

## A COUPLED FLUID-THERMAL-MECHANICAL ANALYSIS OF COMPOSITE STRUCTURES UNDER FIRE CONDITIONS

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**Abstract.** *This paper presents a numerical investigation on a unidirectional procedure joining fluid and thermo-mechanical modeling for assessing composite steel-concrete structures exposed to fire. The first stage of the analysis consists of an evaluation of fire dynamics, which is carried out utilizing a 3D Computational Fluid Dynamics (CFD) model. The temperature variation of fire-exposed members is performed in the second-phase, accounting for the fire elapsed time, obtained CFD heat fluxes and temperature-dependent thermal properties of materials. Finally, the structural behavior is evaluated by a FE-based approach including stress-strain-temperature relationships for steel and concrete, as covered by parts 1.2 of Eurocodes. A real case study is analyzed utilizing the proposed CFD-FEM approach. Obtained results indicate that joined 3D fluid-thermal-mechanical models can be incorporated into the current fire-design analysis, representing a more realistic and economical fire-design verifications.*

### 1 INTRODUCTION

Fire design is used to assure that a structure designed for normal room temperatures can also resist the additional effects induced by fire, for a *fire resistance time*. Simple calculation models are given in some design codes, e.g. parts 1.2 of Eurocodes [1,2]. However, these rules are restricted to a first-order small deflections theory, in which resistance is limited to the plastic or buckling capacity. Therefore, members will have a relatively low survival in fire and would require expensive fire protection. Under fire conditions, large deflections may be tolerated when considerable additional load carrying capacity is designed to avoid structural collapse. In this condition, the structural behavior is mainly governed by the change of geometry, where the behavior of beams will vary from bending to catenary action, and the beneficial tensile membrane action can develop in structural floor slabs. In recent years, a number of researchers have developed and applied complex numerical FE-based modeling to simulate the structural behavior of steel, concrete and composite structures in fire conditions [3,4,5]. On the other hand, dedicated fire CFD (*Computational Fluid Dynamics*) codes have also been widely available (e.g., SMARTFIRE<sup>®</sup> and NIST-FDS<sup>®</sup>) and largely used for fire field modeling, making it more accessible to the fire engineer with limited CFD experience, to simulate the actual fire dynamics and scenarios. CFD model solves numerically the *Navier-Stokes* equations, which are based on the assumption of low speed, thermally-driven flow with a special emphasis on smoke and heat transport from fires [6]. Conversely, very few works are dedicated to the investigation of the development of actual fires in real buildings (works of [7,8] deserve to be specially mentioned). The differences in numerical techniques, spatial and temporal length scales, and the complexity of the computer codes make the development of an efficient coupled analysis of fire-structure interactions not an easy task.

In this context, this paper presents a one-way coupling procedure that has been developed based on the CFD code SMARTFIRE and the FEM code VULCAN, making it possible to perform an accurate global analysis of buildings under fire conditions [9]. The proposed approach for integrating the CFD and solid-phase (thermal and structural response) models are divided in 3-phase steps (1, 2, and 3, respectively denoted as *Fluid*, *Thermo*

and *Mechanical*), which are described in the following sections of this paper. Calculations are considered to be independently, *i.e.*, with a loosely coupling. In this approach, the CFD-fluid results (*phase 1*), *i.e.*, gas temperatures, velocities, chemical species, etc. - within the fluid domain of a building geometry in the presence of a given fire source [10], are used to perform the thermal response (*phase 2*) of exposed members, and finally adopted in the calculation of the associated mechanical response structural elements (*phase 3*). The advantage of the one-way coupling include: (i) computational efficiency - a single CFD simulation to be used for many subsequent structural analysis calculations and (ii) it allows the CFD and structural modeling to be performed independently by experts in their respective disciplines [7]. Two-way coupling should be used in more elaborate treatments where the predicted response of the structure is coupled back to the CFD codes.

