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NUMERICAL AND EXPERIMENTAL RESEARCH IN TAPERED STEEL PLATE GIRDERS SUBJECTED TO SHEAR

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Abstract. *Plate girders are used when it is necessary for a structural element to support high loads, above which a normal rolled section would either not be structurally viable or would become uneconomical. For the sake of achieving structural efficiency, such members are usually designed as tapered. The varying depth of the girders is aimed to provide robustness following the zones where high shear and flexural loads are expected. Presently, rigorous analyses and design provisions on this field are thought to be as scarce. Thus, the structural response of these members for serviceability and/or collapse loads is still unclear. The present research is aimed to thoroughly analyse the structural response of tapered plate girders. The response will be analysed by considering the potential yielding of the plates but also, their potential instability. The study leads to obtain a realistic value of the critical shear buckling load and the ultimate shear resistance for tapered plate girders.*

1 INTRODUCTION

Steel plate girders are used when it is necessary for a structural element to bear high loads, above which a hot rolled section would either not be sufficient or would be uneconomical. Such structural cases are likely in girders which are aimed at bridging long spans or in which the self-weight of the structure governs considerably its design (typically, for steel and composite bridges, industrial buildings).

For the sake of developing an efficient design for a given structural member, plate girders are often designed as non-prismatic i.e. web tapered members. Typically, such design consists of a horizontal flange welded to a web whose height varies linearly from one bearing another. This variation is conceived for the element to resist loads according to the typical bending and shear diagrams. The height of the plate girder is higher in the cross-sections where greater bending moments and shear forces are expected to occur. This variation leads to lighter structural members than traditional prismatic girders.

Moreover, as considerable weight savings are obtained when designing a web plate as tapered, economical savings are usually linked to these material reductions. As a result, the web plates happen to be more slender than the webs in uniform plate girders design. Consequently, local instabilities of the plates are more likely to occur in such type of girders even for relatively low values of shear force, when compared to the shear plastic resistance of the member.

There are however, very few theoretical and experimental investigations into the structural response of tapered steel plate girders under increasing shear load up to failure. Just as Galambos [1] points out, more work is required to develop general design procedures for the ultimate strength of steel panels with variable depth. There are no rules in current steel codes for the design of tapered plate girders.

The pursued objective of this paper is to present a numerical investigation to improve fundamental understanding of the shear buckling phenomenon and the post-buckling response of tapered steel plate girders as well evaluate several formulae proposed for these types of girders. With the intention of studying the structural response of tapered girders and validating the analytical and numerical approaches, experimental tests should be carried out. A series of experimental tests will be conducted at the Laboratory of Structural Technology of the School of Civil Engineering in Barcelona, UPC.

2 SHEAR MODELS FOR TAPERED PLATE GIRDERS

2.1 Introduction

The shear buckling behavior of rectangular plates has been deeply studied during last century and different theories have been developed to describe and analyze the mechanisms that take place during the post-buckling state, to determine the ultimate shear capacity of rectangular plate girders. Some of them are implemented in design codes: the Rotated Stress Field Model and the Tension Field Model [2].

However, these models are based on the assumption of simply supported rectangular plates and do not consider the boundary conditions existing in the flange – web junctions and in the stiffener – web junctions neither the geometry of the tapered steel plate girder. Some authors have demonstrated the importance of these effects ([3], [4] and [5]).

The ultimate shear strength models for tapered plate girders proposed in literature are based on previous presented models for plate girders with constant depth. Several models for tapered girders have been developed by: Falby and Lee [6], Davies and Mandal [7], Takeda and Mikami [8], Roberts and Newmark [9], Zárate and Mirambell [10] and Shanmugam and Min [11].

2.2 Ultimate shear strength for tapered plate girders

It is well known that the structural behavior of a prismatic steel plate girder subjected to an increasing shear load up to failure may be divided into three clearly different phases. Prior to buckling, equal tensile and compressive principal stresses are developed in the web panel. In the post-buckling stage, an inclined tensile membrane stress state is developed. The total stress state is obtained by adding the post-buckling to that induced at buckling. Once the web has yielded, failure of the steel plate girder occurs when plastic hinges are formed in the flanges. The failure load can be determined from the consideration of the mechanism developed in the last stage (upper bound solution) or by the consideration of the equilibrium of forces (lower bound solution) [12].

The behavior of a tapered steel plate girder subjected to increasing shear load is practically identical to that exhibited in a prismatic steel girder. When the web buckles under the action of direct stresses, it does not exhaust the full capacity of the plate. After buckling, a significant increase in the strength of the steel plate girder can be observed. Experimental tests and numerical studies carried out on tapered steel plate girders reveal the existence of post-critical strength, by means of the development of the diagonal tension field anchored in the stiffeners and flanges.

