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# SHEAR STRENGTH OF STEEL PLATE WITH REINFORCED OPENING

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***Abstract:*** *A cost effective way to mitigate the detrimental effects of large web openings in the floor joists of cold-formed steel buildings is to fasten reinforcements. This paper presents the details associated with the finite element analysis of thick/thin-plate, representing the web of a cold-formed steel member, having a large reinforced opening. The study considered simply-supported rectangular plates with opening subjected to in-plane shear loadings until failure (including post-buckling behaviour). The plate and the reinforcements were modelled using geometrically non-linear quadrilateral shell elements, and using experimentally established non-linear stress-strain relationships. The investigation considered three reinforcement schemes, namely, flat, lip, and angle reinforcements. This paper discusses the modelling considerations and presents the results associated with the three reinforcement schemes under consideration. Where possible the paper compares the analysis results with the experimental results.*

## 1 INTRODUCTION

The floor joists of cold-formed steel buildings often require large web openings. A cost effective way to mitigate the detrimental effects of web openings in the floor joists is to fasten reinforcements. Recently, experiments were conducted to establish such reinforcement schemes for web opening in shear and in flexural zones. Since moment always coexists with the shear, it is impossible to create pure shear state in experiments, resulting in moment influenced results. However, Finite element analysis can be conveniently used to investigate the pure shear behaviour of webs including webs with reinforced opening. This paper presents the details associated with the finite element analysis of thick/thin-web having a large reinforced opening. The study considered post-buckling behaviour of simply-supported rectangular plates with reinforced opening subjected to in-plane shear loadings until failure.

Reinforcements for hot-rolled steel member web openings have been studied in detail in the past. Redwood and Shrivastava [1] studied unreinforced and reinforced web openings in hot-rolled W-shaped sections. Based on experiments, Narayanan and Der-Avanessian [2] proposed an equilibrium solution to predict the strength of webs containing reinforced rectangular openings. They considered welded flat reinforcements placed symmetrically above and below the openings. Very limited research can be found on shear reinforcements on cold-formed steel webs. Pennock [3] carried out experimental studies on cold-formed steel joists with reinforced and unreinforced web openings subjected to bending. Both circular and square openings were considered in that study. Such reinforcement was found to be inadequate for openings located both in high bending and shearing zones. Acharya [4] performed an experimental investigation on reinforcement schemes for cold-formed steel joists having large web openings. His studies considered both flexural and shear zones. Three reinforcement schemes were considered in his study for the shear reinforcement. Two of the reinforcement schemes were recommended by the AISI [5], which are a steel plate having the same size and shape of the opening as the main joist and a cold-formed steel stud section having the same size and shape of the opening as the main joist, respectively. The third reinforcement scheme considered by Acharya [4] consisted of four channel sections placed around the opening. His study concluded that only the reinforcement scheme using the channel sections was adequate to restore the shear strength of cold-formed steel joists having web openings.

## 2 FINITE ELEMENT MODEL FOR PLATES WITH REINFORCED OPENING

In this section, a general finite element analysis model is proposed to investigate the behavior of plates with reinforced square openings subjected to pure shear loads. The model consists of two components, the main plate and the reinforcements. The main plate, representing the web of a cold-formed steel joist, is taken to have a length of ‘a’, a width of ‘h’ and a thickness of ‘t’, resulting in a slenderness of (h/t). The plates under consideration had a fixed aspect ratio (a/h=3) and varying slenderness ratios (h/t). Though the aspect ratio can influence the shear strength of plates, analytical studies [6] indicated that only a marginal change exists in plates having a/h >3, thus an aspect ratio of a/h = 3 was used to represent the cold-formed steel joists whose aspect ratio may be substantially higher than 3. The parametric study considered h/t = 50, 100, 150 and 200, representing thick to thin webs. The main plate is assumed to have a centrally located square opening of side dimensions ‘dc’. This investigation considered dc/h = 0.6 representing 60% web opening, which is rather large. Available web width for reinforcement is thus (h-dc).