## 2 NUMERICAL SIMULATION

### 2.1 CFD Simulation (phase 1)

CFD is the analysis of systems involving fluid flow, heat transfer and associate phenomena such as chemical reactions by means of a computer-based simulation [6]. CFD requires the subdivision of the domain into a number of *smaller*, non-overlapping sub-domains, as shown in Fig.1(a), generating thus a mesh (or grid) of cells (elements or control volumes) covering the whole domain. Furthermore, CFD models calculate changes in each cell by using the fundamental equations of fluid dynamics[6]. They consist generally of a set of three-dimensional, time-dependent and nonlinear partial differential equations expressing the: (i) conservation of mass, (ii) momentum and (iii) energy.

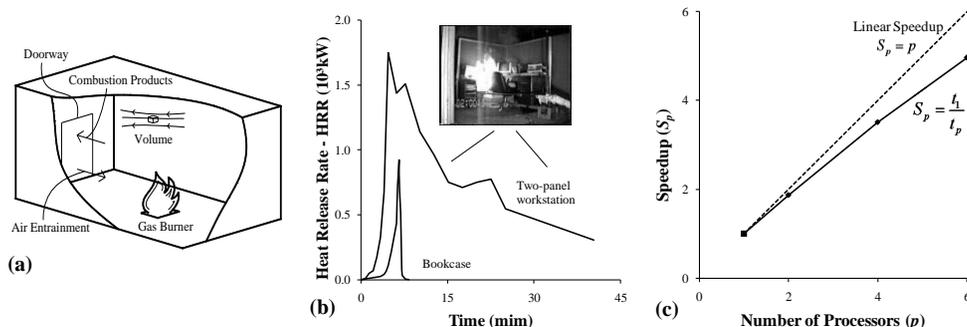


Figure 1: (a) Infinitesimal fluid element approach [6] and (b) HRR curves (Heat Release Rate) used in the present CFD analysis, (c) Speedup ( $S_p$ ) curve of computational job compared to the *linear speedup*.

#### 2.1.1 Heat Release Rate (HRR)

Calculations of fire behavior in buildings are not possible unless the Heat Release Rate (HRR) of the fire is known [11]. This is the rate at which the combustion reactions produce heat. It is thought to be the most important variable in fire hazard, as well as, an essential characteristic that describes quantitatively the fire development [12]. In practice, the HRR is directly obtained through an experimental measuring program, by means of open-burning HRR calorimeters. In the present paper, two main modular furniture units are implemented on the CFD analysis model: (i) a 3-shelf wood *bookcase* with files and (ii) a *two-panel workstation* (office worker cubicles). The corresponding adopted experimental HRR curves for each fire object are shown in Fig.1(b). The bookcase contained about 480 kg of a paper fuel load in shelving units totaling 1.67  $\text{m}^2$  of the floor area [11]. This amount corresponds to a fire of approximately 1 MW lasting about 7 minutes. It should be noted that, in most bookcase storage furniture, the fire hazard is created by the contents, not by the furniture item itself. Office workstations have been tested in several projects at NIST (more details at NIST website: <http://fire.nist.gov/fire/fires/fires.html>). As observed in Fig.1(b), quite severe fire conditions can be generated by this kind of furniture combination. As presented, fires of nearly 1.7 MW were recorded from the burning of a single person's workstation. For the present implementation (Fig.1b) a complete workstation with a combustible mass of about 750 kg is considered.

### 2.1.2 Parallel implementation of CFD codes

The excessive computational resource, required to perform reliable CFD fire simulations, has been considered to be a significant drawback preventing the widespread use of fire-field modeling [13]. In this context, parallel processing has the potential to meet this computational demand at a reasonable processing cost, instead of increasing computational power of single PCs [14]. In the present work, the parallel performance of the CFD-based fire modeling code has been implemented on a homogeneous network (1Gbps) of 6 PENTIUM 4HT (2.4 Ghz, 512Kb, 1Gb RAM, 400 Mhz). This network configuration was tested with a single fire modeling scenario consisting of approximately 67,000 cells. A systematic partitioning of the problem domain into a number of identical sub-domains was carried out. Each sub-domain is computed on a separate processor and runs its own CFD code. At the boundary of the domain partitions each sub-domain needs to communicate with its neighboring sub-domain to exchange necessary data. Fig.1(c) presents the relative speedup of the proposed computational job compared to the speed of a *linear speedup* test (where:  $p$  is the number of processors,  $t_j$  is the execution time of the serial algorithm,  $t_p$  is the execution time of the parallel algorithm with  $p$  processors). As observed (Fig.1c), the increase in speed could dramatically reduce the spent time, or increase the amount of work that can be achieved by the fire engineer. The proposed case required approximately 8h and 46min to run in a single processor, while, this job was completed within 1h 46min with 6 parallel processors. Although, the performed test is related to a simplified case studied [10], the obtained results indicate that the CFD parallel implementation can be assumed to efficiently solve large fire field modeling problems. Moreover, this can be achieved on non-specialized PC equipment which may typically exist in many fire safety engineering offices [13].

