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LATERAL TORSIONAL BUCKLING OF SPACE STRUCTURES WITH I-BEAMS - STRUCTURAL BEHAVIOR AND CALCULATION

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Abstract. *The structural behavior of space structures with I-beams subject of lateral torsional buckling can often be described only insufficiently by plane subsystems. This is caused e. g. by couple effects of stabilizing and destabilizing beams or transmissions of rotations and displacements from cross girders at the connecting points. By means of special finite elements, which are designed for I-beams in space structures, an efficient calculation will be possible. Beam systems with cross-connected structural members, like cross girders or trapezoid sheets will allow a derivation of approximate solutions for standard applications. The intended paper deals with calculation methods for space structures with I-beams and demonstrates structural behavior as well as applications on the basis of typical examples.*

1 INTRODUCTION

Methods for analyzing space structures with beams subject to lateral torsional buckling are mainly restricted to plane systems. Usually, the effect of adjacent structural elements is taken into account by definition of bearings, discrete and continuous translational or rotational restraints. In many cases these simplifications describe the structural behavior of such beams with sufficient accuracy.

Problems may occur when adjacent members do not have a stabilizing, but a destabilizing effect. Often deformations (displacements, angular rotations) are transferred to the beams to be stabilized. In consequence, bearings, discrete and continuous restraints cannot describe the stiffening effect entirely. The following examples will document the difficulties mentioned above.

Figure 1 shows two transversely loaded beams that are connected by a lattice bar at the midspan. In case of lateral torsional buckling with different loading conditions $q_{z,1}$ and $q_{z,2}$ stabilizing forces will be transmitted by the joining member. The more loaded beam will be restrained by the less loaded beam. The interaction can only be recorded considering the entire global system.

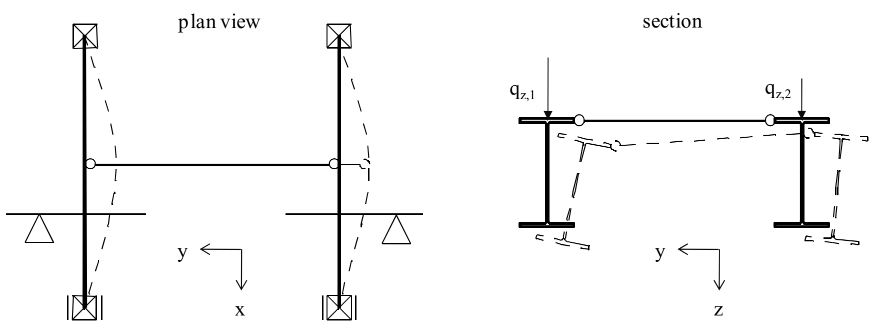


Figure 1: Lateral torsional buckling of cross-connected I-beams

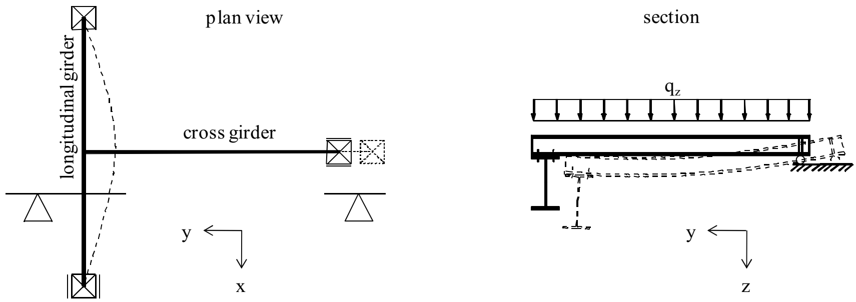


Figure 2: Crossing I-beams with uniformly distributed load

The system in Figure 2 shows a cross girder with uniformly distributed load that is supported by a longitudinal girder. The deformation of both girders at the connecting point can be assumed to be identical. The angular rotation at the end of the cross girder and the vertical deflection of the longitudinal girder result in a three-dimensional deformation shape. In the longitudinal girder bending moments M_z and torsional action effects will occur. They depend on the load level, the stiffness ratio and the dimensions of the system. Connections without stiffeners lead to additional distortions of the cross-section in the area of the connecting point.

