# INTERACTION DIAGRAMS FOR DESIGN OF CONCRETE-FILLED TUBULAR COLUMNS UNDER FIRE

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Abstract. This work presents a procedure for the design of concrete-filled hollow columns under fire action, employing cross-section strength interaction diagrams. Fire action induces thermal strains and modifies the stress-strain relationships for steel, concrete and reinforcement bars. Strength interaction diagrams are tools that relate ultimate values of bending moments and axial force in the cross section. A previously developed algorithm is used for the construction of diagrams, obtained by a stepwise variation of the deformed configuration, under assumptions of conventional ultimate strain values for concrete and steel. The moment magnification factor is used to approximate the second order effects in the column. The procedure is compared with European Committee for Standardization rules for calibration of eccentricities to be used in the calculations.

## **1 INTRODUCTION**

Axial force-bending moment diagrams are useful and popular tools for ambient temperature design of composite beam-column cross sections. A procedure for the construction of such diagrams for arbitrarily shaped steel-concrete composite sections under generic temperature distribution, such as that present in a fire, was developed by Caldas [1]. This author obtained these diagrams for RC sections and compared the results to experimental values. The results showed that the procedure is able to take into account the influence of thermal strains, the ultimate concrete strain variation with temperature and different fire exposure boundary conditions (1, 2, 3 or 4 faces of rectangular sections).

This paper applies the developed procedure [1, 2, 3, 4] to the design of slender concrete filled circular hollow section columns under fire action. The results are then compared to the analytical method adopted by the European Committee for Standardization, EN 1994-1-2:2005 [5].

## 2 COLUMN DESIGN

## 2.1 Resistance of cross section (interaction diagrams)

Under fire action, the thermal strains and the degradation of the mechanical properties of the materials pose challenging difficulties to the limit state analysis of beam-column cross sections. Generic temperature distributions result in unexpected variations of the interaction diagrams. In order to define these limit states it is necessary to establish ultimate strain values for concrete and steel. For concrete, the ultimate compressive strain is conventionally assumed as the value correspondent to the peak stress of the Eurocode 2, EN 1992-1-2:2004 [6], temperature-dependent uniaxial stress-strain relation (figure 1). Concrete tensile stress is not taken into account.

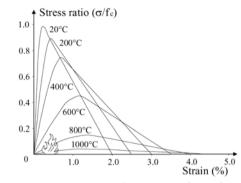


Figure 1: Stress-strain relationships of concrete at elevated temperature [6].

With regard to the steel properties, the ultimate strain value for the hollow section steel and the reinforcement bars is assumed as 1%. The steel stress-strain relation is the bilinear relation with 0.2% reduction factor.

Mechanical or stress-inducing strains are given by

$$\varepsilon(x,y) = \varepsilon_o + k_x y - k_y x - \varepsilon_{th} \quad \text{or} \quad \varepsilon(\xi,\eta) = \varepsilon_o + k_o \eta - \varepsilon_{th} , \qquad (1)$$

where  $_{th}$  is the thermal strain, and the section curvatures are expressed in the coordinate systems shown in figure 2. In this system  $k_x$  and  $k_y$  are the curvatures about x and y axes,  $k_o$  is the curvature about  $\xi$  axis and  $\varepsilon_o$  the centroidal strain.

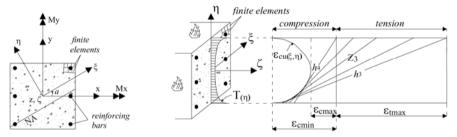


Figure 2: Coordinate systems (NA is the neutral axis) and ultimate configurations.

In the present formulation, an ultimate limit state is conventionally assumed when (x, y) in any point of the cross section reaches its temperature-dependent limit value. If the thermal strains  $_{th}$  and the ultimate concrete compressive strains depend on the temperature field T(x, y), it is possible to evaluate these values for the grid of points that was employed, for instance, in the thermal analysis of the section. This allows one to obtain points of the surface  $_{cu}(\xi, )$  that represent the ultimate compressive concrete strain (figure 2) and therefore have an approximation of this surface. If the plane deformed section touches this surface at any point, an ultimate state is reached and the correspondent axial force and bending moments are points of the interaction diagram for the cross section, under the given temperature profile.