## 2.2 FEM Thermal (phase 2)

The distribution of temperature in a structure is of great importance, not only because of the degradation in material properties in heated zones but also for secondary effects caused by thermal elongation. Coupling between fire dynamics and structural analysis in building fires (*i.e.* between *phases 1* and *3*), is largely due to radiative heat transfer from combustion products to structural elements [7]. Based on this assumption, the present thermal analysis approach states that the radiant heat flux  $\phi$  incident upon the surface of the element is related to the local gas temperature  $T$  (*e.g.*, from CFD) by the formula  $\phi = \epsilon r T^4$  ( $\epsilon$  is the emissivity and  $r$  is the Stefan-Boltzmann constant  $5.67 \cdot 10^{-8} \text{ W/m}^2\text{K}^4$ ). Therefore, given a spatially surrounding temperature and a ‘‘time-temperature curve’’ the thermal environment of the enclosure is specified and attention can be directed to calculate the temperature in the structural elements (*phase 2*). Hence, in order to obtain the members’ temperature evolutions as a function of the elapsed fire duration, two approaches were employed: (*i*) a simplified transient temperature equation [2], and (*ii*) a 2D nonlinear heat transfer finite model incorporated in the code SAFIR [15]. For the applications presented in this paper, temperature evolution obtained by both approaches resulted in very close results; however, FEM results were preferred as they seemed to be more accurate.

## 2.3 FEM Structural (phase 3)

The FEM analysis model reported in this paper – to simulate the 3D structural behavior of reinforced concrete slabs and composite steel-concrete beam-column members exposed to fire – was incorporated within the numerical software VULCAN [3], developed at the University of Sheffield (UK). The beam-column element used in this paper [4] is a three-noded line element with each node having six degrees of freedom in local coordinates, as indicated in Fig.2(a). It is assumed (Fig.2a,b) that the nodes of these different types of element (slabs, beam and shear connectors) are defined in a common reference plane coinciding with the mid-surface of the concrete slab element, whose location is fixed throughout the analysis, as given by Fig.2(b) [5]. Moreover, the following assumptions are established [5]: (*i*) the member is straight and prismatic, (*ii*) plane cross-sections remain plane under flexural deformations and there is no slip between different materials, (*iii*) the twist ( $\theta'$ , as given in Fig.2a) of the beam member is relatively small and there is no distortion of the cross-section, (*iv*) in the present simulations, temperature distribution is assumed to be constant over the member cross-section, (*v*) the interaction of steel beams and concrete slabs within a composite floor is represented by linking two-noded shear-connector elements of zero length, with three translational and two rotational degrees of freedom at each node.

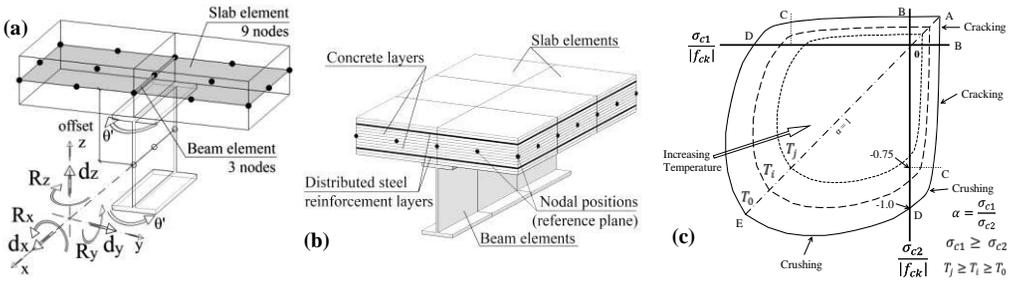


Figure 2: (a) Nodal degrees of freedom in local coordinates for (b) composite beam and slab elements [4,5], (c) concrete failure envelopes at elevated temperatures [3].

