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INELASTIC BEHAVIOUR OF PARTIALLY RESTRAINED STEEL FRAMES

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***Abstract**. The behaviour of beam-column connections for conventional analysis of a structure is simplified to the two idealized extremes of either rigid-joint or pinned-joint behaviour. However most of the connections used in steel frames actually exhibits semi-rigid deformation which influences the global behaviour of structures. This paper presents the development of a finite element for use in second-order inelastic analysis of partially and fully restrained planar steel frames. The finite element considers the spread of plasticity within the cross section and along the member length, several residual stresses distributions, shear deformation of members through the Timoshenko theory and P-δ and P-Δ effects. Nonlinear spring elements are used to include connections. The behaviour of connections is modelled using multilinearized moment-rotation curves. A computer program associated with the finite element model is developed for Advanced Analysis of planar steel frames. Numerical examples are presented.*

1 INTRODUCTION

Conventional analyses of steel frame structures are usually carried out under the assumption that the beam-column connections are either fully rigid or ideally pinned. However, most of the connections used in current practice are semi-rigid type whose behaviour lies between these two extreme cases. The predicted response of the idealized structure may be quite unrealistic compared to that of the actual structure if connection stiffness is ignored in the analysis and design procedures.

The semi-rigid connections have important function in structural steel design, because influence substantially the moment distribution in beams and columns and negatively affect the stability of the frame, since they increase the drift of the frame and cause a decrease in effective stiffness of the member. So, the disregard of the actual behaviour of the connections can lead to unrealistic predictions of response and resistance of structures.

The important attributes that affect the behaviour of semi-rigid steel frames structures are connection, geometric and material nonlinearities. The connection nonlinearity is given by the nonlinear moment-rotation relationship of semi-rigid connections. The geometric nonlinearity includes second-order effects associated with the P-δ and P-Δ effects and geometric imperfections. And finally, material nonlinearity includes spread of yielding or plasticity associated with the influence of residual stresses. The realistic modelling of a steel frame requires the use of these attributes if an accurate response is to be obtained.

One way to account for all these effects in semi-rigid frame design is through the use of an advanced analysis. Advanced Analysis is a method that can sufficiently capture the limit state of strength and stability of a structural system and its individual members, so that separate checks of the capacity of members are not required. With technological advances in computational area has been possible to employ advanced analysis techniques directly in the offices of engineering design.

During the past 20 years, researches efforts have been devoted to the development and validation of several nonlinear inelastic analysis methods for steel frames with semi-rigid connections, as the studies presented by [1]-[9]. The behaviour of semi-rigid connections has been progressively incorporated in structural analyses, resulting in more realistic analysis of the response global of structures, allowing a design accurate and certainly more economical.

This paper presents the development of a finite element for use in the second-order inelastic analysis of partially (PR) and fully (FR) restrained planar steel frames. The finite element considers the spread of plasticity within the cross section and along the member length, several residual stresses distributions, shear deformation of members, P-δ and P-Δ effects. Nonlinear spring elements are used to include partially restrained connections. The behaviour of the connections is modelled using multilinearized moment-rotation curves. The formulation considering Timoshenko theory and self equilibrated residual stresses is based on updated Lagrangian formulation. The Corotacional technique is used to obtain the element's tangent stiffness matrix. A computer program associated with the finite element model is developed. Numerical examples are presented and the results are compared with those previously published by others researchers with the objective to validate the finite element model for the Advanced Inelastic Analysis.

2 BEHAVIOUR OF THE NONLINEAR CONNECTIONS

The knowledge of connections behavior between structural elements is essential for the analysis and design of a structure. Efforts transmitted through the beam-column connections consist of axial force, shear force, bending moment and torsion. The effect of axial and shear forces can be negligible when their deformations are small compared to the rotational deformation of connections. The effect of torsion is excluded of in-plane study. So in this work, only the effect of bending moment in the rotational deformation of the connections will be considered.

The moment-rotation relationship, M-θ_r, depends on the connection type. The rotational deformation is expressed as a function of the moment in the connection. The angle θ_r corresponds to the relative rotation between beam and column at the connection.

Most experiments have shown that the curve M-θ_r is nonlinear in the whole domain and for all connections types. May be observed that a flexible connection has a smaller ultimate moment capacity and a larger rotation, and vice versa for a rigid connection. The behaviour of a simple connection is represented by θ_r-axis with M=0 and the behaviour of a fully-rigid connection is represented by the M-axis with θ_r=0. All semi-rigid connections are represented by curves lying between these two extremes, allowing some moment to be transferred and some rotation to occur in a connection.