For a fixed value of the neutral axis angle one may then define an arbitrary continuous variable that traces the evolution of the deformed configuration of the section such that every consecutive point lies on the ultimate limit state surface [1, 2].

Using the same grid (or mesh) that was employed in the thermal analysis the axial force and bending moments are evaluated by numerical integration:

$$N_{z} = \sum_{i=1}^{n} (\sigma(\varepsilon)A)_{i}, \quad M_{x} = \sum_{i=1}^{n} (\sigma(\varepsilon)yA)_{i} \quad \text{and} \quad M_{y} = \sum_{i=1}^{n} (\sigma(\varepsilon)xA)_{i}.$$
(2)

#### 2.2 Verification of column length

From the *N-M* interaction curve (or alternatively the  $N-M_x-M_y$  interaction surface), a procedure for the verification of columns was developed [1] and adapted in the present work for concrete-filled circular hollow sections. Second order effects along the member are taken into account by the amplification factor

$$\delta = \frac{l}{1 - N/N_{fi,e}},\tag{3}$$

where the buckling load under fire action is evaluated as

$$N_{fi,e} = \frac{\pi^2 E I_{fi}}{\ell^2} \,. \tag{4}$$

The temperature dependent section stiffness  $EI_{\beta}$  is evaluated by numerical integration, using the section grid, considering an averaged temperature on each mesh element,

$$EI_{a} = 0.8EI_{a} + 0.8EI_{s} + 0.5EI_{c},$$
(5)

where subscripts *c*, *a* and *s* refer to concrete, profile steel and reinforcement, respectively. The characteristic value for the concrete secant modulus under fire action is taken as the relation between the peak stress and its correspondent strain for the specified temperature (figure 1). In order to apply the procedure it is necessary to impose an eccentricity on the axial load. Several values of this eccentricity were tested, namely,  $e_{min} = 0.015 + 0.03d$ , where *d* is the section outer diameter, /150, /200, /250, /300, /400, /500 and /1000.

#### **3 EUROCODE 4 ANALYTICAL METHOD**

According to the analytical method of EN 1994-1-2:2005 [5], the axial resistent force of composite columns subjected to fire is given by

$$N_{fi,Rd} = \chi_{fi} N_{fi,p\ell,Rd} ,$$
 (6)

where  $N_{\beta,p,Rd}$  is the plastic axial force under fire conditions and  $f_i$  is the reduction factor related to buckling curve c.

The composite column effective flexural stiffness is given by

$$EI_{fi} = \varphi EI_a + \varphi EI_s + \varphi EI_c , \qquad (7)$$

where is a reduction coefficient depending on the effect of thermal stress, taken as 0.80 in this case. The temperature field in the cross section is required in order to employ the proposed method. In the present work a finite difference scheme, assuming radial symmetry of the temperature distribution in terms of polar coordinates and described in [7] was employed to obtain the temperatures.

### 4 COLUMN ANALYSIS

Employing interaction diagrams and the procedure presented in section 2.2, the load carrying capacity of a group of concrete-filled hollow circular columns was compared to the values obtained by the the Eurocode 4 analytical method [5]. The latter was assumed as a reference value, due to the fact that it has been calibrated by experimental results.

Steel and concrete material properties were taken from [5], considering 4% moisture (in weight) and upper limit thermal conductivity. The hollow section profiles have yield strength of 350 MPa and 200000 MPa elastic modulus. Concrete characteristic compressive strength is 40 MPa and the reinforcement steel has yield strength of 500 MPa and 210000 MPa elastic modulus. Concrete cover was 35 mm. The buckling length of the columns was taken as 2 m, representing columns of intermediate floors with 4 m between lateral restraints in ambient condition.