Some models for the determination of the ultimate shear strength for tapered plate girders have been presented in the last years. All these studies are based on the tension field method, but one determines the ultimate shear load by the lower (equilibrium) bound method [10], other one by the upper (mechanism) bound method [11] and other one by both methods [7].

2.2.1 Lower (equilibrium) bound method

The tension field method assumes that the ultimate shear strength of a plate girder can be obtained as the critical shear buckling force plus the post-buckling shear strength. Zárate and Mirambell [10] developed a shear model for estimating the ultimate shear strength of tapered plate girders

$$V_u = V_{cr} + V_{pcr} = \tau_{cr} \cdot h_0 \cdot t_w + \sigma_{bb} \cdot g \cdot t_w \cdot sin\,\beta \tag{1}$$

where V_{cr} is the critical shear buckling force and V_{pcr} is the post-buckling resistance depending on the magnitude of the tension field (σ_{bb}) and the width and the slope of the tension band (g and β) (see Fig. 1).

In eq. (1) the critical shear buckling stress of the web panel (τ_{cr}) is obtained by using a shear buckling coefficient proposed in [4] that considers the actual boundary conditions of the web panel, the variable geometry of the web panel and the flange panels and the slope of the inclined flange. The tension band is composed of three parts and the width of the tension band g is given by the following expression

$$g = (s_c - a) \cdot sin\,\beta + h_0 \cdot cos\,\beta + s_t \cdot sin(\phi + \beta) \tag{2}$$

where ϕ is the slope of the inclined flange. The magnitude of the tension field (σ_{bb}) is reduced by a factor ρ that considers the effect of the principal compressive stresses during the post-buckling behavior.

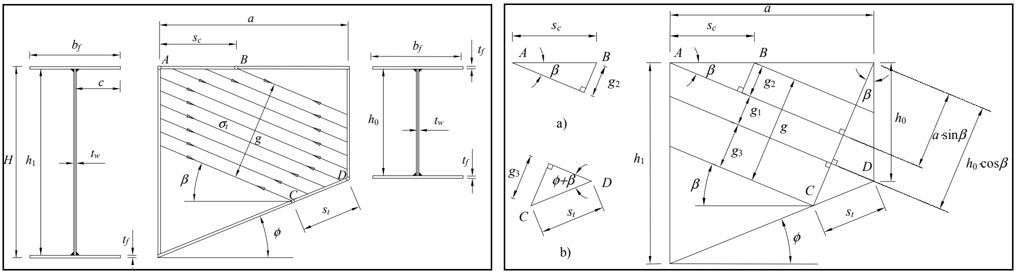


Figure 1: Ultimate shear capacity model of tapered plate girders [10].

It is important to point out that the proposed model has to be used in cases where the diagonal tension field is developed in the shorter geometrical diagonal of the tapered web.

2.2.2 Upper (mechanism) bound method

Other solutions for determining the ultimate shear capacity of plate girders can be obtained by using the upper bound method. Porter et al. [12] proposed a model for rectangular plate girders and Davies and Mandal [7] and Shanmugam and Min [11] proposed other models for tapered plate girders based on numerical and experimental investigations. Shanmugam and Min [11] proposed two models, one to predict the ultimate shear capacity for tapered plate girders when the inclined flange is in tension and the other one when the inclined flange is in compression.

$$V_u = V_{cr} + V_{pcr} = \tau_{cr} \cdot h \cdot t_w + V_{pcr} \tag{3}$$

For both models the critical shear buckling stress (τ_{cr}) was calculated as for a rectangular plate with fixed edges. The web depth is the average value between the smallest depth (h_0) and the largest depth (h_1).

3 NUMERICAL ANALYSIS OF TAPERED PLATE GIRDERS

3.1 Numerical model

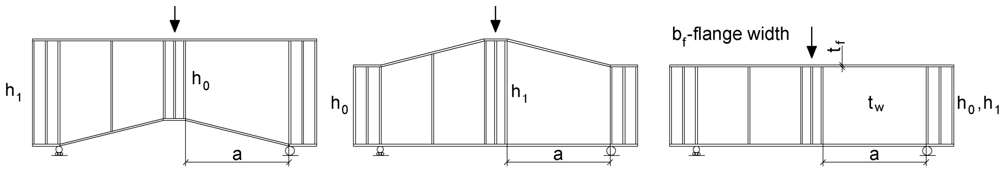
Different numerical analyses have been conducted during the investigation to study the shear buckling phenomenon in tapered plate girders considering both geometric and material nonlinearities. The Abaqus code [13] has been used to carry out such structural analyses. The 4-node shell element S4R was adopted to discretize the geometry and the steel properties were defined as a material with the von Mises criterion for yielding stress and isotropic hardening. The stress–strain relationship is based on the characteristic bi-linear σ–ε diagram of the steel with elastic-plastic behavior.