In the present simulations, concrete floor slabs are modeled as layers of finite plate elements (Fig.2b). The elements used are the quadrilateral nine-noded higher-order isoparametric elements [16]. The plate elements are subdivided into several layers representing concrete and distributed reinforcing steel as shown in Fig.2(b). The main assumptions of the layered approach can be summarized as follows [3]: (i) the slab elements are considered to consist of plain concrete and reinforcing steel layers - there is no slip between layers, (ii) each layer can have a different but uniform temperature, (iii) the initial material properties of each layer may be different and the stress-strain relationships may change independently for each layer, (iv) the reinforcing steel bars in either of the orthogonal mesh directions are modeled by an equivalent, smeared, steel layer with stiffness only in the direction of the reinforcement, (v) concrete layers are in a state of plane stress and considered to be orthotropic after cracking.

The mechanical properties of materials (concrete and steel, including reinforcement bars) are assumed to be temperature dependent in accordance to EC4-p1.2 [2] definitions, as treated by Fig.3(a,b) for concrete and Fig.3(c) for steel, respectively. The concrete model is assumed to be isotropic, homogeneous and elastic before cracking or crushing occurs [3]. Nevertheless, in order to account for fire design conditions, the compressive  $f_{ck}$  and tensile characteristic strengths  $f_{tk}$  of concrete are modified as a function of the temperature [2] (being denoted respectively as  $f_{ck,T}$  and  $f_{tk,T}$ ). The adopted uniaxial stress-strain relationships for concrete at elevated temperature are given by Fig. 3(a,b), respectively for compressive and tensile strengths. When concrete is subjected to tension, a linear elastic behavior is assumed up to its ultimate tensile capacity ( $f_{tk}$ ). The uniaxial tensile and compressive strengths are considered to be related by  $f_{tk} = 0.3321\sqrt{f_{ck}}$  in MPa. Beyond this point, the tensile stress decreases gradually with increasing tensile strain. As observed in Fig. 3(b), the bilinear curve [17] has been used to model tensile strain softening, where the following parameters have been addressed [18]:  $\varepsilon_{cr} = f_{tk}/E_c$ ,  $\varepsilon_{cu} = \alpha_j \varepsilon_{cr}$ , and  $\alpha_j = 10$  to 25, the value of 15 is adopted herein. The proposed model does not consider spalling, the concrete cross-section being assumed to remain intact. When subjected to compression, the nonlinear stress-strain relationship is applied as recommended by EC4-p1.2 [2]. In the present approach, after crushing concrete is assumed to lose all stiffness [3].

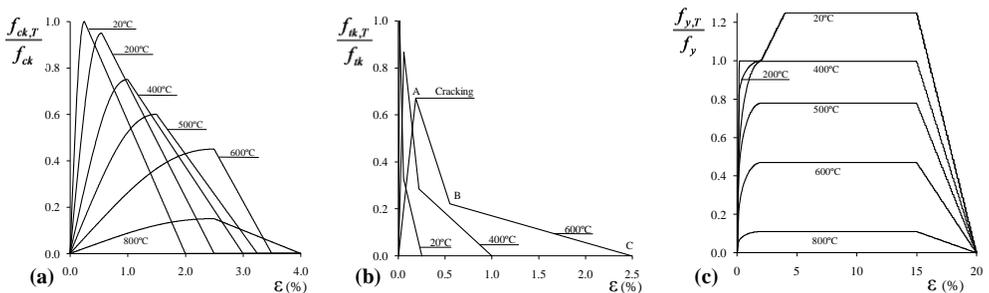


Figure 3: Stress-strain relationships for materials at elevated temperatures [2]: concrete under (a) compression and (b) tension, (c) steel (compression and tension).