Experimental works on connections have been performed, and a large body of moment-rotation data has been collected, as researches of [10]-[11]. Using these databases, researchers have developed several connection models. The main are: linear; bilinear, trilinear, multilinear; polynomial; b-spline; three-parameter power and exponential models.

The multilinear model is proposed in this work to represent moment-rotation curves of partially restrained connections. This model is simple and able to describe the M-θ_r curve with higher precision than the bi and trilinear models. The values of the pair bending moment and rotation are inserted directly as input in the program and the stiffness values for each segment are automatically calculated for a given connection. Unloading and reloading of the connection are assumed to follow the initial stiffness. A representation with five linear segments of the moment-rotation curve is shown in figure 1.

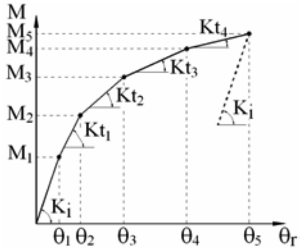

Figure 1: Multilinear moment-rotation curve for partially restrained connections

3 FINITE ELEMENT MODEL

This paper outlines the development of a finite element for use in the second-order inelastic analysis of partially (PR) and fully (FR) restrained planar steel frames. This finite element is shown in figure 2. The structural nodes have three degrees of freedom, namely, the displacements **u** and **v** along the **x** and **y** axes, respectively, and the rotation $\boldsymbol{\theta}$, positive when counter-clockwise. In the reference configuration, the chord between elements nodes has the length $\mathbf{l_r}$. On the chord a local reference coordinate system (x_r,y_r) is placed, with the origin at the center. The angle between the axis **x** and the chord is denoted by $\boldsymbol{\phi_r}$. At current configuration the chord between element nodes has length $\mathbf{l_c}$. A corotational coordinate system (x_c,y_c) is defined on this chord, with the origin at the center, as indicated in the figure 2. The angle between the axis x and the chord is now $\boldsymbol{\phi_c}$ while the angle between the chord and the axis of the bar is denoted by $\boldsymbol{\alpha}$.

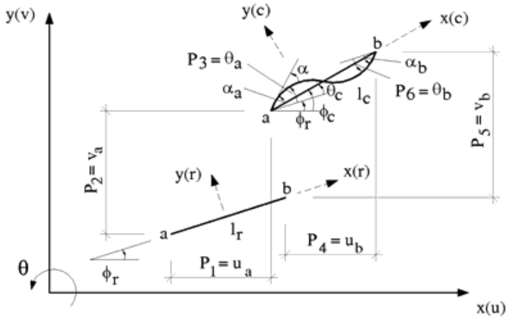

Figure 2: Finite element deformation

The natural and Cartesian degrees of freedom of the element are defined, respectively, by:

$$q_\alpha^T = \{q_1 = l_c - l_r \,;\, q_2 = \alpha_a \,;\, q_3 = \alpha_b\};\;\; p = \{u_a ; v_a ; \theta_a ; u_b ; v_b ; \theta_b\} \tag{1}$$

The relations between natural and Cartesian degrees of freedom are important and listed below:

$$\begin{cases} q_1 = l_c - l_r \\ q_2 = \alpha_a = \theta_a - \theta_c = p_3 - \varphi_c + \varphi_r \\ q_3 = \alpha_b = \theta_b - \theta_c = p_6 - \varphi_c + \varphi_r \end{cases} \tag{2}$$

Longitudinal and shear deformations are, respectively:

$$\varepsilon_x = \frac{du}{dx} = \frac{d\overline{u}}{dx} - y_r \frac{d\theta}{dx} = \overline{\varepsilon}_x - y_r\, \alpha' \tag{3}$$

$$\gamma_{xy} = \frac{du}{dy} + \frac{dv}{dx} = -\theta + \alpha = -\gamma \tag{4}$$

The virtual power theorem is used in the development of the finite-element stiffness:

$$\int_{V_r} \sigma\;\, \delta\varepsilon\;\, dV_r + \int_{V_r} \tau\;\, \delta\gamma\;\, dV_r = P_i\, \delta p_i \tag{5}$$

where dV_r is the volume element in the reference configuration, σ the normal stress, τ the shear stress, δε virtual longitudinal deformation and δγ virtual distortion of a fiber.