Static problems with geometric non-linearity often involve buckling or collapse mechanisms, in which the load-displacement response displays negative stiffness and the structure must release energy in order to maintain equilibrium. The numerical model considers a non-linear analysis algorithm in which the equilibrium states during the unstable response phases are found using the "modified Riks" method. This is useful in the analysis of structures that show non-linear geometric behaviour and also non-linear material behaviour (post-buckling behaviour, softening and collapse).

In order to bring about the phenomenon of buckling, geometric imperfections have been added to the initial geometry. This imperfection corresponds to the first shear buckling mode of the web panel.

3.2 Plate girders analyzed

Previous studies demonstrated that ultimate shear strength in tapered plate girders depends on the inclination of the flange and on the stress state of the inclined flange (tension or compression). In order to evaluate the proposed methods abovementioned, a numerical study has been conducted for different geometries of rectangular and tapered plate girders with the inclined flange subjected to tension or compression (see Fig. 2).



Type I Type II Types III(h_1)-IV(h_0)

Figure 2: a) Plate girder with inclined flange under tension b) Plate girder with inclined flange under compression c) Rectangular plate girders with the largest depth (h_1) and the smallest depth (h_0).

All the girders analyzed have been numerically modeled as simply supported short beams with a point load applied at mid-span to consequently obtain a constant shear law. The steel of the all girders was S275 (f_y= 275 MPa, f_u= 430 MPa). Dimensions of the analyzed girders are presented in table 1.

Table 1: Dimensions of the girders.

Girder	h_0 (mm)	h_1 (mm)	a (mm)	t_w (mm)	b_f (mm)	t_f (mm)
A_525_700_700_4_140_15	525	700	700	4	140	15
B_350_700_1400_4_140_15	350	700	1400	4	140	15
C_350_700_700_4_140_15	350	700	700	4	140	15
A_600_800_800_4_180_15	600	800	800	4	180	15
B_500_800_1200_4_180_15	500	800	1200	4	180	15
C_480_800_800_4_180_15	480	800	800	4	180	15

3.3 Critical shear buckling force. Numerical results

For each plate girder the elastic critical shear force has been obtained by using Abaqus ($V_{cr,\,num}$) [13], the model proposed by Mirambell and Zárate (V_{cr1}) [4] and the approach of a fixed rectangular plate (V_{cr2}) proposed in [11]. Summary of the obtained results is shown in table 2. The design variables considered in this study are obtained through the relationship between several parameters, namely:

$\lambda_f = \dfrac{b_f}{t_f}, \eta = \dfrac{b_f}{h_1}, \alpha = \dfrac{a}{h_1}$ and $tg\phi$.

The analysis of the numerical results ($V_{cr,\,num}$) allows us to conclude that the critical shear buckling force depends on the stress state of the inclined flange (Type I in tension and Type II in compression) although in both cases the buckling of the web occurs in the direction of the shorter diagonal of the web panel (see figure 3). For all cases where inclined flange is in compression (Type II), the critical shear buckling force is much higher than the critical shear buckling force of the plate girders with the inclined flange in tension (Type I). Likewise, the $V_{cr,\,num}$ values for Type I cases are close to the values obtained for rectangular plates considering the highest depth (h_1) (Type III).

Table 2: Critical shear buckling force.

