PLASTIC RESISTANCE OF L-STUBS JOINTS SUBJECTED TO TENSILE FORCES

M.Couchaux*,, I.Ryan* and M.Hjiaj****

* CTICM, Saint-Aubin, France
e-mails: mcouchaux@cticm.com, iryan@cticm.com

** INSA of Rennes, Rennes, France
e-mail: Mohammed.Hjiaj@insa-rennes.fr

Keywords: plastic, contact, bolted, tensile.

***Abstract**. The tensile parts of tubular bolted flange joints can be treated as L-stubs or T-stubs. It has been shown that contact has a significant effect on the elastic behavior of these joints. In the present paper, the effects of contact between flange plates are modeled via a mechanical model. Taking into account the behavior of the flange in the contact area and bolt bending, a model is developed to predict the resistance of L-stub joints. The comparison of analytical results with 3D finite element elasto-plastic analyses indicates that the proposed model provides good predictions. Effects of contact, and therefore of the prying force, are particularly important in the case of relatively thin flanges.*

1 INTRODUCTION

T-stubs and L-stubs (see Figure 1) are commonly used to model the tensile part of many types of bolted joint ([2]-[7]) both in the elastic and in the plastic range of behavior. L-stubs are commonly used to model the tension part of ring flange joints. The pertinence of this approach has been validated by Seidel [4]. The effects of prying force are particularly important for this type of joints as it can increase significantly the value of the bolt force and thus modify the plastic or ultimate resistance of this type of joint.

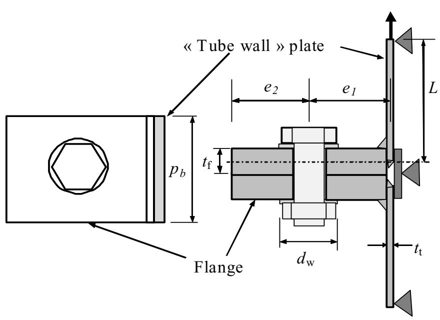


Figure 1: Geometry and main dimensions of an L-stub joint

Prying force is the resultant of contact pressures between the two opposite flanges. For some time now, the prying force has been represented by a concentrated force acting at or near the flange plate edges of T-stubs [5] or L-stubs ([2], [4]). Senda et al [3] postulated a linear distribution of contact pressure. However it was shown in [7] that the contact pressure distribution may depend on the length of the contact area and may not be unique in shape. Couchaux et al [7] proposed to model the flange in contact with the opposite ones via a mechanical model. This model has been applied in the elastic range and the results were in good agreement with numerical ones. Due to contact phenomena the prying force can be significantly increased particularly for thinner flanges. When the plastic resistance of a joint is attained, global plastic deformations are not usually very large so that the distribution of contact pressure can be fairly accurately obtained via an elastic analysis. Following the usual practice of considering plastic resistance models to evaluate the capacity of joints, the authors propose to determine the resistance of L-stub joints while considering the effects of contact. The contact distribution is evaluated for an elastic behavior of the flange just prior to the formation of a plastic mechanism in the flange or yielding of the bolt. As a more accurate determination of the contact area is not easy, a simplified formulation is provided. The results obtained by both models are satisfactorily compared to those obtained by non-linear elasto-plastic 3D finite element analyses.

2 ANALYTICAL MODEL

2.1 General hypothesis

The following four failure modes are considered for the L-stub joint: bending mechanism collapse of the flange, failure of the bolts with contact over a part of the flange width, failure of the bolts with full separation of the flange and finally rupture of the tube wall. Thus, the position of the prying force is important for the first and the second failure modes. The bolt is assumed to apply two equal punctual forces, each at a distance e_w (see Figure 2) from the bolt axis. In conjunction with its axial tensile load a plastic bending moment is usually rapidly attained on the tube wall, noting that in general the tube-wall thickness t_t is much less than that of the flange t_f (here we consider that $t_t < t_f$). The plastic tube-wall bending moment, as a function of the applied axial tensile load $F_{T,pl,MN}$, at the junction with the flange is given by:

$$M_{pl,t,MN} = M_{pl,t}\left[1-\left(\frac{F_{T,pl,t,MN}}{F_{T,pl,t}}\right)^2\right] \tag{1}$$

where:

$M_{pl,t} = \frac{l_{eff}t_t^2}{4}f_{y,t}$, $F_{T,pl,t} = l_{eff}t_tf_{y,t}$ with $f_{y,t}$: Yield strength of the tube wall steel.