Figure 3 shows a roof structure consisting of purlins, trusses and a roof bracing. Lateral displacements of the trusses and the roof bracing are linked by means of the purlins. On the one hand the bracing has a stabilizing effect on the trusses. On the other hand deformation, caused e. g. by wind loads, is transferred into the trusses. This causes deflecting forces in the trusses that lead to additional loadings in the bracing. The assumption that purlins connected to the roof bracing act as rigid supports for the top flange of the trusses is inaccurate. Especially when diagonal bracings are realized as round steel tension bars and the bracings have to span large distances the influence of the deformation may become important.

The preceding examples demonstrate that the structural behavior of space structures with I-beams subject to lateral torsional buckling can often only insufficiently be described by a division into plane subsystems. Although known there is a lack of practical calculation tools that consider the interaction of the involved components with reasonable effort.

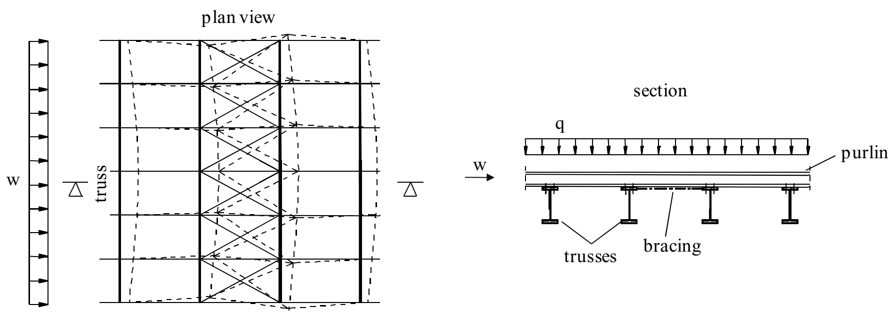


Figure 3: Roof structure with vertical and horizontal loads

2 FINITE ELEMENT MODELING OF SPACE STRUCTURES WITH I-BEAMS

Applying the finite element method the structural system is idealized by the arrangement of structural finite elements. Suitable elements must be available to describe the fundamental mechanical properties of the members and their connections. Multifarious structures can be modeled by continuum elements such as membrane, solid or shell elements. Decisive disadvantages of the analysis of I-beam structures subject to lateral torsional buckling are:

- The input of the structural systems including the definition of load and boundary conditions as well as geometrical imperfections such as initial bow or sway imperfections is very complex and time consuming.
- Generating the numerical model requires a comparatively large number of elements. The resulting meshes have a large number of nodes. To calculate the response of the structure a large system of equations needs to be solved.
- The analysis of the huge data output requires a graphical post processor. The transparency of results is partially lost since action effects are only presented as stresses, bending moments and in-plane forces per unit length.

In practice such calculations remain restricted to special cases, e. g. when manufacturing a large number of identical girders justifies these efforts. However, it is more efficient to use “macro-elements” with modified mechanical properties that represent the essential properties of the components to be modeled. The effort needed for modeling, calculation and analysis can be minimized by using such elements, so that FEM calculations can more widely be used for applications as described here.

Within the framework of the research project [1] tools for the calculation of structural systems with I-beams subject to lateral torsional buckling were developed based on the finite element method. Among other aspects the research aimed at providing a design tool for space structures and thus to remove limitations to plane systems such as single span or continuous I-beams and I-columns. It was intended to find a possibility to describe the impact of adjacent structural members directly by means of suitable elements and not only indirectly through the definition of discrete or continuous restraints and bearings. In addition, it was important to eliminate the assumption that there no change in cross-section shape during buckling, because some section designs with thin webs and support conditions of beam flanges require that web distortions need to be taken into account. The concept was to limit the mechanical properties of the elements that are essential for the modeling of members and connections. Effects such as transverse shear or membrane strains of I-beams were neglected. In a first step, the development of finite elements was limited to linear-elastic material properties and linearized distortion-translation relationship (second order analysis).

For the modeling of beams and columns with double-symmetric I-sections a “macro-“ or “super element” composed of a group of element types (figure 4) was developed. The structural behavior in the plane of web is idealized by a beam element. Perpendicular to the plane of web the I-section consists of two flange members and a web plate.

