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FURTHER RESULTS ON THE APPLICATION OF THE EXTRAPOLATION TECHNIQUES

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***Abstract**. The experimental determination of critical buckling load of structures undergoing lateral buckling has usually been accompanied by the application of certain and just a few deformation characteristics such as lateral displacement and/or twist. This paper explores the possibility of application of various deformation variables such as web transverse and longitudinal strains, vertical deflection, and angles of twist of top and bottom flanges for experimental determination of the critical buckling load of I-beams with different initial geometrical imperfections undergoing elastic lateral-distortional buckling. After demonstrating the linear relationship between lateral displacement and the various aforementioned deformation variables, the four Southwell, Massey, Modified, and Meck extrapolation techniques are applied on these various deformation variables, and consequently satisfactory estimates are acquired for the critical buckling loads.*

1 INTRODUCTION

The extrapolation or plotting techniques are experimental methods developed for determining experimentally the critical buckling load of structures, without having to test them to failure. By plotting the results of a structure test in a certain manner, it would be possible to determine the structure's buckling load.

Southwell [1] initially proposed a plotting method for a concentrically loaded pin-ended column with a sinusoidal initial imperfection. Later on, Massey [2], Trahair [3], and Meck [4] successfully applied this method and variations of it to predict buckling loads for beams.

A search of the literature shows that the extrapolation techniques have mostly been used on certain and just a few deformation characteristics such as lateral displacement and/or twist, and also Mandal and Calladine [5] demonstrated that lateral displacement tends to be proportional to rotation as deformations increase in lateral-torsional buckling mode. In a recent research work reported by the author [6], it was demonstrated that lateral displacement in I-beams undergoing lateral-distortional mode of buckling tends to be directly coupled with the web transverse strains developed as a result of occurrence of web distortion, and accordingly the application of the extrapolation techniques on the web transverse strains yielded good predictions for the critical buckling load.

Based on findings of the previous studies, the possibility of application of various deformation variables for experimental determination of the critical buckling load is explored in this paper. Initially, the relationship between lateral displacement and the various considered deformation variables including web transverse and longitudinal strains, vertical deflection, and angles of twist of top and bottom flanges of I-beams with different initial geometrical imperfections is investigated, and subsequently the applicability of the Southwell, Massey, Modified, and Meck Plots on the aforementioned deformation variables is studied.

Four finite element lateral-distortional buckling solutions are developed using the ABAQUS software system [7]. All of the solutions are of simply-supported steel I-beams subjected to uniform bending moment with identical cross-section dimensions h_w=900 mm, b_f=240 mm, t_w=5 mm, t_f=20 mm, and L=7000 mm. The material properties adopted for the beams are E=200 GPa, ν=0.3, F_y=345 MPa, and G=0.385E. The beam cross-section components, *i.e.* flanges and the web, were modeled using a four-node shell element S4R5. Finally, the details of various geometrical imperfection forms of the beams are provided in Table 1.

Table 1: Details of various geometrical imperfection forms of the beams

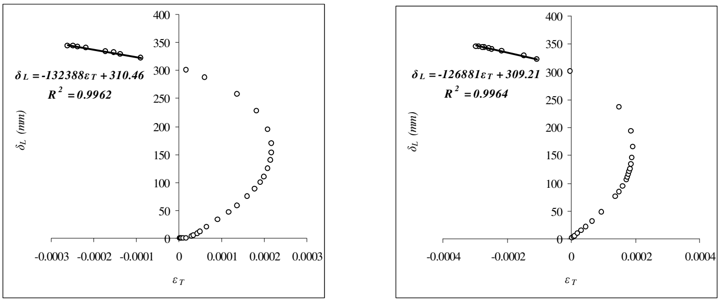
Beam number	Initial imperfection form	Initial imperfections at center of beam	
		Crookedness (mm)	Twist (rad)
1	-	-	-
2	*Half-sine wave along the length*	-	*0.04363*
3	*Half-sine wave along the length*	*20 (lateral displacement)* *20 (vertical displacement)*	-
4	*Half-sine wave along the length*	*20 (lateral displacement)* *20 (vertical displacement)*	*0.04363*