The virtual longitudinal deformation and virtual distortion are respectively:

$$\delta\varepsilon = \varepsilon_{,\alpha}\;\, q_{\alpha,i}\delta p_i\,;\;\; \delta\gamma = \gamma_{,\alpha}\;\, q_{\alpha,i}\delta p_i \tag{6}$$

Therefore, the equilibrium equation of the element is given by:

$$P_i = \left(\int_{V_r} \sigma\, \varepsilon_{,\alpha}\, dV_r + \int_{V_r} \tau\, \gamma_{,\alpha}\, dV_r \right) q_{\alpha,i} = Q_\alpha q_{\alpha,i} \tag{7}$$

Considering an incremental formulation of equilibrium, differentiation of P at time can be given by:

$$\frac{dP}{dt} = \frac{\partial P}{\partial p}\frac{dp}{dt} = \mathbf{k_t}\frac{dp}{dt} \tag{8}$$

where, k_t is the tangential stiffness matrix of element in Cartesian coordinates. The components k_{ij} are obtained through differentiation of P_i with respect to Cartesian coordinate's p_j:

$$\frac{\partial P_i}{\partial p_i} = k_{ij} = q_{\alpha,i}\, Q_{\alpha,\beta}\, q_{\beta,j} + Q_\alpha\, q_{\alpha,ij} \tag{9}$$

$$Q_{\alpha,\beta} = \int_{V_r} \left(\varepsilon_{,\alpha} \frac{d\sigma}{d\varepsilon} \varepsilon_{,\beta} + \sigma\varepsilon_{,\alpha\beta} + \gamma_{,\alpha} \frac{d\tau}{d\gamma} \gamma_{,\beta} + \tau\, \gamma_{,\alpha\beta} \right) dV_r \tag{10}$$

The tangent stiffness matrix is given by:

$$k_{ij} = q_{\alpha,i} \left(\int_{V_r} \left(\varepsilon_{,\alpha} \frac{d\sigma}{d\varepsilon} \varepsilon_{,\beta} + \gamma_{,\alpha} \frac{d\tau}{d\gamma} \gamma_{,\beta} \right) dV_r \right) q_{\beta,j} + q_{\alpha,i} \left(\int_{V_r} \left(\sigma\, \varepsilon_{,\alpha\beta} + \tau\, \gamma_{,\alpha\beta} \right) dV_r \right) q_{\beta,j} + Q_\alpha q_{\alpha,ij} \tag{11}$$

The first term of the equation represents the constitutive part, the second and third parts represent the P-δ and P-Δ effects, respectively.

4 IMPLEMENTATION WITH RESTRAINED CONNECTION

The matrices obtained from the formulation were implemented in the program developed by [12]. The program, written in FORTRAN 90, employs Newton-Raphson method and the displacement control to obtain the nonlinear equilibrium path and to allow the correct determination of the collapse load.

The program considers P-δ and P-Δ effects, partially restrained connections, shear deformations through the Timoshenko theory and spread of plasticity. The frame element, made up of layers, enables to identify the plastic region through the cross section and along the member length and to consider any kind of residual stresses distribution.

The connection behaviour is characterized by moment-rotation curve. Nonlinear spring elements are used for an approximation of the actual connection behaviour. The spring elements have three degrees of freedom, namely, the displacements u and v along the x and y axes, respectively, and the rotation θ, positive when counter-clockwise. Stiffness is given in terms of relative displacements. In this study, the rotational stiffness K_θ is obtained by linearized curves of different types of connections available in the literature. Unloading and reloading can be considered in any segment of the curve.

5 NUMERICAL EXAMPLES

5.1 Portal Frame

This example aims to study the loading and unloading behaviour of the connections of a portal frame, when it's subjected to a lateral force after the total vertical loading to be applied in the structure, as shown in figure 3. The uniformly distributed load is modelled as a set of equivalent nodal loads. The frame is analyzed in second-order elastic theory. Beam and columns are made of profiles I. The area and the moment of inertia of the beam are, respectively, equals to 43 cm^2 and 2770 cm^4. The area and the moment of inertia of columns are equals to 33,4 cm^2 and 1510 cm^4, respectively. The lengths and numbering of bars are also shown in figure 3. The Young's modulus is 21000 kN/cm². Beam and columns were modelled, respectively, with ten and four finite elements and the cross sections were divided into twenty layers.