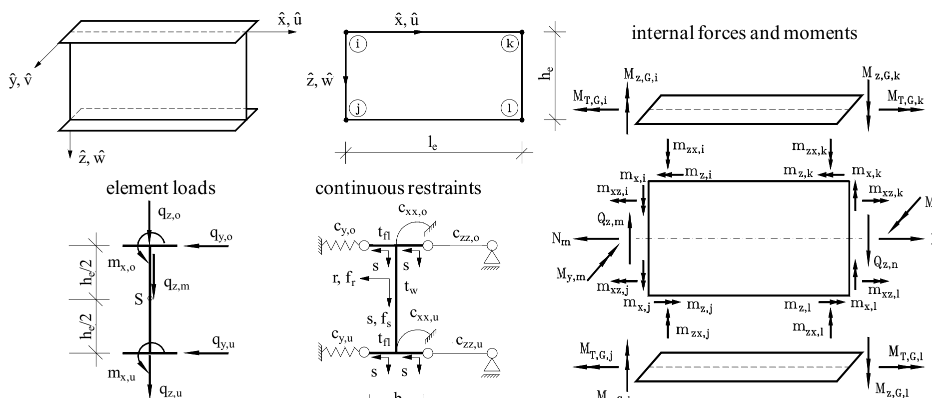


Figure 4: I-Profile-Element – reference system, loads, bedding, internal forces and moments

The splitting of I-sections into different elements has the advantage that web distortions can be described with two-dimensional buckling shapes. The separation of the cross section eased the application of eccentric forces and moments, the modeling of practical supporting conditions and the attachment of further structural elements to the flanges of the beams and columns. A frequently discussed issue in the beam theory is how to model geometric and/or static coupling conditions (keyword transmission of warping deformations) especially in cases when the beams are attached perpendicularly or in a random angle to each other or if they consist of stepped cross sections. The modeling with this type of element eliminates such kind of problems.

Compared to the modeling of I-profiles with shell elements the advantage of the proposed concept is that a relatively small number of elements is required for discretization. This leads to a significant reduction in computing time. Generating the model is very practicable and the numerical results are easier to analyze and interpret. The influence of cross-section fillets in rolled sections that significantly increase torsional stiffness can easily be defined by means of modifying the stiffness values. Modeling with solid elements requires a large number of elements, thus further increasing the effort for calculation and evaluation of the results.

Besides the I-profile element additional elements were developed. A stiffening element can be used for the modeling of stiffeners at load applications and of end plates; a trapezoidal profile element, a beam element for the modeling of bracings and lattice bars as well as different spring elements for the consideration of connection flexibility.

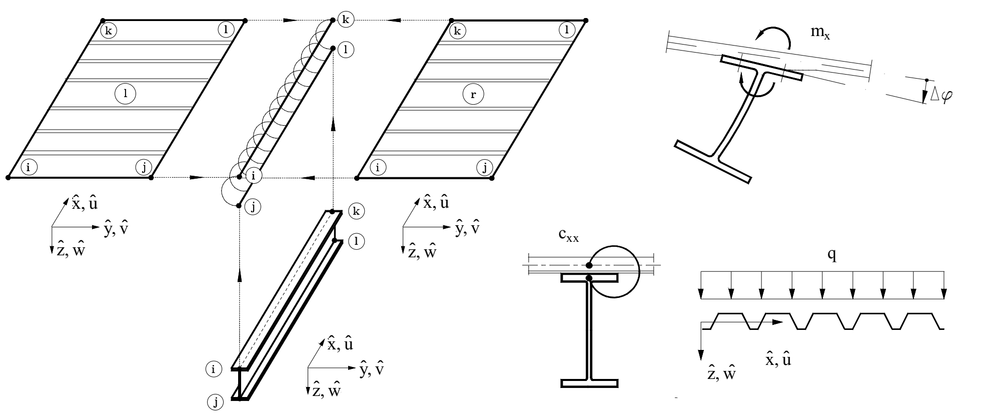


Figure 5: Assembly of the finite elements for trapezoidal sheeting, connection springs and I-beams

3 APPROXIMATION METHODS FOR THE CALCULATION OF CROSS-CONNECTED BEAMS

3.1 Introduction to and survey of calculation methods

In practice the level of utilization of the single beams in structural systems differs often significantly. This is for example the case, when beams with various loads for practical reasons are designed with the same cross-section. Due to varying live loads the level of utilization can be different at a certain point of time.

If different loaded beams are cross-connected, they act together at lateral torsional buckling. The less stressed beams will restrain the more stressed ones (figure 1).