Figure 1 shows the three reinforcement schemes, namely, flat reinforcement, lip reinforcement, and angle reinforcement, under consideration. Accordingly, all four edges of the square opening were reinforced with equal size reinforcements along all four edges. The width of the reinforcement was taken as ‘hr’. Considering the flat reinforcement and assuming that the reinforcement is attached between the opening edge and web edge, the width of the reinforcement hr = (h-dc)/2. Thus, the flat reinforcement consists of flat strips of metal of width hr and tr along all four edges of the opening. In the lip reinforcement arrangement all edges of the square opening were considered to be reinforced with lip plate of width hr and tr. The third reinforcement scheme under consideration consists of equal leg angle fastened along all edges of the square opening. As shown in Figure 1, one leg fastened to the web, some what similar to the flat reinforcement, thus the other leg acts like the lip reinforcement. The width of each leg however, was taken as hr/2, thus the total width of all three reinforcement schemes is hr. The reinforcements are assumed to be fully attached to the main plate with no additional constraints. Thus, though screws are widely used in construction practice, here, no screw will be modeled. In the ensuing parametric studies the thickness of the reinforcements tr was taken to be multiple of main plate thickness (tr= n.t).

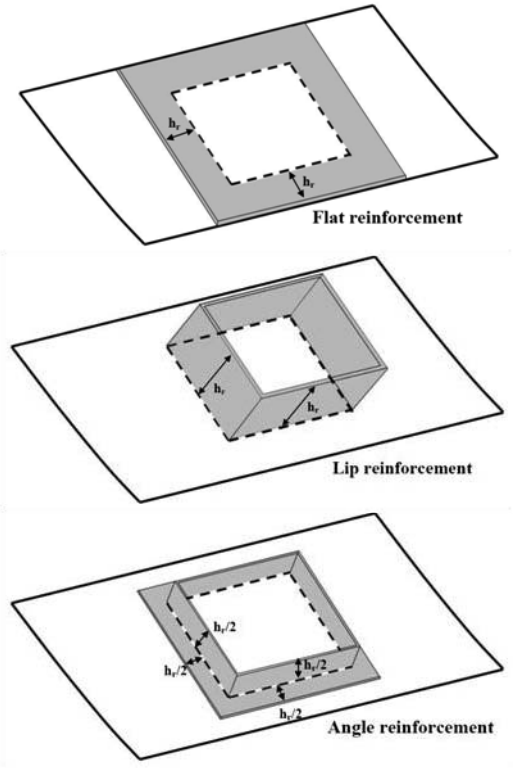

Figure 1: The three reinforcement schemes

<u>Finite Elements:</u> The finite element models of the above presented plate and reinforcement schemes were created and analysed using software ADINA [7]. In the current study the quadrilateral four-node shell elements for the non-linear analysis, which are capable of representing both flat and curved surfaces, were used. Each node of the shell element has six degrees of freedoms, namely three displacements and three rotations. This four-node element is capable of simulating both the membrane and the flexural behaviors of plates. The default 2 by 2 integration point arrangement for the 4-node element is used in the r-s element mid-plane. In the through-thickness direction ‘t’ the Newton-Cotes rule for integration points

is preferred rather than the default Gauss quadrature rule because, instead of having integration points only within the thickness of plates, the Newton-Cotes rule also has the integration points lying at both the top and bottom surfaces. This allows one to capture the gradually yielding response of the plate starting from the boundaries. Though the program default integration point number for Newton-Cotes rule is 3, in order to improve the accuracy of the model, a through thickness integration of 7 was used in the analyses. The quadrilateral four-node shell elements were used for both the main plate and the reinforcements, since both of them are plated elements.