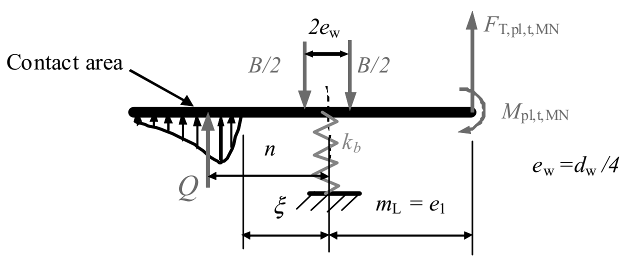


Figure 2: L-stub contact/prying forces just prior to Mode 1 or 2 failures

The behavior of steel is assumed to be elastic perfectly plastic. The flange is modeled by beam elements while the half-length bolt is modeled as a linear elastic axial spring connected to rigid supports at each end, that at the head or nut side being to the center of a rigid bar of width $2e_w$ resting on the flange. The area extending from the bolt to the free edge is considered as a potential contact area. In this model, the behavior of the flange "beam" in the contact area is considered via the mechanical model described in [7]. In the non-contact area of the flange, the Bernoulli beam theory is adopted so the elastic stiffness is $I_f = t_t^3 l_{eff}/12$ where l_{eff} is similar to that defined in EN1993-1-8 [5]. The elastic axial stiffness of the half-length bolt is $k_b = 2EA_s/L_b$, where L_b is similar to that defined in EN1993-1-8 [5]. The absence of a bolt hole in the flange plate is considered to compensate for the effect of any bending in the bolt.

2.2 Behavior in the contact area

The mechanical model, developed in [7] for a beam in contact with an infinitely rigid foundation, is applied to the case of a flange in the contact area. The contact pressure is assumed to be constant across the width of the beam (i.e. effective length of the L flange). Loading and boundary conditions of the flange on the contact area are shown in Figure 3.

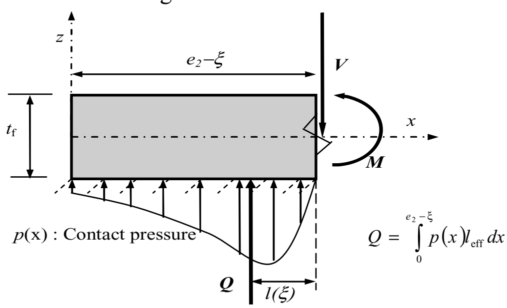


Figure 3: Boundary conditions in the contact area

Considering the boundary conditions and the mechanical model, Couchaux et al [7] obtained the relation between shear force V and bending moment M. The exact solution being quite involved, the following simplified relation has been proposed:

$$l(\xi) = \frac{M}{V} = \frac{1}{\left\{1 + \left(\dfrac{e_2 - \xi}{1{,}11 \cdot t_f}\right)^6\right\}^{\frac{1}{6}}} \frac{2(e_2 - \xi)}{3} \tag{2}$$

2.3 Evaluation of the separation length

In the non contact area, the flange is loaded by the tube wall and the bolt. Furthermore, when the separation length from the bolt axis, ξ, is less than the edge distance e_2, the flange is fully clamped at its junction with the contact area. The expression for the flange deflection at the bolt axis, δ_B, is then:

$$\delta_B = F_{\mathrm{T,pl,t,MN}} \frac{\xi^2}{EI_f}\left(m_L / 2 + \xi / 3\right) - B\frac{\xi^3}{3EI_f} - M_{\mathrm{t,pl,MN}} \frac{\xi^2}{2EI_f} \tag{3}$$

The contribution of bolt force is simplified here by considering the action of a unique bolt force equal to B at a distance ξ from the fully clamped section. Since the flange deflection δ_B and the bolt elongation are equal, we obtain the following expression:

$$B\left[\frac{1}{k_b} + \frac{\xi^3}{3EI_f}\right] = F_{\mathrm{T,pl,t,MN}} \frac{\xi^2}{EI_f}\left(m_L / 2 + \xi / 3\right) - M_{\mathrm{t,pl,MN}} \frac{\xi^2}{2EI_f} \tag{4}$$

Global equilibrium considerations also lead to the following expression for the bolt force:

$$B = \frac{F_{\mathrm{T,pl,t,MN}}\left(m_L + n\right) - M_{\mathrm{t,pl,MN}}}{n} \tag{5}$$

where: $n = \xi + l(\xi)$.

