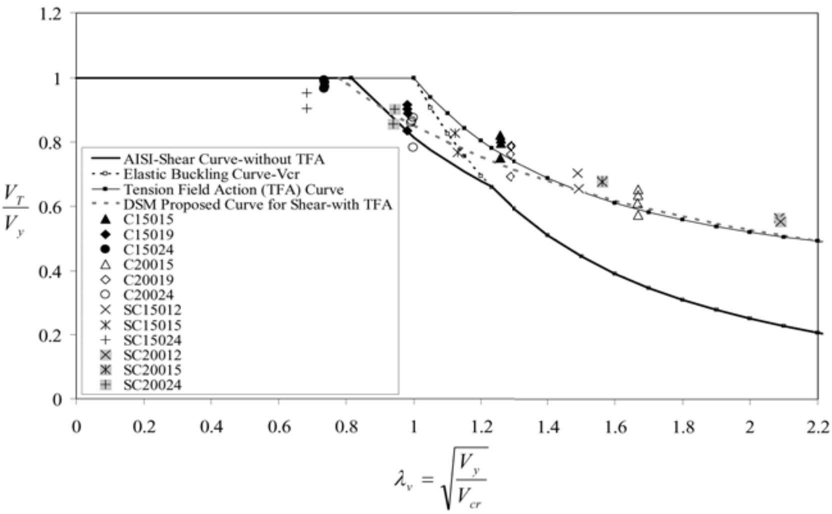


Figure 8. DSM Proposed Nominal Shear Curves and Shear Test Data

4.3 Proposed DSM Design Rules for Combined Bending and Shear

In limit states design standards, the interaction is expressed in terms of bending moment and shear force so that the interaction formula for combined bending and shear of a section with an unstiffened web is given in Clause 3.3.5 of AS/NZS 4600:2005 {Section C 3.3.2 of AISI [2]}:

$$\left(\frac{M^*}{M_s}\right)^2 + \left(\frac{V^*}{V_v}\right)^2 = 1 \tag{12}$$

where M^* is bending action,
M_s is the bending section capacity in pure bending,
V^* is the shear action, and
V_v is the shear capacity in pure shear.

The equation for combined bending and shear with stiffened webs is also given in Clause 3.3.5 of AS/NZS 4600:2005 {Section C 3.3.2 of AISI [2]}:

$$0.6\left(\frac{M^*}{M_s}\right) + \frac{V^*}{V_v} = 1.3 \tag{13}$$

The choice of M_s, the bending section capacity, needs further investigation. As discussed earlier, it can be based on the lesser of M_{sl} from Eq. 2 and M_{sd} from Eq. 4 (called Proposal 1) or simply M_{sl} from Eq. 2 (called Proposal 2). Both proposals are investigated thoroughly in Pham and Hancock [9], [10]. A selection of the results is given here to demonstrate the differences with and without tension field action. Only the tests without the straps are included here as these always give lower values for the test loads and hence are a lower bound. Fig. 9 shows the full set of test data compared with Equations 12 and 13 when V_v is based on Eqs. 5 – 7 without tension field action, and Fig. 10 shows the same data and comparisons when V_v is based on Eq. 11 with tension field action. In each case, the upper figure is Proposal 2 where M_s is based on M_{sl} and the lower figure is Proposal 1 where M_s is based on the lesser of M_{sl} and M_{sd}.

It is clear from the graphs that the use of M_{sl} is acceptable in all cases provided the circular interaction equation (Eq. 12) is used for combined bending and shear. It is also clear that tension field action plays a large part in the tests so that the use of Eq. 11 which includes tension field action as shown in Fig. 8 produces more accurate estimates of the combined bending and shear strength.

5 CONCLUSION

The paper has presented proposals for the design of cold-formed steel sections by the Direct Strength method as in the North American Specification and Australian/ New Zealand Standard accounting for shear, and combined bending and shear. The proposals are compared with tests on C-sections in predominantly shear, combined bending and shear, and pure bending. One feature of the DSM is that the full section buckling including intermediate stiffeners under shear has been included rather than simple web buckling in shear. This proposal produces better correlation with the test data but requires special computer software such as the Spline Finite Strip Method (SFSM) to determine the shear buckling loads of full sections. The general conclusion is that the use of M_{sl} in the interaction equations for combined bending and shear is acceptable in all cases provided the circular interaction equation (Eq. 12) is used for combined bending and shear.