The reinforcement is modeled using a modified layered orthotropic slab element, where a perfect bond between steel and concrete layers is assumed [3]. The stress–strain curve of reinforcing steel at elevated temperatures, originally proposed by EC4-p1.2 [2], is considered as given in Fig.3(c). The calculation of effective-stiffness factors is based on the theory of elastic beam bending and is determined by the dimensions of the cross section of the composite slab. The details of this modified layered procedure have been described by Huang *et al.* [3]. The biaxial strength envelope formulation of concrete [18] was adopted in this paper, *i.e.* VULCAN *FE*-procedure, as given by Fig.2(c) [3]. In this approach, compressive and tensile stresses are denoted to be negative and positive, respectively. The principal directions are chosen so that  $|\sigma_{c1}| \geq |\sigma_{c2}|$ , as indicated in Fig.2(c). The initiation of cracking or crushing at any location occurs when the concrete principal stresses reach one of the failure boundaries. After cracking a smeared model is adopted [3] in which a crack at any *Gauss point* is identified either in the biaxial tension region (*segment AB*) or in the combined tension–compression region (*segment BC*), as shown in Fig.2(c). After the initiation of cracking in a single direction, the concrete is treated as an orthotropic material with principal axes, normal and parallel to the crack direction. The concrete parallel to the crack is capable of resisting both tensile and compressive stresses [3].

### 3 NUMERICAL APPLICATIONS

In order to show how a coupled model works, such as the structure of the small office, given by Fig.4, is verified for fire conditions, assuming two distinct temperature evolutions: (i) CFD performed analysis and (ii) standard ISO-834 fire design curve [1]. The general layout of the simulated area is given in Fig.4(a), where the correspondent fire objects are presented in Fig.4(b). These fire objects are represented by the HRR curves, previously given in Fig.1(b).

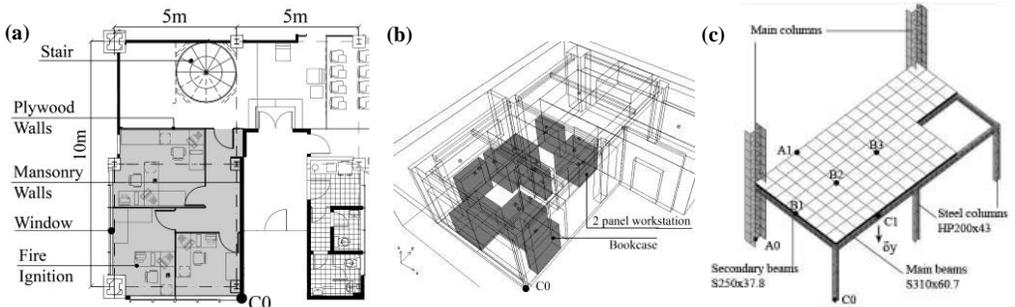


Figure 4: (a) Layout for the fire compartment region and the (b) respective indication of the proposed fire objects for CFD analysis, (c) proposed measure points for CFD gas temperature evaluation.

As indicated in Fig.4(a), the fire compartment measured 10m x 5m in plan and 3.20 m in height and contained standard wooden doors of dimension 0.70 m x 2.10m. The walls of the compartment were also specified in Fig.4(a), made of masonry blocks with a thickness of 0.15m and double 9mm plywood walls, which were considered as combustible material. The initial air temperature was assumed to be 20°C and the fire ignition was postulated on a 2-panel workstation, as pointed out in Fig.4(a). The gas temperatures variation was collected from the CFD model on the several points indicated in Fig.4(c). The structural arrange for the floor is indicated in Fig.4(c), being composed of 5 secondary beams (S250x37.8) and 4 main beams (S310x60.7), where 3 one-way spanning slab are also specified in Fig.4(c). Each slab panel is meshed into 32 shell-elements. It consists of two steel reinforcement layers, representing the main and the secondary reinforcement directions of the slab. The bottom layer meshes with a total reinforcement distributed steel area of  $2.5 \text{ cm}^2/\text{m}^2$  are modeled (a general view of the FEM model will be given in Fig.7). The reinforcement was considered as a 0.25 mm thick smeared steel layer and a 15 mm thick concrete cover. The elastic moduli were assumed to be  $E_s=210 \text{ kN/mm}^2$  and  $E_c=18 \text{ kN/mm}^2$ , respectively for steel and concrete at room

temperature. Material constitutive properties assumed the EC4-p1.2 [2] model as previously indicated in Fig.3, with correspondent compressive strength of  $21 \text{ N/mm}^2$  ( $f_{ck}$ ), and yield strength of  $500 \text{ N/mm}^2$  and  $250 \text{ N/mm}^2$  ( $f_y$ ), related to reinforcement and structural (profiles) steels at room temperature respectively. Due to the original non-composite structural design, a superimposed uniform fire combination loading [1,2] of  $5.77 \text{ kN/m}^2$  is incrementally increased and applied to the slabs, which corresponds to a linear distributed load of  $14.42 \text{ kN/m}$  to be postulated for beams.