The separation length ξ, which depends on the relative stiffness of the bolt and the flange and also on the relative resistances of the joint and the tube wall, is comprised between the values e_w and e_2.

2.4 Failure mode 1: yielding of the flange

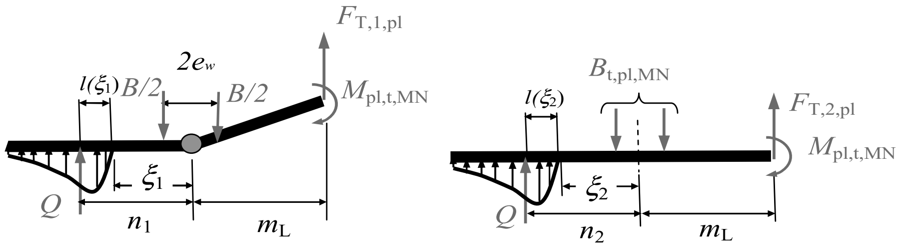
Due to the interaction between tensile force and bending moment in the tube wall (given by (1)), the tensile force which leads to the formation of a plastic bending moment in the flange at the bolt line is:

$$F_{T,1,\mathrm{pl}} = F_{T,1,\mathrm{pl},(0)}\, g(f_1) \tag{6}$$

where:

$$F_{\mathrm{T,1,pl,(0)}} = \frac{2n_1 M_{\mathrm{pl,f}} + M_{\mathrm{pl,t}}\left(2n_1 - e_{\mathrm{w}}\right)}{2n_1 m_{\mathrm{L}} - e_{\mathrm{w}}\left(m_{\mathrm{L}} + n_1\right)},\ M_{\mathrm{pl,f}} = l_{eff}\frac{t_f^2 f_{y,f}}{4}, l_{eff,1} = \min\left(l_{eff}; p_b - d\right),$$

$$g(f_1) = \left[\frac{\sqrt{1+4f_1}-1}{2f_1}\right],\ f_1 = \left(\frac{F_{T,1,\mathrm{pl},(0)}}{F_{\mathrm{T,pl,t}}}\right)^2 \frac{1}{1+\left(\frac{t_{\mathrm{f}}}{t_{\mathrm{t}}}\right)^2 \frac{2n_1}{2n_1 - e_{\mathrm{w}}}}.$$



Mode 1: *Yielding of the flange* **Mode 2:** *Failure of the bolt*

Figure 4: Failure modes 1 and 2

Substituting (5) and (6) into (4), we get an equation from which the separation length ξ_1 can be obtained using an iterative procedure. The separation length depends both on the bolt and flange elastic stiffness and also on the plastic resistances of the flange and of the tube wall. The joint resistance corresponding to mode 1 is a plastic failure mechanism which can now be obtained from expression (6).

2.5 Failure mode 2: Failure of the bolt with prying action

2.5.1 Effect of bolt bending on the tensile resistance of the bolt

Experimental evidence shows that bolt bending is fairly significant both in the elastic and the elasto-plastic ranges of behavior. However, most of the existing analytical models for bolted joints don't consider this aspect explicitly. In the following, a simple evaluation of the effect of bending moment on the resistance of bolt is presented. The interaction between bending moment and tensile force in the bolt at the plastic limit can be approximated by the expression:

$$\frac{M_{\mathrm{pl,b,MN}}}{M_{\mathrm{pl,b}}} + \left(\frac{B_{\mathrm{t,pl,MN}}}{B_{\mathrm{t,pl}}}\right)^2 = 1 \tag{7}$$

where:

$$B_{\mathrm{t,pl}} = A_s f_{\mathrm{yb}},\ M_{pl,b} = \frac{4R_s^3}{3} f_{yb},\ R_s = \sqrt{\frac{A_s}{\pi}} \approx 0{,}45\, d.$$