2 RELATIONSHIP BETWEEN LATERAL DISPLACEMENT AND OTHER DEFORMATION VARIABLES

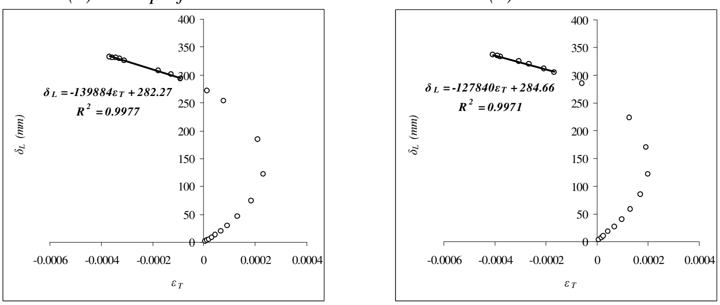
Before applying the various deformation variables for experimental determination of the critical buckling load, the relationship between lateral displacement and the considered deformation variables is explored. It should be noted that the lateral displacement of the top flange is applied in this study.

Lateral displacement vs. web transverse strain: In this case, the relationship between lateral displacement and web transverse strain representing web distortion and measured at midspan and midheight is investigated. The acquired data are plotted straightforwardly as lateral displacement against web transverse strain, as shown in Figure 1. The linear equations obtained using the least squares method and the respective R-squared values are displayed in the figures. As seen in the figures, it is evident that after the initial stages of loading the two deformation characteristics become proportional to each other.

Lateral displacement vs. web longitudinal strain: In addition to the web transverse strain, the relationship between lateral displacement and web longitudinal strain is investigated as well. As seen in Figure 2, lateral displacement is plotted against web longitudinal strain (measured at midspan and midheight), and the obtained linear equations as well as the R-squared values are displayed on the chart. It is clearly observed that after the initial loading stages, the data points align with the linear portion near the latter loading stages and the direct coupling of the two deformation characteristics becomes evident.

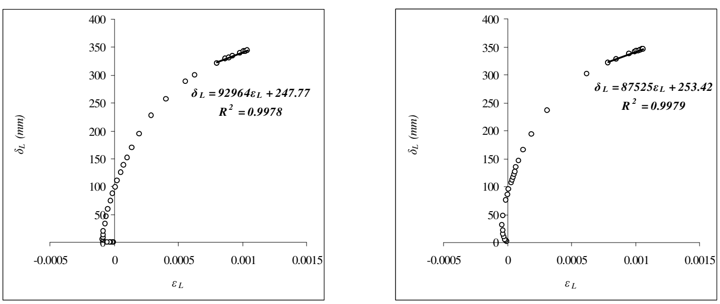


(a) No imperfection *(b) Initial twist*

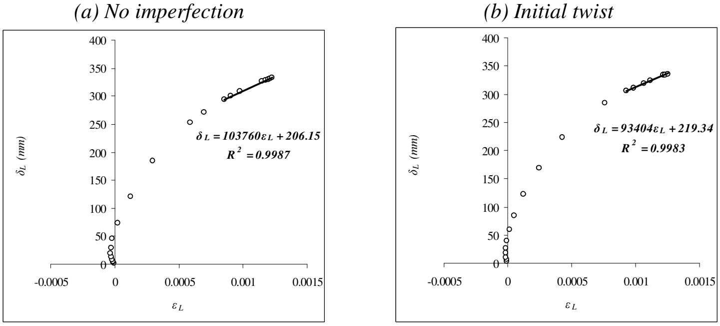


(c) Initial crookedness *(d) Initial twist and crookedness*

Figure 1: Plot of lateral displacement against web transverse strain



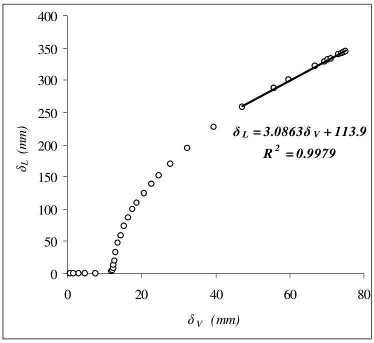
(a) No imperfection *(b) Initial twist*



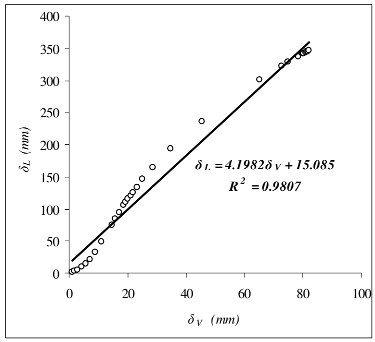
(c) Initial crookedness *(d) Initial twist and crookedness*