Mesh Quality: The mesh configurations for the three reinforcement schemes, which are not shown in this paper, are based on the convergence study [6]. Accordingly, 24-division mesh configuration is adequate for plates with an aspect ratio of 3 (a/b=3), resulting in 3456 elements for the main plate with an opening. Flat portions of the reinforcements in flat reinforcement and angle reinforcement also used similar mesh configuration. Lip reinforcement contained 24 by 24 divisions, which gave a ratio of the longest element edge to the shortest element edge of 3. However, height of the lips associated with angle reinforcement was divided into 20 elements resulting in 24 by 20 divisions, which gave a ratio of the longest element edge to the shortest element edge of 5.

Initial Geometric Imperfections and the Residual Stresses: The models included geometrical initial imperfections; however, possible residual stresses were ignored in this study, primarily based on the study by Rondal [8] which concluded that the flexural residual stress has negligible or no effect on the ultimate strength of cold-formed steel sections. The main plate contained a double sine function imperfection and the amplitude of the imperfection was taken as $w_0 = 0.003h$, where h is the width of the main plate. The flat surfaces and the edges of the lips of the reinforcements contained compatible imperfections.

Boundary Conditions and the Loading Conditions: The main plate is assumed to be simply supported along the four edges and is subjected to uniformly distributed shear loads applied along all four edges. The edges of the opening are left to move freely. The reinforcements were assumed to be fully attached to the main plate with no other additional constraints.

Material Model: The von Mises yield criterion is adopted as the yielding criterion for steel. Sivakumaran and Abdel-Rahman [9] has shown that, instead of a well-defined yield point, cold-formed steel possesses a gradual yielding behavior followed by a certain level of strain hardening. Within a cold-formed steel section, the yield strength and ultimate strength differs between the corner area and the flat area. Since this study focuses only on the flat plates, the stress-strain relationship for the flat area will be used for all models in this research. For analysis purpose, Sivakumaran and Abdel-Rahman [9] proposed an idealized multi-linear stress-strain relationship for cold-formed steel material. Figure 2 shows this stress-strain relationship, where 'Fy' is the yield stress of steel, which depends on the steel grade selected in the analysis. In this research, the commonly used 350MPa yield strength is used as the value of 'Fy'. The above idealized multi-linear stress-strain relationship for cold-formed steel was used for both the main plate and the reinforcement plate. To simulate the non-linear material stress-strain relationship discussed above, plastic-multi-linear isothermal plasticity material models were chosen for the analysis. It assumes the material to be elastic–plastic with strain hardening, following the isotropic hardening rule.

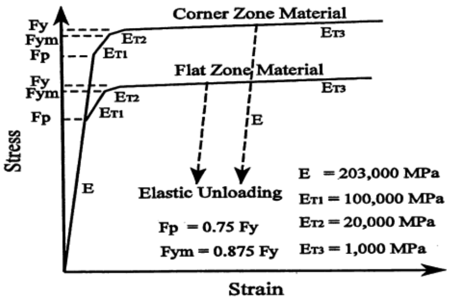

Figure 2: Idealized material model

Analysis Technique: The analysis technique must capture the pre-buckling, post-buckling and the ultimate load level behavior of the models under consideration, as ultimate strength of plates with reinforced opening is of interest. Though ADINA [7] features included the automatic-time-stepping (ATS) method and the load-displacement-control (LDC) method, here the ATS was used which requires prescription of a load and time steps. If no convergence can be obtained through the user-defined load steps, the ADINA will automatically subdivide the time steps until the convergence is achieved.