In most cases when failure of bolts occurs before flange or tube wall, the prying force is often at or very close to the flange edge. If the flange curvature is neglected, we have (see Figure 5):

$$\theta_{\mathrm{B}} \approx \frac{\delta_{\mathrm{B}}}{e_2} \tag{8}$$

Considering the bolt as a cantilever beam subjected to a tensile force $B_{\mathrm{t,pl,MN}}$ and bending moment $M_{\mathrm{pl,b,MN}}$ we get:

$$B_{\mathrm{t,pl,MN}} = \frac{2EA_{\mathrm{s}}}{L_{\mathrm{b}}}\delta_B \tag{9.1}$$

$$M_{\text{pl,b,MN}} = \frac{2EI_s}{L_b}\theta_B \tag{9.2}$$

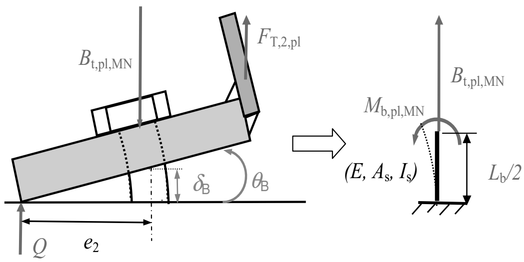


Figure 5: A Deformation of a L-stub when bolt yields first

Introducing (8), (9.1) and (9.2) in (7) we get:

$$B_{\text{t,pl,MN}} = B_{\text{t,pl}}\frac{\sqrt{4+\alpha^2}-\alpha}{2} \approx B_{\text{t,pl}}\left[1-0{,}48\ \alpha\right] \tag{10}$$

where:

$$\alpha = \frac{3\pi R_s}{16e_2} \approx 0{,}27\frac{d}{e_2}.$$

If the usual lower limit is adopted for e_2 , i.e. e_2= 1,2d_0 ≈1,32d where d_0 is the bolt hole diameter, one obtains $B_{\text{t,pl,MN}}$=0,9$B_{\text{t,pl}}$, which seems to justify the use of the 0,90 factor on the bolt axial design resistance given in many standards and to explain how bolt bending can be neglected.

2.5.2 Joint resistance involving bolt failure with prying

Due to the interaction between tensile force and bending moment in the tube wall (given by (1)), the tensile force (see Figure 4) which leads to the failure of the bolt is:

$$F_{\text{T,2,pl}} = F_{\text{T,2,pl,(0)}}\ g\left(f_2\right) \tag{11}$$

where:

$$F_{\text{T,2,pl,(0)}} = \frac{M_{\text{pl,t}} + n_2 B_{\text{t,pl,MN}}}{m_L + n_2},\quad g\left(f_2\right) = \left[\frac{\sqrt{1+4f_2}-1}{2f_2}\right],\quad f_2 = \left(\frac{F_{\text{T,2,pl,(0)}}}{F_{\text{T,pl,t}}}\right)^2 \frac{1}{1+\dfrac{n_2 B_{\text{t,pl,MN}}}{M_{\text{pl,t}}}}.$$

Substituting (11) into (4), we obtain an equation from which the separation length ξ_2 can be determined by iteration. As for the mode 1, the separation length depends on the bolt and flange elastic stiffness and also on the resistances of the bolt and the tube wall. The joint resistance corresponding to mode 2 is by bolt failure which can now be obtained from expression (11).

2.6 Failure mode 3: Failure of the bolt without prying action

If flanges fully separate, rotations will be less than those assumed in § 2.5.1 and there is no prying action. Therefore the joint resistance will be equal to that of the bolt in tension which can safely be taken as:

$$F_{\text{T,3,pl}} = B_{\text{t,pl,MN}} \tag{12}$$

2.7 Failure mode 4: Rupture of the tube wall

As the axial load applied to the tube-wall increases and tends towards the axial plastic resistance of the tube wall, yielding progresses across the wall section to such an extent that the associated tube-wall moment tends towards zero at the junction with the flange. Therefore the joint resistance corresponding to rupture of the tube wall can be given as:

$$F_{\mathrm{T,4,pl}} = F_{\mathrm{T,pl,t}} = l_{eff} t_{\mathrm{t}} f_{\mathrm{y,t}}$$

2.8 Plastic resistance of L-stubs

The tensile resistance of L-stub joint is taken as the smallest of the values for the four failure modes:

$$F_{\mathrm{T,pl}} = \min\left(F_{\mathrm{T,1,pl}};\ F_{\mathrm{T,2,pl}};\ F_{\mathrm{T,3,pl}};\ F_{\mathrm{T,4,pl}}\right) \tag{13}$$