The variation of the  $400^\circ\text{C}$  isotherm for different fire times is presented in Fig.5. Accordingly, in a very early stage of the fire development (4.8 min), it is possible to identify a heat exchanging between the compartment and the exterior, due to the breakage of the glass in the left-hand window. After this point, due to the new ventilation condition caused by the opening of the window, the fire dynamics become more severe, and the fire continues to spread over the defined area, consuming most of the proposed furniture (fire objects) and plywood walls. At approximately 5.6 min, the  $400^\circ\text{C}$  isotherm reaches the stair opening, providing additional air exchange, with the upper floor. This fire will develop up to the maximum temperature in about 7 minutes and will continue to consume all possible fuel up to nearly 13 minutes, after which the cooling down phase will take place.

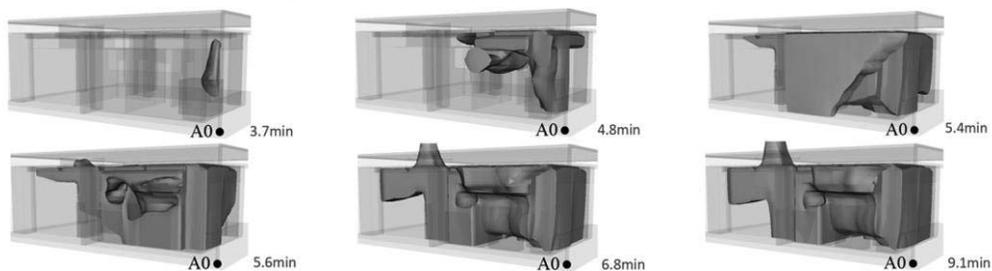


Figure 5:  $400^\circ\text{C}$  isotherm surface on the fire compartment for different fire time.

The members' temperature variations as a function of the fire elapsed time are illustrated in Fig.6. CFD gas temperature results are compared to standard ISO-834 curve [1] in Fig6(c). The mean temperatures for steel member profile (primary and secondary) beams are compared in Fig6(a). The maximum observed temperature reaches nearly  $1,000^\circ\text{C}$ , when the gas temperature is almost equal to that of the steel profile. The temperature variation across the thickness of the solid slab is presented in Fig.6(b), also assuming a ISO-834 fire curve. For the CFD model simulation, the flashover is assumed to occur at about 3 minutes (Fig.6c). After this point, temperatures rapidly increase to reach  $1,000^\circ\text{C}$  at a fire elapsed time of nearly 7 minutes.

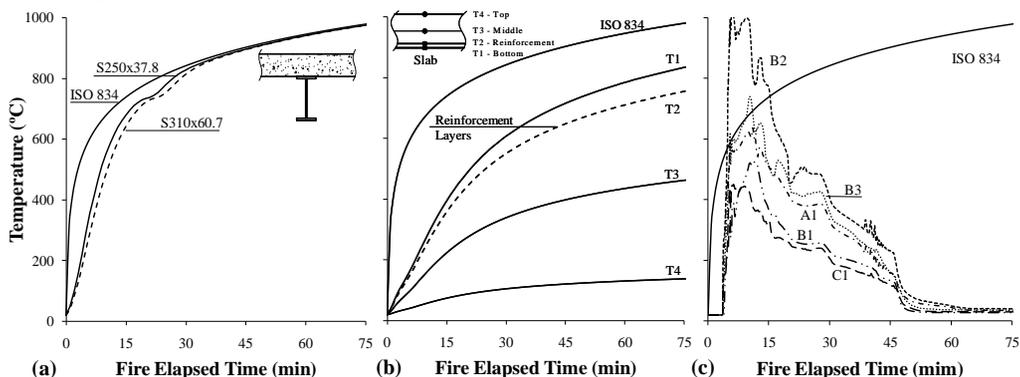


Figure 6: Variation of temperatures as a function of the fire time: (a) mean temperatures of structural elements, (b) idem across the slab element, (c) gas temperature variation (CFD and ISO-834 curves).