The colums are identified by (diameter)x(thickness)-(number of reinforcement bars)x(reinforcement bar diameter).

Figure 3 exemplifies the procedure employing the interaction diagram for an eccentricity of /200 for the 219.1x8.2-0 column.

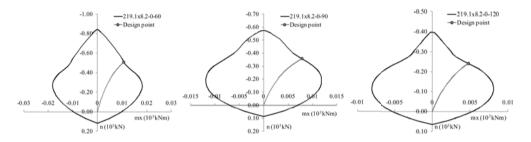


Figure 3: Interaction diagrams for 219.1x8.2-0 columns for 60, 90 and 120 min fire exposure.

Figure 4 displays the graphic results for the column design loads obtained employing the interaction diagrams for ambient and elevated temperature (zero, 30, 60, 90 and 120 minutes of standard fire exposure) and different eccentricity values along with the Eurocode ( $N_{Rd,fi,Eurocode}$ ) analytical method results. It may be observed that the eccentricities  $e_{min}$  and /150 lead to the smallest values for column resistance. The value  $e_{min}$  was employed by Caldas [1] in the analysis of reinforced concrete columns under fire action and is prescribed by the american [8] and brazilian [9] design codes under ambient temperature. The utilization of these values is therefore discouraged. The values of eccentricity between /200 and /1000 gave the closest results to the analytical Eurocode method. The former led to results on the safe side, with the largest error happening for the 30-minute fire exposure, but decreasing for superior times.

It may be noted that the influence of the eccentricity value is less pronounced when the column slenderness increases, as well as for higher exposure times. This behaviour is evident with the 219.1x8.2-0 column where the curves converge to the Eurocode curve for larger times. In contrast, the 355.6x9.5-8x20 column displays a different behaviour with a strong dependency on the eccentricity for the complete range of exposure times. The behaviour of the columns with and without reinforcement are similar.

Table 1 displays the relationship between points from the interaction diagram and values obtained according to EN 1994-1-2:2005 [5].  $N_{max}$  is the axial compressive resistant force with zero moment which is compared to the axial plastic resistant load  $N_{fi,p}$  *Eurocode* [5].  $M_{max}$  is the maximum moment from the interaction diagram which is compared with the maximum plastic moment  $M_{fi,p}$  *max,Eurocode* from EN 1994-1-1:2004 [10].

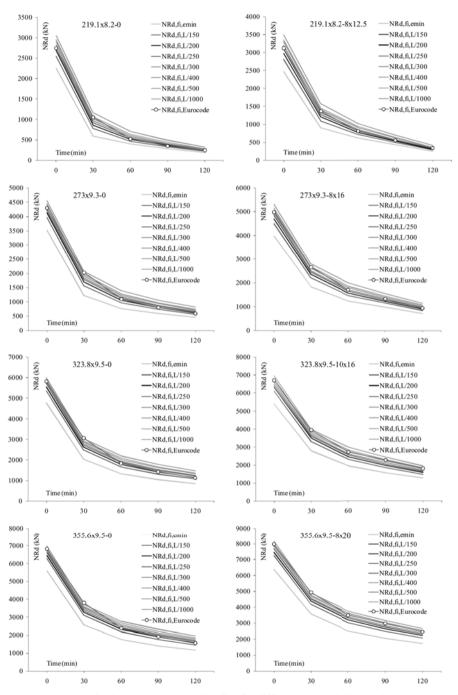


Figura 4: Assessment of column resistance under fire for different eccentricities and comparison with Eurocode.