## 3 ANALYSIS RESULTS

The plates with reinforced opening under consideration had a length of 300 mm (a=300 mm) and a width of 100 mm (h=100 mm), which gave an aspect ratio of 3 (a/h=3). Thus, the amplitude of the initial geometric imperfection used was 0.3 mm. Furthermore, this study considered a 60% square opening with dc=60 mm. Therefore, the total width of the reinforcement plate hr was taken to be constant as 20mm, which is the one half of the remaining width of the plate above and below the opening. For the angle-reinforcement configuration, the reinforcement plates were assumed to be bent into angles with the width of each leg of the angle equals to (1/2).hr =10 mm. As indicated the slenderness ratios considered in this investigation are h/t = 50, 100, 150 and 200, which covers from thick plates to the thinnest plates allowed in the AISI code [5]. In this study, considering the plate with different h/t one by one, reinforcements with increasing thickness were applied on plates to investigate the influence of the thickness of the reinforcements on the behavior and the ultimate shear capacity of such plates. Thus, for plates with each

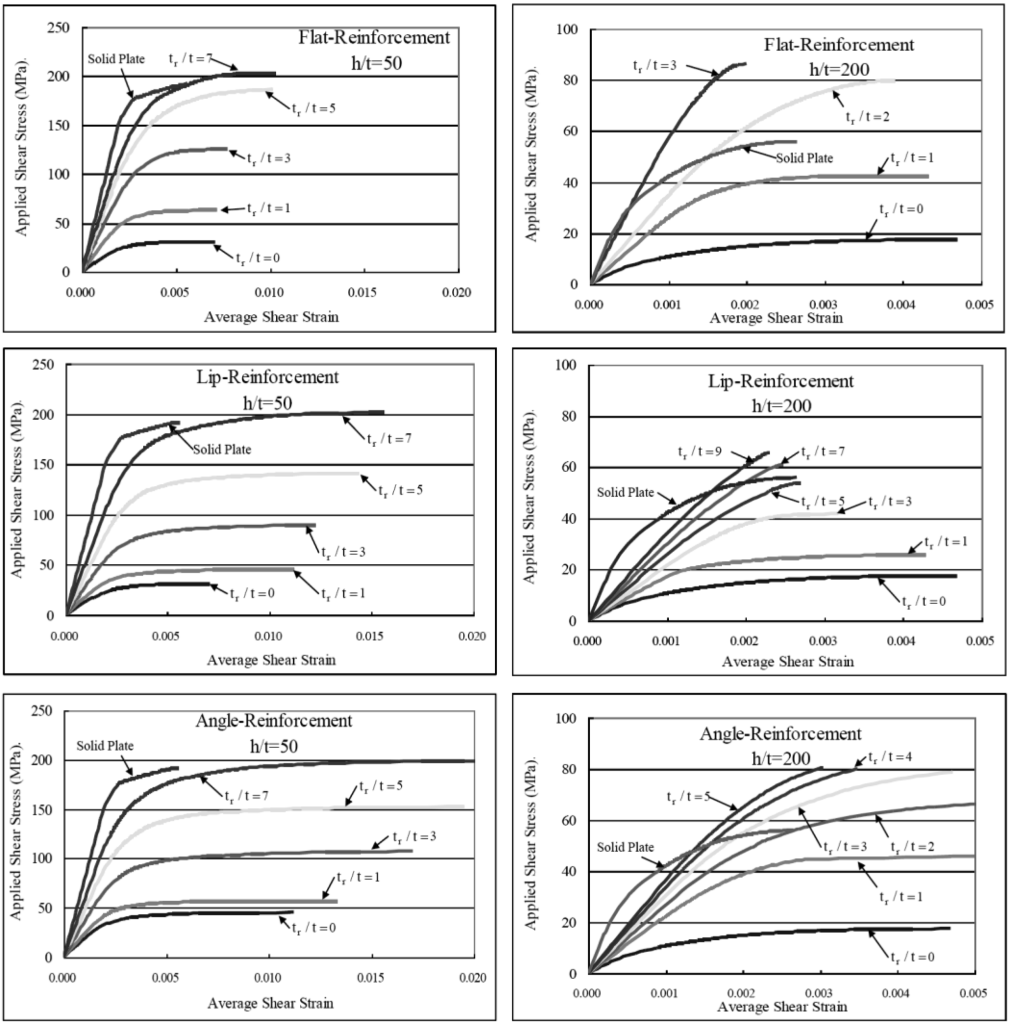

Figure 3: Applied shear stress versus average shear strain for plates with reinforced opening

‘h/t’ value, the reinforcement thickness to the main plate thickness ratio (tr/t) increases from zero until the increase in the ultimate shear strength of plates is less than 1.0%. Generally, (tr/t) was increased at an incremental step of one for each (h/t) value. However, intermediate steps may also be applied when needed. The multi-linear material model with Fy = 350 MPa and ν = 0.3 was used as the model material property of the plate as well as the reinforcements.