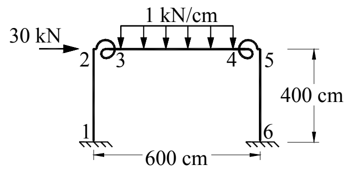

Figure 3: Portal frame with semi-rigid connections

The top and seat angle with double web angle connections, C23 and C45, are identical at the ends of the beam. The behaviour and data of the connections are presented by [11], considering the three-parameter model. At this paper, the behaviour of the semi-rigid connections is represented by multilinear curves, according figure 1, with data shown in table 1.

Table 1: Parameters connection

Segments (i)	1	2	3	4	5
M_i (kNcm)	2316	4632	6176	6948	7566
θ_{ri} (rad)	0,00097	0,00366	0,01046	0,02510	0,15745

Figure 4 shows the graphic of moment versus relative rotation for the windward connection (C23) and leeward connection (C45), for all loading increments. It is observed that, when the frame is subjected firstly to uniformly distributed load, the connections presented same behaviour. The bending moment is equal to 7245 kNcm and the relative rotation is equal to 0,08880 rad, for total vertical loading. When lateral load is applied, the windward connection (C23) unloads, showing a linear behaviour with slope equal to the initial stiffness and the leeward connection (C45) continues load, ie, continues to rotate in the same direction, with slope based on tangent stiffness, as shown in Figure 4. For the structure completely loaded, the connection C45 presents moment equal to 7437 kNcm and relative rotation equal to 0,12982 rad. The connection C23, due to the unloading caused by the lateral force, presents moment equal to 2395 kNcm and relative rotation of 0,08676 rad. It can be concluded that, the loading characteristics are very different from the unloading characteristics of the connections. The connection behaviour is very much affected by the history and direction of the loads applied sequentially.

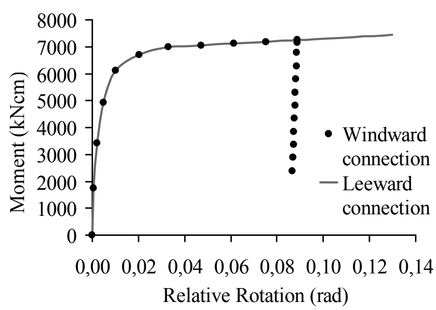

Figure 4: Behaviour of partially restrained connection under loading and unloading

5.2 Ten-story and two bay frame

Consider the ten-story and two-bay frame with endplate connections shown in figure 5. The frame has been analyzed and designed previously by [13], for loads and dimensions shown in figure. The nodes of structural model have the same semi-rigid connection type. The frame is investigated to demonstrate the effect of semi-rigid connections on structural response up to failure. The second-order inelastic analysis, by the plastic-zone method is performed. Two cases are analyzed: semi-rigid and fully-rigid frames.

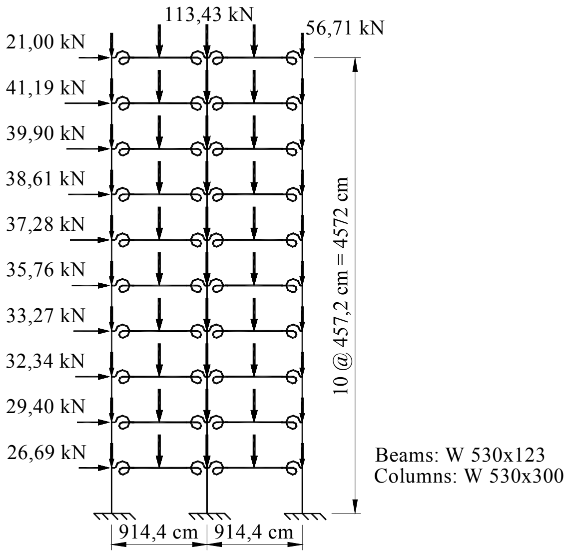

Figure 5: Ten-story and two-bay frame with endplate connections

The vertical and lateral loadings were considered incrementally in the numerical analysis until the failure. The yielding strength and Young's modulus of the steel are assumed equals to 25 kN/cm^2 and 20000 kN/cm^2, respectively, in the elastic-perfectly plastic behaviour. The initial geometric imperfections and the residual stresses are not considered in the analysis. Beams and columns were modelled with four finite elements in the structural model and the cross sections were divided into twenty layers. Three endplate connections types are considered, whose parameters can be found in [13]. At this paper, the connections behaviour is represented by multilinear curves with data shown in table 2, according figure 1.