Based on the performed gas-temperature variation (Fig.6), the structural behavior the proposed small office are compared as indicated in Fig.7(a,b). A general view of the deformed configuration for CFD Fire conditions and ISO-834 fire curve at 15 minutes of fire elapsed time is given in Fig.7(c). The structural behavior associated to CFD results (Fig.7a) is implicitly assumed to be more realistic than the results given in the ISO-834 curve (Fig.7b). Since the CFD curves presented a limited heat release, the related vertical (gravitational) displacements for distinct points, along the secondary and main beams, are consequently low when compared to the related ISO-834 curve[1]. In this, structural elements are subjected to an initial external loading, which remains constant, followed by a progressive temperature increase. The performed analysis accounts for: (i) the variation of the steel or concrete stress-strain curve, (ii) the thermal strains caused by temperature rise and (iii) the continuous presence of the external fire combination loading. The proposed results also implicitly accounts for the contribution of catenary and the tensile membrane actions, on beams and slabs, respectively. These effects can significantly improve the load-carrying capacity of members subjected to fire. Hence, practical exploration of catenary action in beam may involve optimization of restraint stiffness that would give acceptable lateral deflections yet not place too much burden on the adjacent structure [20].

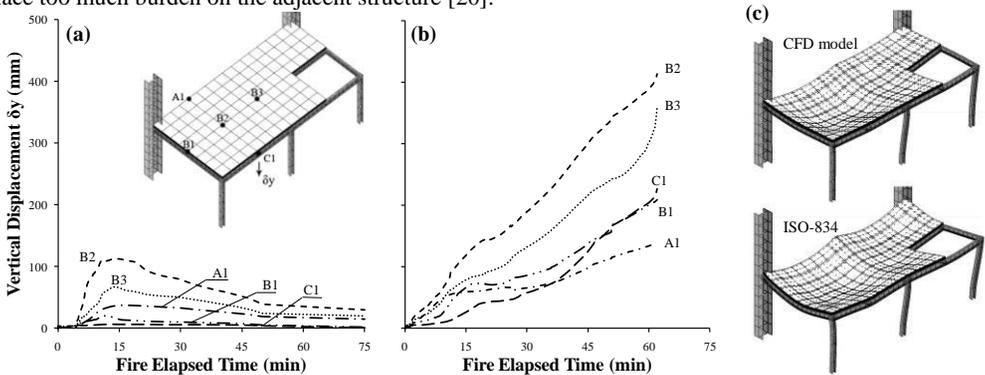


Figure 7: Structural behavior of the proposed small office for different fire conditions. Variation of vertical displacement as a function of the fire elapsed time for (a) CFD and (b) standard ISO-834, (c) general configuration of deformed configuration model for 15 minutes.

#### 4 CONCLUDING REMARKS

A one-way coupling approach was presented in this paper, which can perform numerical simulations, integrating three phases of fire design processes: *Fluid*, *Thermo* and *Mechanical*. Essential attributes for the proposed coupling procedure, as well as, significant characteristics of each step of the integrated analysis were also investigated. With regards to the CFD models, a parallel implementation, allowing efficient solutions of large fire field modeling problems indicating that this procedure can be performed on a regular basis. The member temperature evolution was examined by a heat-transfer *FE*-based model and simplified code recommendations, accounting for thermal-dependent properties of materials. In this verification, temperature results due to radiative heat transfer of combustion products were considered for structural elements. Structural behavior were verified by means of second-order inelastic models, accounting for catenary and tensile membrane actions, representing significant additional structural survival for members exposed to fire. The obtained results indicate that coupled 3D fluid-thermal-mechanical models, similar to those presented herein, can be incorporated into current fire-design analyses, providing a more realistic and economical fire-design verifications.