In steel structures often girders with cross connections are used. Trusses, for example, can be coupled by purlins or secondary girders. Roof, wall and ceiling coverings provide a more or less continuous coupling of beams or columns. Due to the structural detailing diaphragm actions cannot always be taken into account.

The following description will briefly present different methods of calculating buckling loads of lateral coupled beam systems. Knowing the ideal buckling load a simplified verification of buckling resistance, e.g. according to EN 1993-1-1 [3], is possible.

One option to perform the structural analysis of coupled beam systems is to derive stiffness matrices of the beams separately (based on second order analysis) and use suitable coupling conditions to obtain the matrix formulation for the overall structure. Rigid couplings can be described by kinematic constraints and semi-rigid ones by coupling matrices. Connections can be at discrete points or in closely spaced intervals along the beams. The influence of constraining effects against twist rotations at the connecting areas can be considered by discrete or continuous torsional restraints.

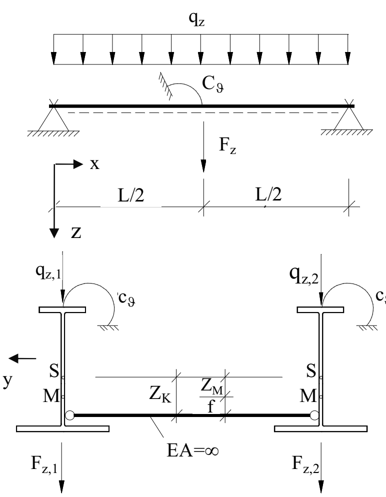
For simple structures approximation formulas and diagrams can be derived to determine the buckling load of coupled beams. Moreover, making use of programs for the calculation of single-span and continuous beams the lateral torsional buckling load of rigid coupled beam systems may by determined according to second order analysis of lateral torsional buckling and non-linear spring characteristics of the beams. A detailed description of the methods is given in [4].

3.2 Approximation formulas for rigid coupled I-beams

In case of simply supported beams with uniform distributed and single loads the assumption of buckling shapes in the form of half sinus waves for twist rotations ϑ and lateral displacements v_M of shear centre axis leads to an acceptable approximation for the buckling load. When the beams are rigidly coupled, the kinematic constraints provide the transformation rules of the stiffness matrices. In case of two coupled beams the degrees of freedom will be reduced from four to three. The assembly of beam stiffness matrices to the global stiffness matrix and the derivation of the determinantal equation result in a characteristic cubic polynomial with three eigenvalue. A closed-form solution is possible.

The approximation formulas given in figure 6 provide buckling load values that in most cases deviate by less than 5 % compared to calculations using more significant buckling shapes. Systems with a high stiffness of discrete and continuous torsional restraints show larger differences. If four, six or more

$$a_0 + a_1 \cdot \eta + a_2 \cdot \eta^2 + a_3 \cdot \eta^3 = 0 \quad \rightarrow \eta_{Ki}$$



$$a_0 = 2 \cdot c_1 \cdot c_2 \cdot (f^2 \cdot c_1 + c_2)$$

$$a_1 = 2 \cdot c_1 \cdot c_2 \cdot [f \cdot (g_1 + g_3) + g_2 + g_4] + (f \cdot c_1)^2 \cdot (g_2 + g_4)$$

$$a_2 = -(f^2 \cdot c_1 + c_2) \cdot (g_1^2 + g_3^2) + 2 \cdot c_1 \cdot [f \cdot (g_1 \cdot g_4 + g_2 \cdot g_3) + g_2 \cdot g_4]$$

$$a_3 = -2 \cdot f \cdot g_1 \cdot g_3 \cdot (g_1 + g_3) - g_1^2 \cdot g_4 - g_2 \cdot g_3^2$$

$$M_{q,i} = \frac{q_{z,i} \cdot L^2}{8} \qquad M_{F,i} = \frac{F_{z,i} \cdot L}{4} \qquad r_{M_y} = \frac{1}{I_y} \int_A (y^2 + z^2) \cdot z \cdot dA - 2 \cdot z_M$$

$$c_1 = \frac{EI_z \cdot \pi^4}{2 \cdot L^3} \qquad c_2 = \frac{EC_M \cdot \pi^4}{2 \cdot L^3} + \frac{GI_T \cdot \pi^2}{2 \cdot L} + c_\vartheta \cdot \frac{L}{2} + C_\vartheta \qquad f = Z_K - Z_M$$