Girder	Type	α	tgϕ	λf	η	$V_{cr,\,num}$ [kN]	V_{cr1} [kN]	$Diff_{.[1]}$ [%]	V_{cr2} [kN]	$Diff_{.[2]}$ [%]
A_525_700_700_4_140_15	I	1,00	0,25	9,33	0,20	209,7	214,4	2,2	263,1	25,5
	II	1,00	0,25	9,33	0,20	266,0	214,4	-19,4	263,1	-1,1
	III	1,00	0,00	9,33	0,20	206,2	210,3	2,0	253,0	22,7
	IV	1,33	0,00	9,33	0,27	241,8	247,7	2,4	280,7	16,1
B_350_700_1400_4_140_15	I	2,00	0,25	9,33	0,20	159,0	160,3	0,8	226,0	42,1
	II	2,00	0,25	9,33	0,20	291,8	160,3	-45,1	226,0	-22,5
	III	2,00	0,00	9,33	0,20	157,3	158,2	0,6	180,1	14,5
	IV	4,00	0,00	9,33	0,40	307,1	314,5	2,4	323,8	5,4
C_350_700_700_4_140_15	I	1,00	0,5	9,33	0,20	214,7	222,4	3,6	280,7	30,7
	II	1,00	0,5	9,33	0,20	383,0	222,4	-41,9	280,7	-26,7
	III	1,00	0,00	9,33	0,20	206,2	210,3	2,0	253,0	22,7
	IV	2,00	0,00	9,33	0,40	335,8	340,5	1,4	360,3	7,3
A_600_800_800_4_180_15	I	1,00	0,25	12,0	0,225	191,6	187,3	-2,2	230,2	20,1
	II	1,00	0,25	12,0	0,225	245,6	187,3	-23,7	230,2	-6,3
	III	1,00	0,00	12,0	0,225	194,9	183,8	-5,7	221,4	13,6
	IV	1,33	0,00	12,0	0,300	221,3	216,9	-2,0	245,6	11,0
B_500_800_1200_4_180_15	I	1,5	0,25	12,0	0,667	152,2	150,7	-1,0	198,5	30,4
	II	1,5	0,25	12,0	0,667	232,9	150,7	-35,3	198,5	-14,8
	III	1,5	0,00	12,0	0,667	154,4	150,1	-2,8	174,1	12,8
	IV	2,4	0,00	12,0	0,417	226,4	226,9	0,2	241,8	6,8
C_480_800_800_4_180_15	I	1,00	0,4	12,0	0,225	190,7	190,6	-0,1	238,5	25,1
	II	1,00	0,4	12,0	0,225	297,4	190,6	-35,9	238,5	-19,8
	III	1,00	0,00	12,0	0,225	194,9	183,8	-5,7	221,4	13,6
	IV	1,67	0,00	12,0	0,375	261,7	256,2	-2,1	278,3	6,3

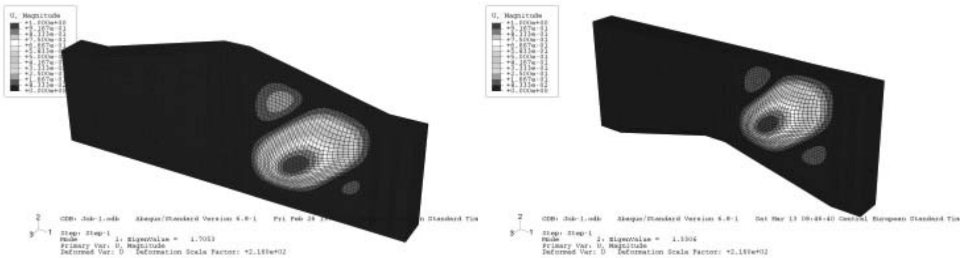


Figure 3: Deformation of the web panel for the elastic critical shear buckling force (numerical analysis).

A detailed analysis of the results shows that the model proposed by Mirambell and Zárate (V_{cr1}) [4] approximates the critical shear buckling force very satisfactorily when the inclined flange is in tension (Type I) and also for rectangular plate girders (Types III and IV). However, it provides lower results of the critical shear buckling force when the inclined flange is in compression (Type II). That is due to the fact that the model was developed for tapered plate girders with flanges in tension and for rectangular plate girders. In order to obtain the critical shear buckling force, the approach of a fixed rectangular plate (V_{cr2}) [11] adopts an average value for the structural depth of the girder and therefore, it does not consider the taper effect in an explicit way neither the real boundary conditions of the flanges.

Then, from the analysis of the obtained results, it can be concluded that a new expression for determining the critical shear buckling force for the case of tapered girders when the inclined flange is in compression must be developed. Moreover, cases where web buckling occurs in the direction of the large diagonal of the web panel should be considered.

3.4 Ultimate shear strength. Numerical results

In this section, the ultimate shear strength results obtained by the numerical simulation are compared with the ones obtained by using the ultimate shear model proposed by Zárate and Mirambell (V_{u1}) [10] presented in section 2.2.1. In this paper the analysis is focused on the ultimate response of tapered plate girders when the inclined flange is in compression (Type II). This type of tapered girders would reproduce the most common design situation for intermediate supports in continuous steel girders.

Table 3 shows the ultimate shear force values obtained with the numerical model ($V_{u,\ num}$) [13] and with the ultimate shear model proposed by Zárate and Mirambell [10].