Figure 2: Plot of lateral displacement against web longitudinal strain

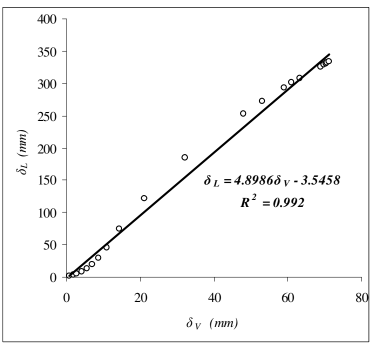
Lateral displacement vs. vertical deflection: The proportionality between the lateral displacement and vertical or in-plane deflection at midspan and midheight of the analyzed I-beams undergoing lateral-distortional buckling is investigated in here. The plots of lateral displacement against vertical deflection are shown in Figure 3. The linear equations and R-squared values are displayed in the figures as well. To a fair approximation, lateral displacement and vertical deflection seem to be proportional to each other and the linearity range is comparatively large in this case.



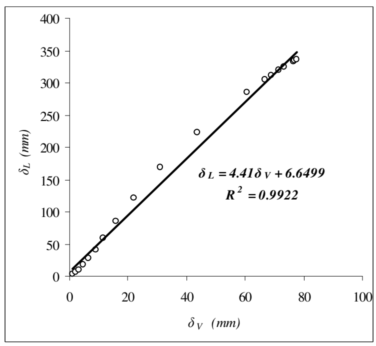
(a) No imperfection



(b) Initial twist



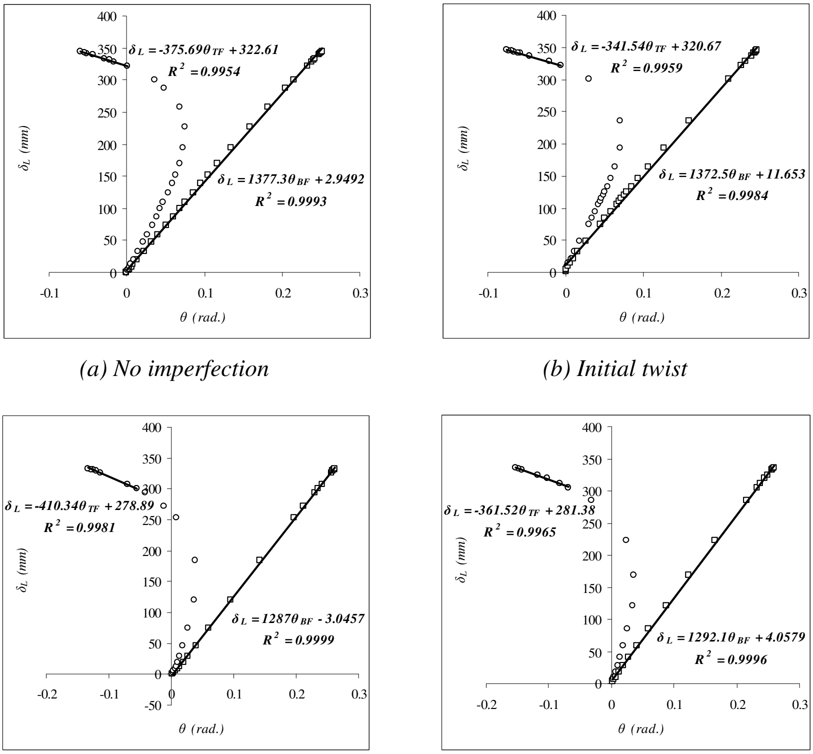
(c) Initial crookedness



(d) Initial twist and crookedness

Figure 3: Plot of lateral displacement against vertical deflection

Lateral displacement vs. angle of twist: Lastly, the relationship between the lateral displacement and angles of twist of top and bottom flanges of the I-beams is examined. Unlike the lateral-torsional mode of buckling, in lateral-distortional buckling mode top and bottom flanges have different angles of twist, hence the two angles of twist are taken into consideration in this study. Plots of lateral displacement against angles of twist of top and bottom flanges are made and shown in Figure 4. It is evident that lateral displacement and angles of twist of the two flanges are directly coupled.