3 SIMPLIFIED MODEL

For the analytical model proposed above for the determination of the separation length, required for failure modes 1 and 2, an incremental resolution method is needed. To overcome this inconvenience a simplified expression has been developed to provide the separation length and which gives good results in the elastic range of behavior [8] and is shown here to be acceptable in the plastic range as well. The simplified expression for the distance n from the bolt axis to the prying force is:

$$n = \min\left((2e_2 + \xi)/3;\ (\xi + 0,74t_{\mathrm{f}})\right) \tag{14}$$

where the separation length is:

$$\xi = e_2.\min\left(1; \sqrt{\frac{\alpha_{\mathrm{R,0}}}{\alpha_{\mathrm{R}}}}\right) \geq e_{\mathrm{w}},\ \ with\ \ \alpha_{\mathrm{R}} = 4\left(\frac{m_{\mathrm{L}}}{t_{\mathrm{f}}}\right)^3 \frac{A_{\mathrm{s}}}{L_{\mathrm{b}} l_{\mathrm{eff}}},\ \ \alpha_{\mathrm{R,0}} = \frac{e_2/m_{\mathrm{L}} + 1}{\left(e_2/m_{\mathrm{L}}\right)^3}.$$

The formula (14) for n can be used to determine both lengths n_1 (i.e. for failure mode 1) and n_2 (i.e. for failure mode 2). To account simply for the reduction of the plastic bending moment in the tube wall as its axial load increases, the joint resistance are calculated via (6) and (11) for mode 1 with $M_{\mathrm{pl,t,MN}} = 0,5M_{\mathrm{pl,t}}$ and for mode 2 with $M_{\mathrm{pl,t,MN}} = 0$.

4 COMPARISONS WITH NUMERICAL RESULTS

4.1 Finite element model and interpretation

The model is using the Finite element code ANSYS V11.0 and is quite similar to that used by Seidel [4] and Couchaux et al [7]. Joints are generated with three dimensional elements, which are hexahedral. A bolt of diameter d of a constant bolt cross-section, which is equal to the tensile stress as defined for EN1993-1-8 [5]. Only a quarter of the flange to flange joint needs to be modeled due to symmetry.

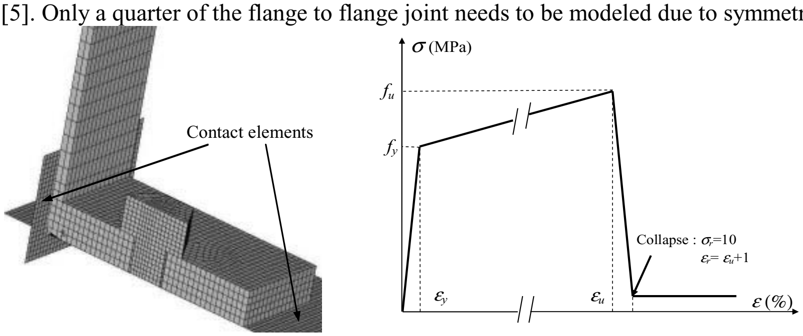


Figure 6: Meshing and behavior of steel considered in the model

Two types of contact elements are also used: a) Flexible contact elements between the flange and the bolt and b) Rigid contact element between one flange and two plane of symmetry. The first rigid plane is horizontal and models the fictitious flange. The second rigid plan is vertical and models the resistance to tube dilation provided by a circular flange (see Figure 1 and 6). An isotropic Coulomb friction law (μ = 0,25) is used to reproduce sliding/sticking conditions between the flange and the bolt. Friction is neglected between the two flanges because of the symmetry.

A vertical displacement is applied at the end of the tube wall. The stress-strain relationship for the steel (flange, tube and bolts) is assumed to be multi-linear (see Figure 6). Large deformations are also considered. As soon as the deformation level reaches ε_u, the stress drops to 10 MPa in order to model the failure of the element. This phenomena leads either to a drop-off of the force applied to the joint or to the termination of the calculation. This latter state is assumed to be the ultimate state for the joint. The criterion of the yielding surface is that by Von-Mises. Finally, the ECCS method [1] is used to determine the plastic resistance from the numerical results as presented by the force-displacement curves.