Table 2: Parameters connections

	C1 Connection		C2 Connection		C3 Connection	
Segments	M (kNcm)	θ_r (rad)	M (kNcm)	θ_r (rad)	M (kNcm)	θ_r (rad)
1	19108	0,00092	24996	0,00085	36635	0,00070
2	57325	0,00703	49991	0,00284	73269	0,00245
3	76433	0,01490	74987	0,00688	109904	0,00613
4	95542	0,03083	99982	0,01496	146538	0,01375
5	107771	0,04979	124978	0,03177	183173	0,03015

The figure 6 shows the load-deflection behaviour of the frame with endplate connections, with different rigidity and moment capacity, namely C1, C2 and C3 connections, until the failure. In graphics of this figure, the abscissa axis represents the maximum lateral sway of the top of the frame and the ordinate axis denotes the load level. It is observed that, the load originally proposed, according figure 5, was gradually expanded until the strain of steel reached the limit of $21\varepsilon_y$. Results of the load-deflection behaviour, obtained by the developed program, are compared with results of [13], obtained by a computer program able of performing a second-order inelastic analysis of planar steel structures based on the refined plastic hinge method. It can be noted that the results of the program, developed based on the plastic-zone method, showed a good correlation with the results obtained by [13]. A difference less than 5% in the ultimate load between the analyses was obtained for all structural models.

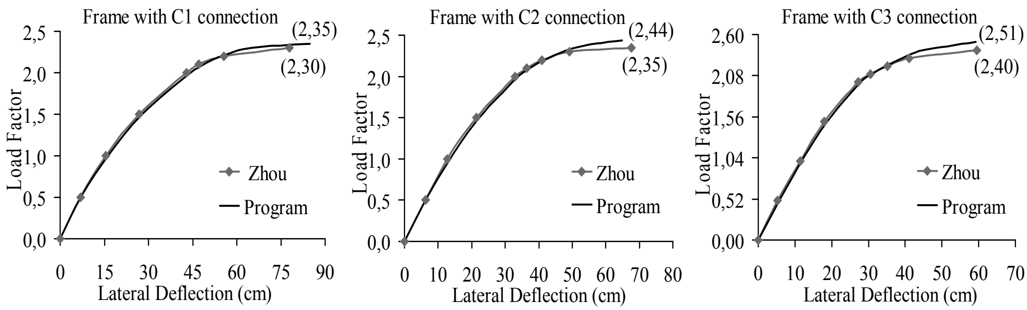

Figure 6: Load-displacement behaviour at the top of the frame

Figure 7 shows, comparatively, the load-deflection behaviour at the top of frame, obtained by the program developed, considering conventional rigid connections and the C1, C2 and C3 connections. The results show that the frame with the C1 connection has larger deflection, resulting in the more flexible structure between models analyzed. The frame with the C3 connection presents deflection and load factor values very close to the conventional model with perfectly rigid connections. It can be concluded that the properties of connections have significant influence on the strength, stiffness, and ductility of the frame.

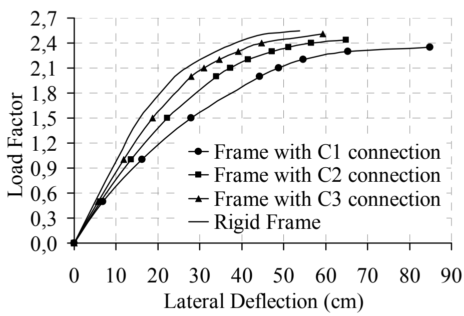

Figure 7: Load-displacement behaviour at the top of frame obtained by the program developed

It can be noted that, when connections in a frame become stiffer, the response of the semi-rigid frame get close to the rigid frame. So, endplate connections can be regarded semi-rigid or rigid depending on their rigidity. A frame with endplate connections can be regarded as a rigid frame if its connections are rigid enough.

6 CONCLUSION

A computer program for Advanced Inelastic Analysis of partially (PR) and fully (FR) restrained planar steel frames, considering the geometric, material and connections nonlinearities, is developed. The finite element considers P-δ and P-Δ effects, shear deformations of members through the Timoshenko theory and spread of plasticity. The frame element, made up of layers, enables to identify the plastic region through the cross section and along member length and to consider any kind of residual stresses distribution.

The method of advanced analysis showed to be very efficient in the analysis of the behaviour of steel structures involving semi-rigid connections from the initial stage of loading until the final stage of collapse. The results indicate that semi-rigid connections in steel structures have fundamental importance, since greatly affect the behaviour of the structure. The developed method considering the nonlinear behaviour of the connections through M-θ_r multilinear curve showed to be suitable for these analyses.

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