(a) No imperfection *(b) Initial twist*

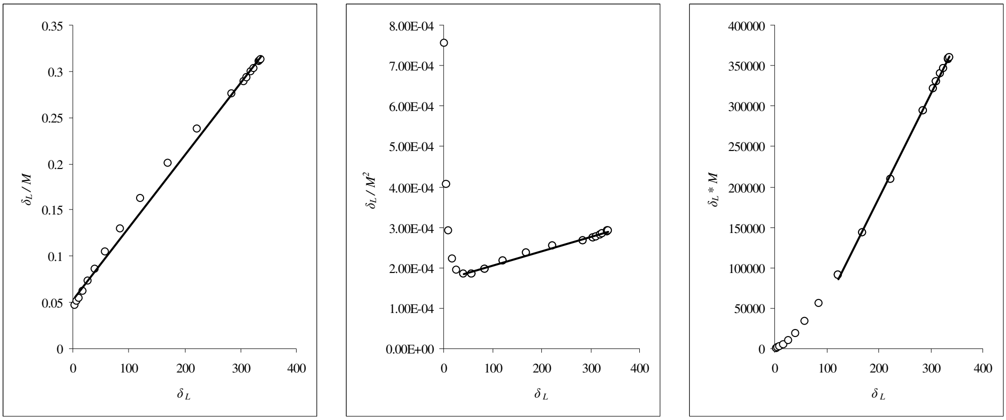
(c) Initial crookedness *(d) Initial twist and crookedness*

Figure 4: Plot of lateral displacement against angle of twist

3 APPLICATION

Based on the linear relationship between lateral displacement and the other considered deformation variables, it seems logical to obtain straight lines by applying the extrapolation techniques on the various deformation variables, and also acquire favorable predictions for the buckling load. To prove this, the following five case studies are performed.

Lateral displacement: In this case, Southwell, Massey, and Modified Plots are applied on the lateral displacement of the top flange at midspan. As an example, the Southwell, Massey, and Modified Plots for beam 4 are shown in Figure 5. The extrapolated-to-ultimate failure moment ratios for the three methods are also given in Table 2.



(a) Southwell Plot *(b) Massey Plot* *(c) Modified Plot*

Figure 5: Southwell, Massey, and Modified Plots (beam 4)

Table 2: Comparison of the ultimate failure moments with the extrapolated moments for the case of lateral displacement

Beam number	$M_{Southwell}/M_u$	M_{Massey}/M_u	$M_{Modified}/M_u$
1	*1.038*	*0.992*	*1.025*
2	*1.069*	*1.352*	*1.051*
3	*1.137*	*1.174*	*1.189*
4	*1.165*	*1.474*	*1.232*
Average Discrepancy (%)	*9.07*	*18.44*	*10.50*

In spite of some scatter in the results, it can be seen from the table that the extrapolated moments are generally in good agreement with the ultimate failure moments. Moreover, as it is seen, the lowest average discrepancy is found in the case of the Southwell Plot.

Web transverse strain: The four Southwell, Massey, Modified, and Meck Plot methods are applied on the web transverse strains captured at the mid-height and midspan of the analyzed beams. The extrapolated-to-ultimate failure moment ratios for the plotting methods are given in Table 3.

Table 3: Comparison of the ultimate failure moments with the extrapolated moments for the case of web transverse strain

Beam number	$M_{Southwell}/M_u$	M_{Massey}/M_u	$M_{Modified}/M_u$	M_{Meck}/M_u [a]
1	*1.038*	*0.992*	*0.995*	*1.040*
2	*0.950*	*1.022*	*1.003*	*1.033*
3	*1.010*	*1.017*	*1.002*	*1.009*
4	*1.036*	*1.042*	*1.008*	*0.992*
Average Discrepancy (%)	*3.29*	*2.15*	*0.45*	*2.18*

[a] *M_{Meck} is obtained as a result of the use of Meck Plot method on lateral displacement and web transverse strain.*

As it is seen in Table 3, some scatter in the results is present in this case as well. In general, the agreement between the extrapolated and the ultimate failure moments is satisfactory in all cases. Lastly, in this case, the lowest average discrepancy is found in the case of the Modified Plot.

Web longitudinal strain: In this case, the use of Southwell, Massey, Modified, and Meck Plots on the web longitudinal strains captured at the mid-height and midspan of the considered I-beams, is investigated. The extrapolated-to-ultimate failure moment ratios for the four considered Plot methods are presented in Table 4.