It must be pointed out that, for determining the ultimate shear strength with the analytical model, the critical shear buckling force has been determined by using the expression proposed in [4] (V_{cr1}) and by using the numerical results ($V_{cr,\ num}$). Then, V_{u1} is the ultimate shear force for the first case and V_{u2} is the ultimate shear force for the second case, respectively.

Table 3: Values of ultimate shear force and differences with the numerical model.

Girder	V_{cr1} [kN]	$V_{cr,\ num}$ [kN]	V_{u1} [kN]	$V_{u,\ num}$ [kN]	$Diff_{[1]}$ [%]	V_{u2} [kN]	$Diff_{[2]}$ [%]
A_525_700_700_3_140_15	90,5	121,9	180,8	237,6	-23,9	190,9	-19,7
A_525_700_700_4_140_15	214,4	266,0	272,4	332,5	-18,1	308,0	-7,4
A_600_800_800_4_180_15	187,3	245,6	282,1	363,7	-22,5	311,2	-14,4
B_350_700_1400_3_140_15	67,6	131,2	99,5	175,7	-43,4	148,4	-15,6
B_500_800_1200_4_180_15	150,7	232,9	211,8	311,4	-32,0	270,7	-13,1
C_480_800_800_4_180_15	190,6	297,4	246,3	363,1	-32,2	311,0	-14,3

The ultimate shear strength values obtained using the model proposed by Zárate and Mirambell [10] underestimates the shear capacity of the girders when the inclined flange is in compression with differences around 30%, but using the critical shear stress obtained from the numerical model, an improvement of around 15% of the results can be observed. Further ultimate shear models for tapered steel plate girders need to be developed in order to accurately evaluate the post-buckling resistance. These models should include the taper effect in accordance with the orientation of the tension band and the collapse mechanism forming plastic hinges in the flanges. Figure 4 shows the tension band and the location of plastic hinges in flanges for the cases studied (Type II inclined flange in compression and Type I inclined flange in tension).

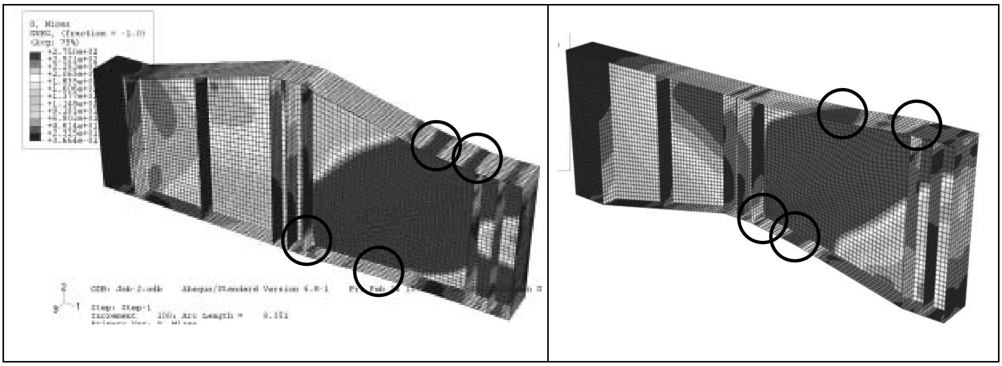


Figure 4: Ultimate shear response for Type II and Type I girders. Von Mises stresses.

4 CONCLUSIONS

For the sake of achieving structural efficiency, steel plate girders are sometimes designed as tapered. The varying depth of the girders is aimed to provide robustness following the zones where high shear and flexural loads are expected. In order to obtain an efficient solution for design purposes in tapered plate girders, significant slender web panels are designed. Due to this fact, attention should be paid to instability phenomena in order to assess the ultimate shear capacity of the tapered girder.

In this paper, the structural response until failure of several tapered steel plate girders has been studied. Numerical analyses have been conducted using Abaqus code and the results obtained have been compared with the results derived from the application of an analytical ultimate shear model for tapered girders.

The analysis of the numerical results shows that the critical shear buckling force is higher for the case of inclined flange in compression than for the case of inclined flange in tension. This effect is not well reproduced by the analytical models considered in this paper and further analytical models should consider it properly. The ultimate shear model developed by Zárate and Mirambell might be extended to the case of tapered girders with inclined flanges in compression. Moreover, tapered girders where web buckling occurs in the direction of the largest diagonal of the web panel should also be studied.

An experimental campaign over tapered steel plate girders subjected to shear loads is planned to assess a new ultimate shear model that takes into account the actual boundary conditions and the taper effect in accordance with the orientation of the tension band and the geometry of the girder.

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