Results: The behaviors of the plates considered in this section are illustrated through the shear stress versus the average shear strain diagrams. The average shear strain shown in the horizontal axis was obtained by dividing the y-displacement of the lower right corner of the plate by the length of the plate. Figure 3 shows the applied shear stress versus the average shear strain diagrams for both solid plates and plates with reinforced openings. In order to be able to compare the behavior of the reinforced plates with that of solid plates, the behavior of the solid plate is also shown in these diagrams. These figures also include the behavior of plates with unreinforced openings, which corresponds to the case of (tr/t) = 0. Essentially, thickness of the reinforcement tr=0 mm indicates no reinforcements. Figure 3 shows the shear stress-strain diagrams for plates with flat-reinforcements, for plates with lip-reinforcements, and for plates with angle-reinforcements. For illustration purposes only the two extreme slenderness analyzed (i.e. h/t=50 and h/t=200) are shown in this figure. Each figure shows the shear stress-strain relationships as the size of the reinforcement (tr/t) increases. It may be noted that only selected ‘tr/t’ values are plotted in Figure 3. It can be observed from Figure 3 that solid plates are generally stiffer than plates with both unreinforced and reinforced openings. As anticipated, openings in plates tend to decrease the stiffness of plates. Increasing reinforcement thickness $t_r$ increases the stiffness as well as the strength of the system. Also, the ultimate strengths of plates with reinforced opening can be extracted from these graphs.

Table 1 shows the ultimate shear capacities of plates with reinforced openings obtained from the finite element analysis corresponding to h/t= 100 and 150. The table shows the strengths of corresponding solid plate, ultimate shear strength of plate with unreinforced opening (row 1, tr/t =0) and the strengths for the three reinforcement schemes under consideration. The first column indicates the thickness of the reinforcement (tr/t). As stated before, the ‘tr/t’ values were generally increased at an incremental step of one, however, in some analyses additional half steps were made in order to obtain enough data points for the later analysis. The analyses were carried until the percentage increase in the ultimate shear strength of plates is less than 1.0%, essentially, no further increase in shear strength. At this point, the reinforcement has recaptured the loss in strength due to opening and any increase in reinforcement thickness increases the strength of opening region that the member begin to fail in regions outside the opening.

Table 1: Ultimate shear strength of plates with reinforced opening

| (h/t=100) | Ultimate Strength of the Corresponding Solid Plate: $\tau_{ul(solid)}$ =113.1 MPa (h/t=100) | | |
|---|---|---|---|
| $t_r$ / t | Flat-Reinforcement $\tau_{ul(reinf.)}$ (MPa) | Lip-Reinforcement $\tau_{ul(reinf.)}$ (MPa) | Angle-Reinforcement $\tau_{ul(reinf.)}$ (MPa) |
| **0.0** | **23.6** | **23.6** | **23.6** |
| **1.0** | **62.9** | **39.1** | **53.9** |
| **1.5** | | | |
| **2.0** | **100.0** | **52.6** | **75.7** |
| **2.5** | | | |
| **3.0** | **128.8** | **68.8** | **96.3** |
| **3.5** | | | |
| **4.0** | **151.0** | **85.5** | **114.3** |
| **5.0** | **152.4** | **102.9** | **130.8** |
| **6.0** | | **121.3** | **144.2** |
| **7.0** | | **135.1** | **145.8** |
| **8.0** | | **140.1** | **146.6** |
| **9.0** | | **140.6** | |