Table 4: Comparison of the ultimate failure moments with the extrapolated moments for the case of web longitudinal strain

Beam number	$M_{Southwell}/M_u$	M_{Massey}/M_u	$M_{Modified}/M_u$	M_{Meck}/M_u [a]
1	*1.038*	*0.992*	*1.044*	*1.014*
2	*1.069*	*1.022*	*1.051*	*1.092*
3	*1.010*	*1.087*	*1.044*	*1.018*
4	*1.036*	*1.114*	*1.059*	*1.091*
Average Discrepancy (%)	*3.64*	*5.28*	*4.73*	*4.96*

[a] *M_{Meck} is obtained as a result of the use of Meck Plot method on lateral displacement and web longitudinal strain.*

As can be seen from the results in Table 4, despite some scatter, the agreement between the extrapolated and the ultimate failure moments is satisfactory. Furthermore, in this case, the lowest average discrepancy is found in the case of the Southwell Plot.

Vertical deflection: The applicability of the extrapolation techniques on the beam midspan vertical deflection is studied in this case. The extrapolated-to-ultimate failure moment ratios for the four applied plotting methods are given in Table 5.

Table 5: Comparison of the ultimate failure moments with the extrapolated moments for the case of vertical deflection

Beam number	$M_{Southwell}/M_u$	M_{Massey}/M_u	$M_{Modified}/M_u$	M_{Meck}/M_u [a]
1	*1.038*	*1.072*	*1.025*	*1.022*
2	*1.069*	*1.209*	*1.040*	*1.104*
3	*1.137*	*1.174*	*1.126*	*1.010*
4	*1.165*	*1.318*	*1.154*	*1.077*
Average Discrepancy (%)	*9.07*	*15.74*	*7.72*	*4.92*

[a] *M_{Meck} is obtained as a result of the use of Meck Plot method on lateral displacement and vertical deflection.*

In spite of the scatter in the results, it may be concluded that the extrapolated and the ultimate failure moments are generally in good agreement in all cases. Also, the lowest average discrepancy in the present case study is interestingly found in the case of the *modified* Meck Plot.

Angle of twist: In the last case study, the four considered extrapolation techniques are applied on the angles of twist of both top and bottom flanges captured at midspan of the analyzed beams. The extrapolated-to-ultimate failure moment ratios for the Southwell, Massey, Modified, and Meck Plot methods are presented in Table 6.

Table 6: Comparison of the ultimate failure moments with the extrapolated moments for the case of angle of twist

Beam number	Location[a]	$M_{Southwell}/M_u$	M_{Massey}/M_u	$M_{Modified}/M_u$	M_{Meck}/M_u [b]
1	*TF*	*1.038*	*0.992*	*1.001*	*0.948*
	BF	*1.038*	*0.992*	*1.022*	*1.007*
2	*TF*	*0.950*	*1.022*	*1.002*	*1.037*
	BF	*1.069*	*1.209*	*1.042*	*1.046*
3	*TF*	*1.010*	*1.017*	*1.007*	*0.956*
	BF	*1.137*	*1.286*	*1.181*	*1.043*
4	*TF*	*1.036*	*1.042*	*1.011*	*1.025*
	BF	*1.165*	*1.474*	*1.209*	*1.066*
Average Discrepancy (%)	*TF*	*3.29*	*2.15*	*0.50*	*3.92*
	BF	*9.07*	*18.11*	*9.72*	*3.86*

[a] *TF and BF stand for "top flange" and "bottom flange", respectively.*
[b] M_{Meck} *is obtained as a result of the use of Meck Plot method on lateral displacement and angle of twist.*

As it is seen in Table 6, despite some scatter in the results, the agreement between the extrapolated and the ultimate failure moments is generally satisfactory in both cases. Finally, in this case study, the lowest average discrepancies for the cases of angles of twist of top and bottom flanges are found in the cases of Modified and Meck Plots, respectively.

4 CONCLUSION

Based on the key findings regarding the proportionality between lateral displacement and other considered deformation variables including web transverse and longitudinal strains, vertical deflection, and angles of twist of top and bottom flanges of the I-beams undergoing elastic lateral-distortional buckling, the applicability of the Southwell, Massey, Modified, and Meck extrapolation techniques on the various deformation variables was investigated in this paper and generally satisfactory and reliable results were obtained. The results of this study may be considered as an indication of a great extension in the application of the extrapolation techniques.

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