#### ACKNOWLEDGMENT

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## REFERENCES

- [1] Comité Européen de Normalisation, CEN. *Eurocode 1 - Actions on Structures - Part 1-2: General Actions - Actions on structures exposed to fire*. Brussels, 2002.
- [2] Comité Européen de Normalisation, CEN. *Eurocode 4 - Design of composite steel and concrete structures - Part 1-2: General Rules - Structural Fire Design*. Brussels, 2004.
- [3] Huang, Z., Burgess, I.W. and Plank, R.J., "Modeling Membrane Action of Concrete Slabs in Composite Buildings in Fire. I: Theoretical Development". *Journal of Structural Engineering*. **129**(8), 2003.
- [4] Huang, Z., Burgess, I.W. and Plank, R.J., "3D Modelling of Beam-Columns with General Cross-Section in Fire", *Proceedings of Structures in Fire*, Ottawa, 2004.
- [5] Huang, Z., Burgess, I.W. and Plank, R.J., "Fire resistance of composite floors subject to compartment fires". *Journal of Constructional Steel Research*. **60**(2), 339-360, 2004.
- [6] Yeoh, G.H. and Yuen, K.K., *Computational Fluid Dynamics in Fire Engineering*, Elsevier, 2009.
- [7] Kumar, S., Miles, S., Welch, S., Vassart, O., Zhao, B., Lemaire, A.D. *et al.*, "FIRESTRUC - Integrating advanced three-dimensional modelling methodologies for predicting thermo-mechanical behavior of steel and composite structures subjected to natural fires", *RFS Research Programme of the Research Fund for Coal and Steel*, 2006.
- [8] Prasad, K. and Hamins, A., "Fire Structure Interface and the Thermal Response of the World Trade Center Towers", *Proceedings of Structures in Fire*, Aveiro, 499-509, 2006.
- [9] Silva, J.C. and Landesmann, A., "Application of a 3D Fluid-Thermal-Mechanical Model for Performance-based Analysis of Composite Structures under Fire", *Proceedings of Structural Stability Research Conference*, Orlando, 2010.
- [10] Silva, J.C. and Landesmann A., "Application of CFD model to compartment fire dynamics analysis", *Proceedings of Iberian-Latin-American Congress on Computational Methods in Engineering*, Rio de Janeiro, Brazil, 2009 (in portuguese).
- [11] National Fire Protection Association, NFPA. *Handbook of Fire Protection Engineering*. Quincy, Massachusetts, 2002.
- [12] Babrauskas, V. and Peacock, R.D., "Heat Release Rate: The Single Most Important Variable in Fire Hazard". *Fire Safety Journal*. **18**(3), 255-272, 1992.
- [13] Grandison, A.J., Galea, E.R., Patel, M.K. and Ewer, J., "The Development of Parallel Implementation for a CFD based fire field model utilising conventional office based PCs". *Journal of Applied Fire Science*. **12**(2), 137-157, 2003-2004.
- [14] Hoffmann, N., Berhane, D. and Galea, E.R., "Large-Scale Fire Field Modelling - the Route to General Use via Parallel Processing", *Proceedings of INTERFLAM '93*, Oxford, 1993.
- [15] Franssen, J.M., Kodur, V.K.R. and Mason, J., *User's manual for SAFIR 20001 a computer program for analysis of structures submitted to the fire*, 2000.
- [16] Bathe, K.-J., *Finite element procedures*. Englewood, Prentice-Hall, 1996.
- [17] Rots, J.G., Kuster, G.M.A. and Blaauwendraad, J., "The need for fracture mechanics options in finite element models for concrete structures", *Proceedings of International Conference on Computer Aided Analysis and Design of Concrete Structures*, Pineridge Press, Part 1, 19-32, 1984.
- [18] Barzegar-Jamshidi, F., "Non-linear finite element analysis of reinforced concrete under short term monotonic loading". PhD Thesis. Urbana-Champaign, University of Illinois at Urbana-Champaign, 1987.
- [19] Bailey, C.G. e Toh, W.S., "Behaviour of concrete floor slabs at ambient and elevated temperatures". *Fire Safety Journal* **42**(6-7), 425-436, 2007.
- [20] Yin, Y.Z. and Wang, Y.C., "A numerical study of large deflection behaviour of restrained steel beams at elevated temperatures". *Journal of Constructional Steel Research* **60**(7), 1029-1047, 2004.