| (h/t=150) | Ultimate Strength of the Corresponding Solid Plate: $\tau_{ul(solid)}$ =74.5 MPa (h/t=150) | | |
|---|---|---|---|
| $t_r$ / t | Flat-Reinforcement $\tau_{ul(reinf.)}$ (MPa) | Lip-Reinforcement $\tau_{ul(reinf.)}$ (MPa) | Angle-Reinforcement $\tau_{ul(reinf.)}$ (MPa) |
| **0.0** | **19.3** | **19.3** | **19.3** |
| **1.0** | **58.7** | **32.0** | **51.2** |
| **1.5** | **77.8** | | |
| **2.0** | **94.3** | **42.5** | **75.7** |
| **2.5** | **107.0** | | |
| **3.0** | **108.1** | **53.9** | **96.3** |
| **3.5** | **108.9** | | |
| **4.0** | | **66.4** | **101.2** |
| **5.0** | | **78.1** | **102.6** |
| **6.0** | | **87.3** | **103.3** |
| **7.0** | | **91.1** | |
| **8.0** | | **94.3** | |
| **9.0** | | **95.0** | |

Figure 4 shows the deformed shapes (magnified by 10%) of plates at the failure load levels. The figures are for plates with h/t=200 (thin plate), however, Figure 4 [A] shows the plate with unreinforced opening, and Figures 4[B], 4[C], and 4[D] are plates with adequate flat, lip and angle reinforcements, respectively, to restore the shear strength reduced by the opening. It can be seen from Figure 4 that before reinforcements are applied, plates with 60% openings (dc/h=0.6) fail at the four corners of the openings. With adequate reinforcements, plates fail in diagonal shear failure outside of the opening region.