$$g_1 = -\frac{M_{q,1}}{L} \cdot (1 + \frac{\pi^2}{3}) - \frac{M_{F,1}}{L} \cdot (1 + \frac{\pi^2}{4})$$

$$g_2 = -\frac{M_{q,1} \cdot r_{M_y}}{L} \cdot (1 - \frac{\pi^2}{3}) + \frac{q_{z,1} \cdot L}{2} \cdot (z_{q,1} - z_M) - \frac{M_{F,1} \cdot r_{M_y}}{L} \cdot (1 - \frac{\pi^2}{4}) + F_{z,1} \cdot (z_{F,1} - z_M)$$

$$g_3 = -\frac{M_{q,2}}{L} \cdot (1 + \frac{\pi^2}{3}) - \frac{M_{F,2}}{L} \cdot (1 + \frac{\pi^2}{4})$$

$$g_4 = -\frac{M_{q,2} \cdot r_{M_y}}{L} \cdot (1 - \frac{\pi^2}{3}) + \frac{q_{z,2} \cdot L}{2} \cdot (z_{q,2} - z_M) - \frac{M_{F,2} \cdot r_{M_y}}{L} \cdot (1 - \frac{\pi^2}{4}) + F_{z,2} \cdot (z_{F,2} - z_M)$$

Figure 6: Approximation formulas for calculation of cross-connected Beams

beams are rigidly coupled and respectively half of them have the same load intensity the buckling load of these systems can obviously be calculated as well using the formulas given in figure 6.

3.3 Diagrams for calculation of lateral torsional buckling moments

Using the program *PROFIL* [2] diagrams were created for the determination of lateral torsional buckling moments of girder systems with discrete and continuous rigid couplings (see figure 7). The diagrams are valid for systems of simply supported rolled I-beams. The double-symmetric cross sections of the respective I-beams are identical. The application points of the uniformly distributed loads and the level of couplings are placed to the centroid from the top flanges of the beams. At the determination of the diagrams web distortions were excluded. The application is as follows:
The number of coupled beams determines the type of diagram to be used. Curve parameters are given by the moment distribution M_y and the load relation q_2/q_1. Depending on torsion coefficient χ (see equation (1)) the diagram provides the coefficient k to determine the lateral torsional buckling moment of the whole system (equation (2)). The value $M_{y,Ki}$ refers to the total maximum moment M_y, that means either the span or support moment. Lateral torsional buckling moments of single beams j will be determined with equation (3) by the relation of beam load and system load.