Column	Nmax	Mmax	$M_{N=0}$	Nfi.c
identification	Nfi,p ,Eurocode	Mfi,p ,max,Eurocoće	Mfi,p ,Eurocode	Nfi,e,Eurocode
219.1x8.2-0-0	1.00	0.87	0.92	0.93
219.1x8.2-0-30	0.80			0.87
219.1x8.2-0-60	0.94			0.91
219.1x8.2-0-90	0.94			0.95
219.1x8.2-0-120	0.96			0.96
219.1x8.2-8x12.5-0	1.00	0.84	0.87	0.94
219.1x8.2-8x12.5-30	0.83			0.90
219.1x8.2-8x12.5-60	0.86			0.93
219.1x8.2-8x12.5-90	0.83			0.91
219.1x8.2-8x12.5-120	0.82			0.93
273x9.3-0-0	1.00	0.87	0.93	0.93
273x9.3-0-30	0.82			0.87
273x9.3-0-60	0.95			0.87
273x9.3-0-90	0.95			0.90
273x9.3-0-120	0.94			0.93
273x9.3-8x16-0	1.00	0.84	0.87	0.93
273x9.3-8x16-30	0.86			0.90
273x9.3-8x16-60	0.89			0.92
273x9.3-8x16-90	0.86			0.95
273x9.3-8x16-120	0.86			0.95
323.8x9.5-0-0	1.00	0.87	0.93	0.92
323.8x9.5-0-30	0.84			0.85
323.8x9.5-0-60	0.95			0.83
323.8x9.5-0-90	0.95			0.86
323.8x9.5-0-120	0.95			0.88
323.8x9.5-10x16-0	1.00	0.84	0.88	0.92
323.8x9.5-10x16-30	0.87			0.89
323.8x9.5-10x16-60	0.90			0.89
323.8x9.5-10x16-90	0.86			0.91
323.8x9.5-10x16-120	0.86			0.88
355.6x9.5-0-0	1.00	0.86	0.93	0.91
355.6x9.5-0-30	0.86			0.84
355.6x9.5-0-60	0.95			0.81
355.6x9.5-0-90	0.96			0.83
355.6x9.5-0-120	0.95			0.84
355.6x9.5-8x20-0	1.00	0.84	0.88	0.92
355.6x9.5-8x20-30	0.88			0.88
355.6x9.5-8x20-60	0.91			0.89
355.6x9.5-8x20-90	0.88			0.92
355.6x9.5-8x20-120	0.88			0.93
Minimum=	0.80	0.84	0.87	0.81
Maximum=	1.00	0.87	0.93	0.96

Table 1: Comparison of interaction diagram solution with EN 1994-1-2:2005 [5].

 $M_{N=0}$  is the moment from the interaction diagram for which the normal force is zero. This value is compared to the plastic moment  $M_{fi,p}$ , *Eurocode* [10]. These values were obtained for ambient temperature. Also presented are the elastic buckling loads  $N_{fi,e}$  calculated according to Section 2.2, and according to EN 1994-1-2:2005 [5].

The relation  $N_{max} / N_{fi,p}$ , *Eurocode* presents a minimum value of 0.80 suggesting that complete section plastification does not occur according to the interaction diagram procedure. This is related to the ultimate concrete strain variation with respect to the temperature variation along the radius (figure 1). For ambient temperature this relation is equal to 1.00 as the ultimate concrete strain is constant.

The values of  $M_{max} / M_{fi,p, max,Eurocode}$  attain a minimum of 0.84, which is coherent with the 0.80 coefficient employed by the brazilian code ABNT NBR 8800:2008 [11]. For the relation  $M_{N=0} / M_{fi,p, Eurocode}$  the minimum value is 0.87, which agrees well with the value 0.90 prescribed by ABNT NBR 8800:2008 [11] and EN 1994-1-1:2004 [10]. Caldas *et al.* [12] provide an explanation for these coefficients.

The relation between the elastic buckling loads varies between 0.81 and 0.96 due to the differences in the evaluation of the effective stiffness in Section 2.2 and EN 1994-1-2:2005 [5], Section 3. The effective stiffness was chosen to be evaluated according to Section 2.2 in order to have a unified procedure for reinforced concrete columns [1] and composite columns, with the only difference being the eccentricity to be employed.