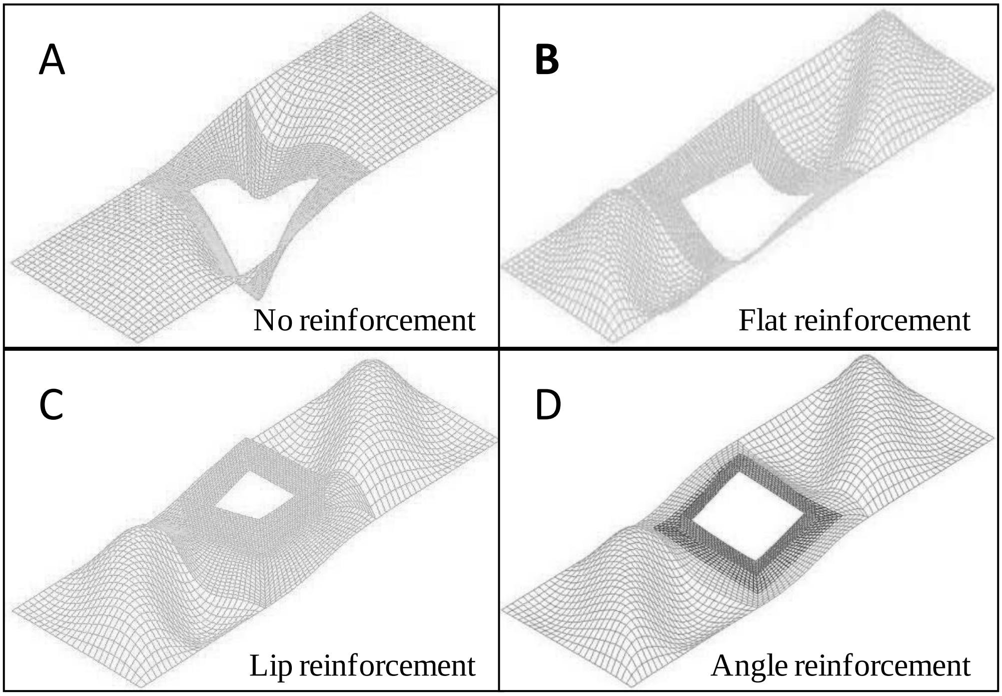

Figure 4: Deformed shapes of plates containing adequate opening reinforcement

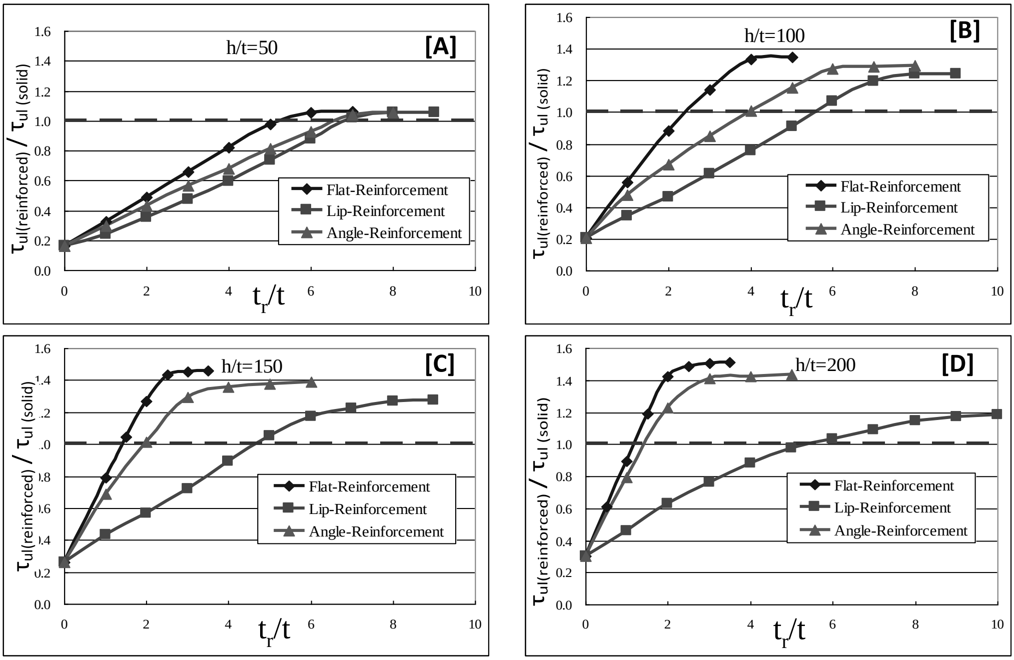

Figure 5: Relations between ultimate shear strength and reinforcement thickness

Figure 5 shows the plots of the ultimate shear strength of plates with reinforced openings normalized by the ultimate shear strength of the corresponding solid plates versus ‘tr/t’ for plates with flat, lip, and the angle reinforcement schemes. Normalized strength greater than 1.0 indicates that the reinforced plate has recaptured the original strength. By observing the normalized shear strengths, it can be seen from the Figure 5 that all three reinforcement schemes are capable of restoring the shear strength of plates which was compromised due to the presence of centrally located square opening. When an adequate amount of reinforcements is applied the ultimate shear strength of a plate with a square opening can even increase beyond its original shear strength when the plate is solid. Especially for thin plates, for example, when h/t=200, Figure 5 [D] shows that the ultimate shear strength of a plate with flat-reinforcement around the opening can be as high as about 1.5 times the original shear strength of a solid plate. This can be attributed to the fact that the reinforcement actually divides the plate into three panels. Two panels are on either side of the opening, and the third region is the reinforced opening. When enough reinforcements are provided the plate fails in the regions outside of the opening. Thus, the failure load of the plate is governed by the outside square panels. Chen [6] has shown her studies that the ultimate shear strength of a plate increases with decreasing aspect ratio (a/h). Thus, with proper opening reinforcement, the effective a/h of the plate actually decreases, causing the ultimate shear strength to increase relative to the original solid plate. Furthermore, the reinforced opening edges may provide rotational restraints to the outer panels, thus, one edge of these outer panels is not simply supported, and thus experiences higher load than that of a simply supported plate. For example, when h/t=200, with enough reinforcement (tr/t=3.5), $\tau_{ul(reinf.)}$ = 84.7MPa. The shear strength of a simply supported plate with h/t=200 and a/h=1 would be $\tau_{ul(a/h = 1)}$ =81.50MPa. Thus, when enough reinforcement is applied, the ultimate shear strength of a plate with a/h=3 and reinforced opening is comparable to the ultimate shear strength of a solid plate with a/h=1.