$$\chi = \frac{EI_W}{GI_T \cdot L^2} \tag{1}$$

$$M_{y,Ki} = \frac{k}{L} \cdot \sqrt{GI_T \cdot EI_z} \tag{2}$$

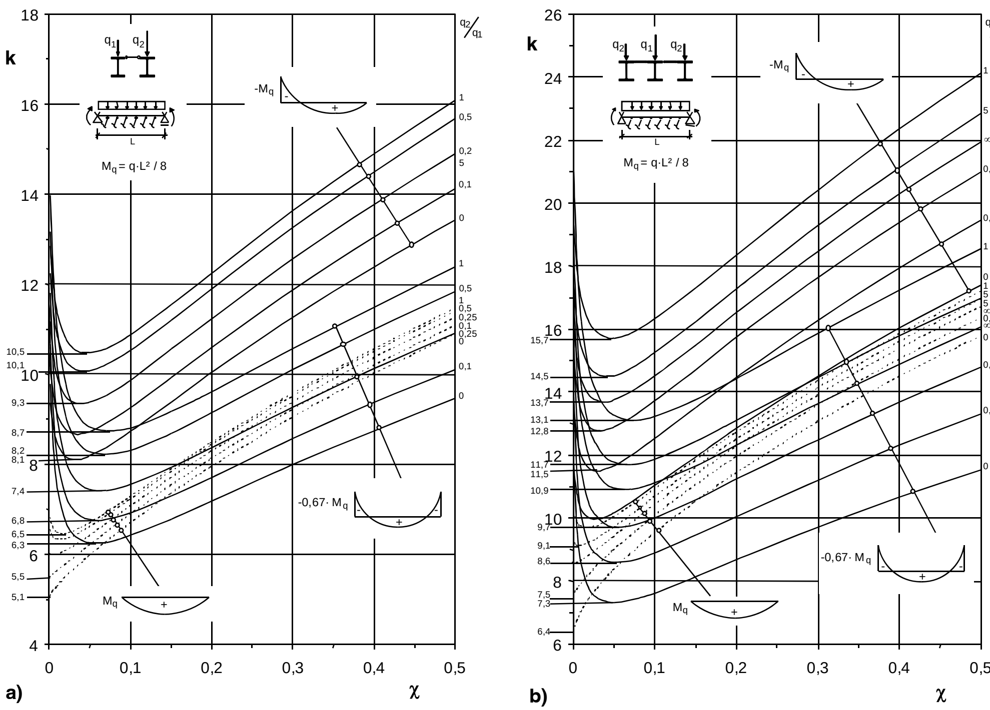


Figure 7: Coefficients k for the calculation of lateral torsional buckling moments

$$M_{y,j,Ki} = M_{y,Ki} \cdot \frac{q_j}{\sum q} \tag{3}$$

The coefficient k is nearly independent on section series and size. The diagrams in figure 7 are based on minimum values of the IPE series. In good approximation the application is possible for other rolled beams with doubly symmetric I-section. In the diagrams for three coupled beams it has to be considered that the coefficient k increases from load relationships of zero to one and thereafter k decreases again. Under certain conditions the diagrams may also be used for determining lateral torsional buckling moments of systems of more than three beams. This requires that only two different values of transverse load are present and that the relation between the number of beams with the same load is n_1:n_2 = 1:1 or 1:2.

4 EXAMPLES

In order to estimate the buckling loads of discrete coupled beam systems an iterative determination can be performed. The aim of such determination is to find the load level η_{Ki} at which the sum of stabilizing and destabilizing forces that are transferred to the coupling beams reach equilibrium. Figure 8 demonstrates the principle by an example. Three differently loaded beams are rigidly connected at midspan to the top flange. If the spring characteristic C_y for lateral displacement of the I-beams at the connecting point is known from a previous calculation performed according to second order analysis the iteration can be carried out by using the diagram presented.

Figure 9 shows a girder grillage consisting of purlins and trusses, respectively with same cross-section, on which a uniformly distributed load of q=5.00 kN/m^2 acts. Lateral torsional buckling of the purlins is prevented by restraints, e. g. due to trapezoidal sheeting. Caused by the structural system, the load transfer of the inner trusses is approximately 2.75 times higher than that of the outer trusses. By the connection with the purlins the trusses are torsionally restrained and coupled in transverse direction at the top flanges.