## 5 CONCLUSION

The authors of this paper, previous developed a procedure for the construction of interaction diagrams for generic concrete and steel cross sections subjected to arbitrary temperature distributions [1, 2, 3, 4]. The consideration of initial imperfection and  $2^{nd}$  order local effects allows the diagrams to be used in the analysis of slender columns under fire. In this work the procedure was applied to the analysis of hollow circular concrete filled columns and different eccentricities were analyzed and compared with results obtained from the Eurocode 4, EN 1994-1-2:2005 [5].

The eccentricities  $e_{min}$  and /150 produce excessively conservative results and are not recommended for the analyzed columns, and eccentricities smaller than /200 led to unsafe results, always compared with the Eurocode. Therefore the authors recommend the proposed design method of Section 2.2 with the value of /200 for the eccentricity.

The comparison of the points from the interaction diagram with the EN 1994-1-2:2005 [5] lead to interesting results. The relation  $N_{max} / N_{fi.p. ,Eurocode}$  presents a minimum value of 0.80 suggesting that complete section plastification does not occur according to the interaction diagram procedure. The relations  $M_{max} / M_{fi.p. ,max,Eurocode}$  and  $M_{N=0} / M_{fi.p. ,Eurocode}$  present minimum values that agree well with the brazilian [11] and european [10] design codes for composite columns under ambient temperature.

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### REFERENCES

- Caldas, R.B. *ENumerical analysis of steel, concrete and composite structures under fire*S, PhD thesis (in Portuguese), DEES, Universidade Federal de Minas Gerais, 2008.
- [2] Caldas, R.B., Sousa Jr., J.B.M., Fakury, R.H. Enteraction diagrams for reinforced concrete sections subjected to fireS, submitted to *Engineering Structures*.
- [3] Sousa Jr., J.B.M., Caldas, R.B., Fakury, R.H. Enteraction diagrams for concrete-filled tubular sections under fireŠ, *Proc. of Tubular Structures XII*, Shen, Chen and Zhao (eds.), Taylor and Francis Group, London, 635-639, 2009.
- [4] Caldas, R.B., Fakury, R.H., Sousa Jr., J.B.M. ÈAnálise de seções transversais de concreto armado em situação de incêndioŠ (*in Portuguese*), Proc. of 49° Congresso Brasileiro do Concreto, 2007.
- [5] European Committee for Standardization. Eurocode 4: Design of Composite Steel and Concrete Structures, Part 1-2: General Rules, Structural Fire Design, EN 1994-1-2:2005.
- [6] European Committee for Standardization. Eurocode 2: Design of Concrete Structures, Part 1.2: General Rules, Structural Fire Design. EN 1992-1-2:2004

- [7] Caldas, R.B., Fakury, R.H., Sousa Jr., J.B.M. Araújo, A.H.M. ÈCapacidade resistente à compressão de pilares tubulares circulares de aço preenchidos com concreto em situação de incêndioŠ, *Proc. of* 30° CILAMCE, Congresso Ibero-Latino-Americano de Métodos Numéricos em Engenharia, 2009.
- [8] American Concrete Institute. Building Code Requirements for Structural Concrete. ACI 318, 2005.
- [9] Associação Brasileira de Normas Técnicas (Brazilian Association of Technical Standards). Design of Structural Concrete. ABNT NBR 6118:2007.
- [10] European Committee for Standardization. Eurocode 4: Design of Composite Steel and Concrete Structures, Part 1.1: General Rules and Rules for Buildings. EN 1994-1-1:2004.
- [11] Associação Brasileira de Normas Técnicas (Brazilian Association of Technical Standards). Design of Steel and Composite Structures for Buildings. ABNT NBR 8800:2008.
- [12] Caldas, R.B., Fakury, R.H., Sousa Jr., J.B.M. ÈBases do dimensionamento de pilares mistos de aço e concreto segundo o projeto de revisão da NBR 8800Š, *Revista da Escola de Minas*, 60(2), 271-276.