4.2 Comparison of the FEM model results with those by the analytical models

A limited parametric study has been carried out to confront results of the analytical and the simplified models with those by the FEM model. All the joints studied have the dimensions presented in Figure 7. The thickness of the flange, t_f, which is comprised between 8 and 35 mm is the only variable parameter considered for the present study. The mechanical characteristics of steels for bolts, flanges and tube walls used are given in Table 1.

Table 1 Mechanical characteristics of steel

Elements	f_y	f_u	ε_u
-	*MPa*	*MPa*	*%*
Bolt M20 class 10.9	900	1050	5
Flange/Tube wall S355	355	480×1,5	30

The ratio of the joint resistance to the bolt resistance calculated by the analytical, simplified and numerical models are presented in Figure 7. When greater than 18mm, the flange thickness has almost no influence on the resistance of the joints considered because failure is by mode 2. The mode 2 failure involves significant prying forces in L-stubs leading to yielding/failure of the bolt. For lower flange thickness values, the joint resistance decreases with the thickness since it depends on the flange yielding only. The results obtained via the analytical and simplified models are in good agreement with those obtained by the numerical one. The simplified model usually underestimates the joint resistance. In the cases studied, the tube-wall failure mode is never attained.

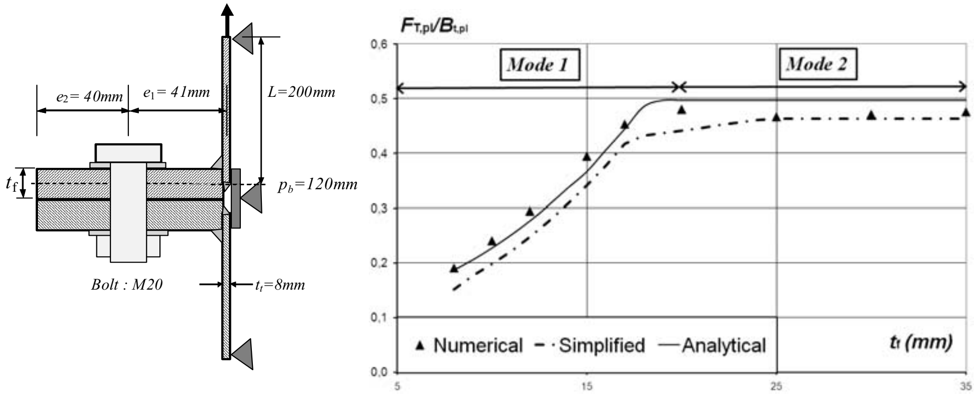


Figure 7: Joint configuration and ratio of joint resistance to the bolt resistance

5 SUMMARY

In the present paper an analytical model is proposed for calculating the plastic resistance of L-stubs joints subjected to tensile force by considering four failure modes not much unlike those described by EN1993-1-8 for T-stubs. The effects of contact between the two opposite flanges, which often leads to a significant additional prying force in the bolt, and bolt bending are also considered explicitly. Contact effects and particularly the position of the prying force can be determined considering an elastic behavior of the joint just prior to the formation of the failure mechanism.

REFERENCES

[1] ECCS, Recommended testing procedures for assessing the behaviour of structural elements under cyclic loads, European Convention for Constructional Steelwork, Technical Committee 1, TWG 13 – Seismic Design, No.45, 1986.

[2] Petersen, Ch., Stahlbau (Steel Construction), Braunschweig: Vieweg-Verlag, 1988.

[3] Senda, H., Suzuki, T., Ogawa, T., Inelastic behaviour of bolted T-stub connections, *Journal of structural and construction engineering, Transaction of AIJ, **476***, 159-168, 1996 (in Japanese).

[4] Seidel, M., Zur Bemessung geschraubter Ringflanschverbindungen von Windenergieanlangen, Institut für Stahlbau, Dissertation, Heft 20, Universität Hannover, 2001.

[5] Eurocode 3, Design of steel structures – Part 1 – 8: Designs of joints, 2005.

[6] Lemonis, M.E., Gantes, C.J., Incremental modelling of T-stub connections, *Journal of Mechanics of materials and structures*, **1**(7), 1135-1157, 2006.

[7] Couchaux, M., Hjiaj, M., Ryan, I., Bureau, A., Effect of contact on the elastic behaviour of bolted flange connections, *11th Nordic Steel Construction Conference*, Malmö, Sweden, 2009.

[8] Couchaux, M., Behaviour of bolted circular flange joints, PhD Thesis, INSA of Rennes, 2010 (in preparation in French).