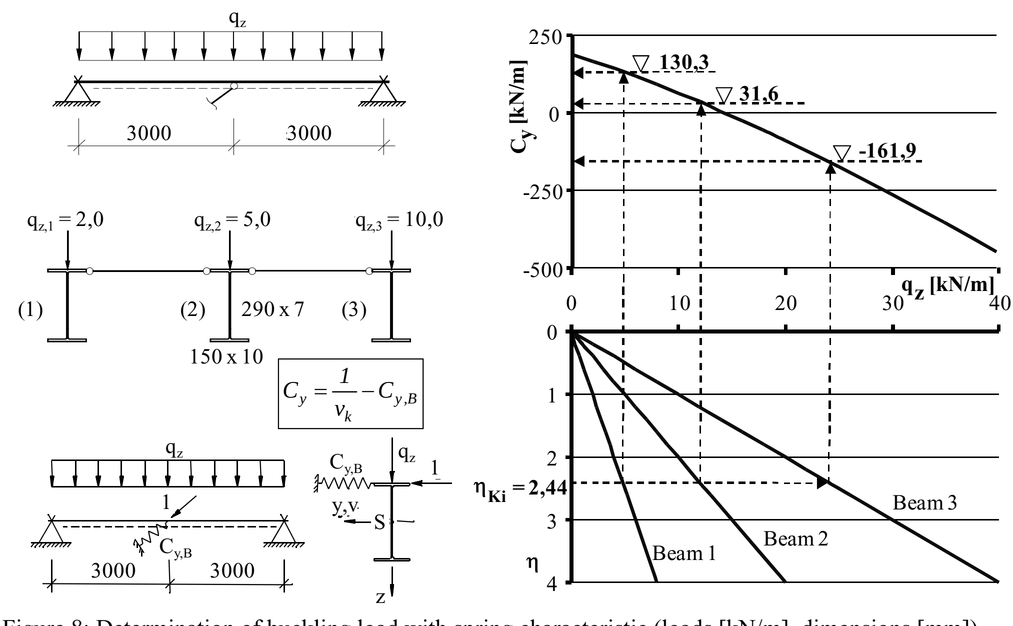


Figure 8: Determination of buckling load with spring characteristic (loads [kN/m], dimensions [mm])

The stability of the structural system was calculated by the program *PROFIL* [2] and various approximation methods as documented in [4]. Moreover, various effects were investigated by means of different calculations. Disregarding the coupling effect to the outer trusses and the torsional restraints from the purlins the buckling load factor of the inner trusses is η_{Ki}=0.39. With torsional restraints this factor increases to η_{Ki}=1.24. Additionally, the consideration of the coupling effect to the outer trusses results in a buckling load factor of η_{Ki}=1.71. In this particular case it is necessary to consider both effects, to verify the structural safety of the inner trusses.

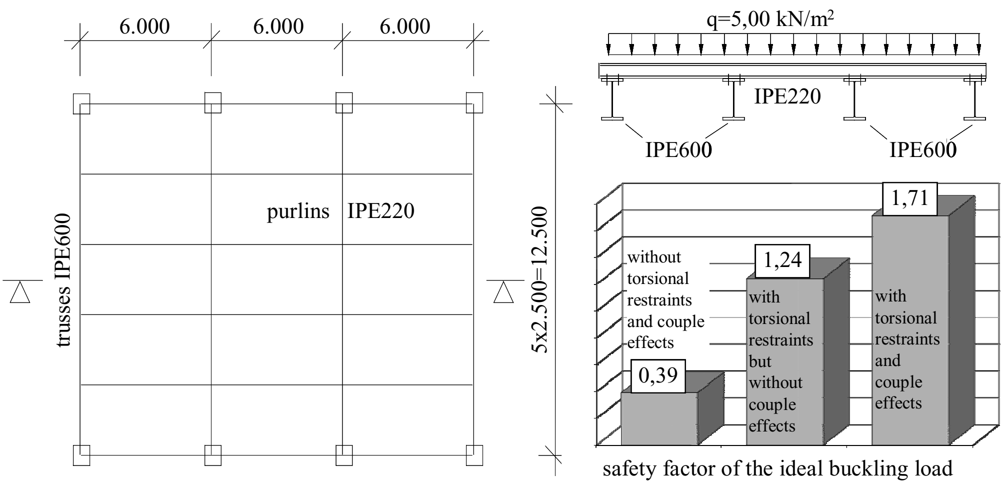


Figure 9: Girder grillage with trusses and purlins – influence of different stabilization effects

5 CONCLUSION

The stability of space structures is often only insufficiently assessed by the analysis of plane subsystems. Specific finite element formulations allow system analyses that include essential effects with regard to the overall structural behavior. Besides the possibility to consider the transmission of deformation of adjacent structural members, stabilizing forces of bracing systems can be directly determined. By restriction to the effects that are essential for the structural behavior the efforts for modeling, calculation and interpretation are minimized. In this way, the finite element method can economically be applied to space structures in practice.

REFERENCES

[1] Stroetmann, R., “Zur Stabilitätsberechnung räumlicher Tragsysteme mit I-Profilen nach der Methode der finiten Elemente“, Veröffentlichung des Instituts für Stahlbau und Werkstoffmechanik der Technischen Universität Darmstadt, Heft 61, 1999.

[2] Stroetmann, R., “PROFIL – FEM-Program for the structural analysis of space structures with I-Profiles“, Fachgebiet Stahlbau und Werkstoffmechanik, TU-Darmstadt, 1999.

[3] EN 1993-1-1: “Eurocode 3 – Design of steel structures, Part 1-1 – General rules and rules for buildings“, 2005.

[4] Stroetmann R., “Zur Stabilität von in Querrichtung gekoppelten Biegeträgern“, *Der Stahlbau* **69**, Verlag Ernst & Sohn, Berlin, 391-408, 2000.