Effects of Reinforcement Thickness: Table 1 shows that for all three reinforcement schemes, as the thickness of the reinforcement (tr/t) increases, the ultimate shear strength of the reinforced plate increases. From Figure 5, it can be seen that, at the initial portion of the diagrams, the ultimate shear strength of plates with reinforced openings increases approximately linearly with increasing ‘tr/t’, but the increase in the ultimate shear strength reduces for higher values of ‘tr/t’. Essentially, there is no strength gain beyond an optimal reinforcement thickness. From Figure 5, it can be noted that it is easier to restore the shear capacity of slender plates by any one of the three reinforcement schemes. For example, for a plate with h/t=50 a flat-reinforcement of tr/t=6 is needed to restore the shear capacity of the plate with opening to the shear capacity of a solid plate. However, for a plate with h/t=200 a flat-reinforcement of tr/t=1.5 is enough to restore the shear capacity of that plate with opening to the shear capacity of a solid plate. Similar observations can be made with respect to the lip-reinforcement and the angle-reinforcement schemes.

Effects of Reinforcement Configuration: Since the total width (hr) was fixed as 20mm, and since the length of the reinforcement is approximately equal to the perimeter of the opening for all three reinforcement configurations, the reinforcement scheme with the least ‘tr/t’ value which can restore the shear strength of plates with openings to the shear strength of solid plates may be considered as the most effective reinforcement scheme. Figure 5 compares the ultimate shear strength of plates with different reinforcement configurations as the size of the reinforcements (tr/t) increases. It can be seen that for plates under consideration with ‘h/t’ values 50, 100, 150 and 200, the flat-reinforcement scheme can restore the ultimate shear strength of a plate with an opening to that of a solid plate with the least ‘tr/t’ value. For example, Figure 5 [D] indicates that for plate with h/t=200, the flat-reinforcement can restore the shear strength of plates with openings to the shear strength of solid plates with tr/t ≈1.1; the angle-reinforcement can restore the shear strength of plates with openings to the shear strength of solid plates with tr/t ≈1.3; the lip-reinforcement can restore the shear strength of plates with openings to the shear strength of solid plates with tr/t ≈4.3. Similar trend can be noticed for plates with h/t=50, 100 and 150. Thus the flat-reinforcement is considered as the most effective way to reinforce a square opening in a plate comparing with the other two reinforcement schemes. In the same way, the lip-reinforcement is the least effective reinforcement configuration to restore the shear capacity of plates with square opening, and the angle-reinforcement configuration works between the flat and the lip-reinforcement configuration.

## 5 CONCLUSIONS

This paper considered the ultimate shear strength of plates with reinforced openings. The plate analyzed in this chapter has an aspect ratio of 3 (a/h=3) and a centrally located 60% square opening (dc/h=0.6). The four slenderness ratios (h/t) considered in this chapter are h/t=50, 100, 150 and 200, which cover from thick plate to the thinnest plate allowed in the AISI code [5]. Three reinforcement schemes, namely the flat-reinforcement, the lip-reinforcement and the angle-reinforcement, were applied on the plates to compare and evaluate the effectiveness of these three reinforcement schemes. It was shown from the study that with an adequate amount of reinforcement material, all three reinforcement schemes are capable of restoring the ultimate shear strength of a plate with a square opening to that of a solid plate. However, the flat-reinforcement is the most effective reinforcement scheme as compared to the other two reinforcement schemes.

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