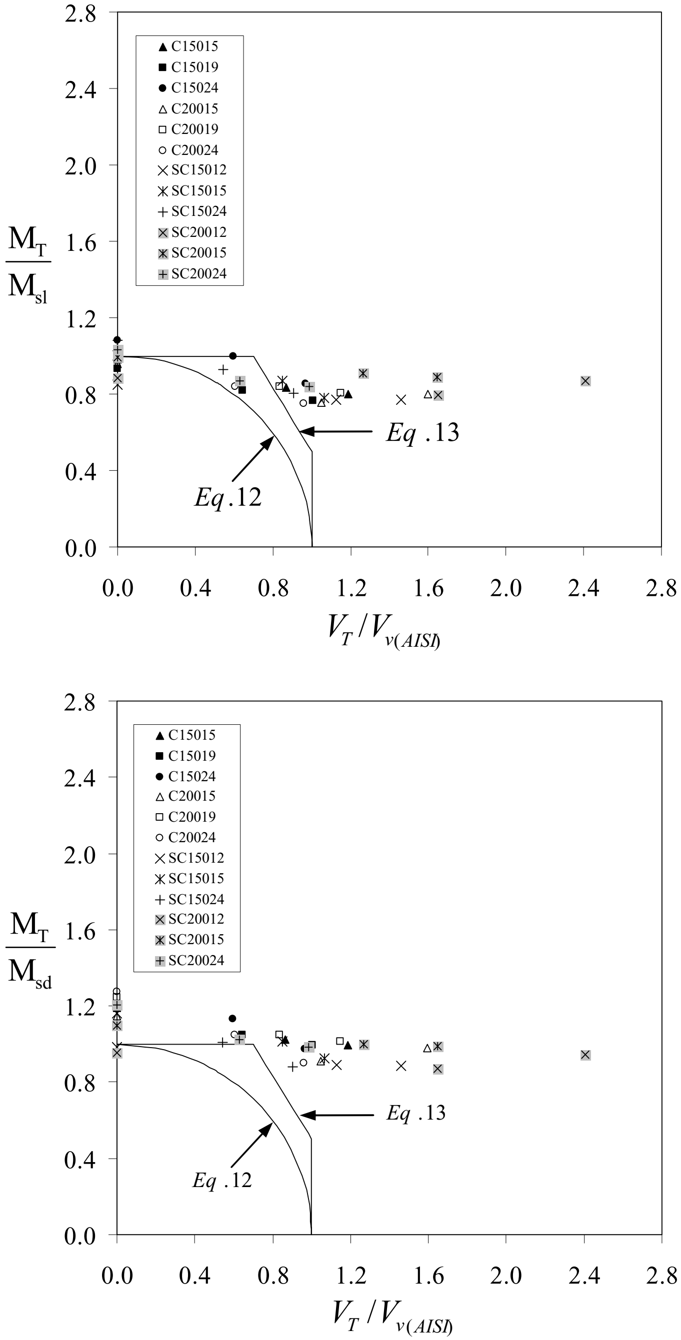


Figure 9. Comparison of Tests with Design Proposals for Combined Bending and Shear excluding TFA

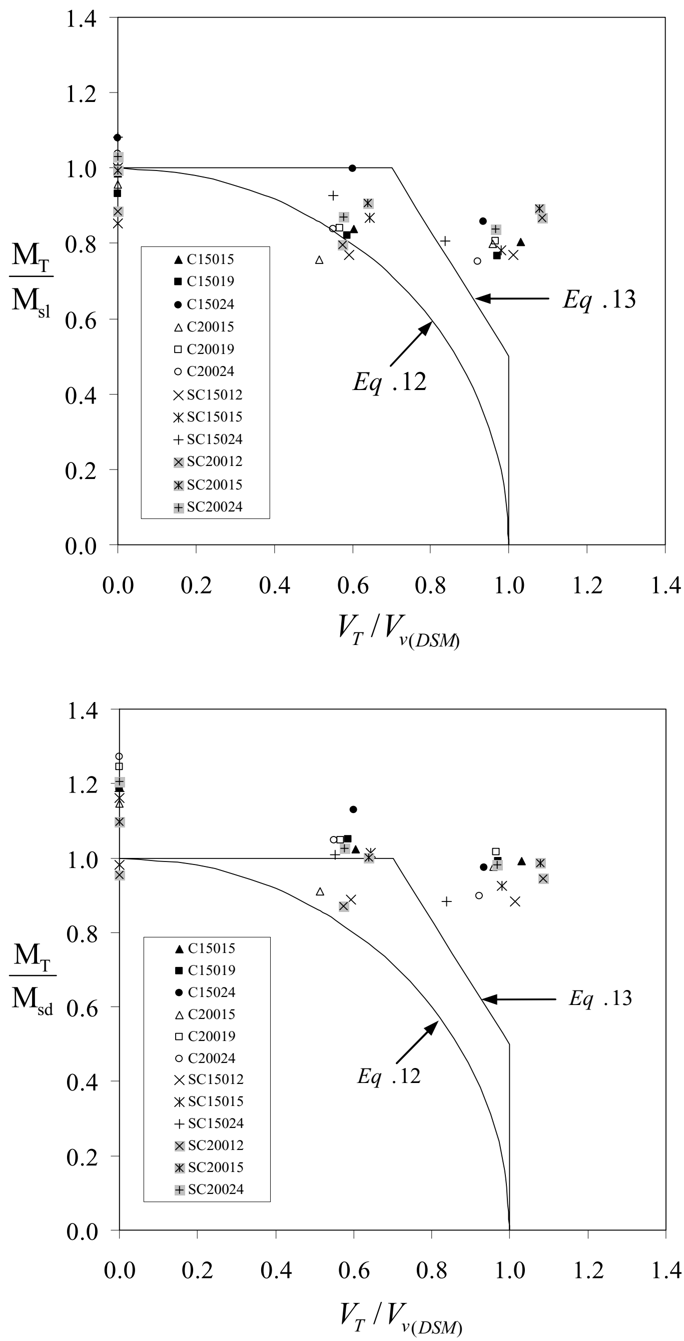


Figure 10. Comparison of Tests with Design Proposals for Combined Bending and Shear including TFA

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