SDSS'Rio 2010 STABILITY AND DUCTILITY OF STEEL STRUCTURES
E. Batista, P. Vellasco, L. de Lima (Eds.)
Rio de Janeiro, Brazil, September 8 - 10, 2010

INFLUENCE OF GEOMETRY ON THE DYNAMIC BUCKLING AND BIFURCATIONS OF CYLINDRICAL SHELLS

Zenon N. del Prado* and Paulo B. Gonçalves**

* School of Civil Engineering, Federal University of Goiás, UFG, 74605-200, Goiânia, GO, Brazil
zenon@eec.ufg.br

** Civil Engineering Department, Catholic University, PUC-Rio, 22453-900, Rio de Janeiro, RJ, Brazil
paulo@puc-rio.br

Keywords: Cylindrical Shells, Dynamic Instability, Shell Geometry, Frequency-Amplitude Relation.

Abstract. *In this work, Donnell's nonlinear shallow shell equations are used to study the dynamic buckling and bifurcations of simply supported cylindrical shells subjected to axial or lateral load. A modal expansion with eight degrees of freedom containing the fundamental, companion, axially asymmetric and five axi-symmetric modes is used to describe the lateral displacement of the shell. The Galerkin method is used to obtain the nonlinear equations of motion which are, in turn, solved by the Runge-Kutta method. Several studies on the nonlinear dynamics of cylindrical shells are found in literature but they are restricted to specific geometries. In this paper we intend to study through a detailed parametric analysis the influence of the shell geometry, specifically Batdorf's parameter, length to radius ratio and radius to thickness ratio on the main nonlinear dynamic characteristics of the shell.*

1 INTRODUCTION

The combination of a simple geometry and its efficiency as a load carrying member, particularly for axial loads and lateral pressure, makes cylindrical shells one of the most common shell geometries in industrial applications and in nature. The buckling and vibration analysis of cylindrical shells under various loading conditions has thus become an important research area in applied mechanics. Also, the adequate selection of geometric characteristics is fundamental in designing against instability.

Amabili and Païdoussis [1] and Karagiosis [2] present extensive literature reviews related to the nonlinear dynamics of shells in vacuum, and shells filled with or surrounded by quiescent or flowing fluid. These topics are also presented in detail in a book by Paidoüssis [3] on fluid-structure interactions and a book by Amabili [4] on nonlinear vibrations and stability of plates and shells. Here only a few key contributions are mentioned.

The seminal works of Evensen [5] and Dowell and Ventres [6] gave the original idea to the modal expansions for the shell flexural displacement involving symmetric and asymmetric modes. Later, the studies by Ginsberg [7] and Chen and Babcock [8] contributed to the understanding of the influence of the companion mode on the behavior of cylindrical shells. These works showed that cylindrical shells usually display a softening behavior. Gonçalves and Batista [9] found that the presence of a dense fluid increases the softening characteristics of the frequency-amplitude relation when compared with the results for the same shell in vacuum. In a series of important papers Amabili *et al.* [10-13] the nonlinear free and forced vibrations of a simply supported, circular cylindrical shell in contact with an incompressible and non-viscous, quiescent or flowing dense fluid are studied using the Donnell's nonlinear shallow-shell theory. However most of these investigations are concerned with the analysis of elastic isotropic shells with fixed geometric characteristics and there are no specifics works related to the effect of geometry on the instability of cylindrical shells. Other interesting works on nonlinear dynamics of cylindrical shells can be seen in [14-17].

In this work, an eight-degrees-of-freedom model is used to study the nonlinear vibrations of perfect circular cylindrical shells with both axial and lateral loads. To discretize the shell, Donnell shallow shell equations are used together with the Galerkin method to derive a set of coupled ordinary differential equations in time domain. In order to study the effect of the geometric characteristics of the shell, several analyses are developed to understand their influence on the natural frequencies, critical loads, circumferential wave number and nonlinear frequency-amplitude relation. The obtained results can be used as a design tool by engineers and scientist to select adequate shell geometries. To the authors' knowledge, such an investigation has not been presented so far.

2 MATHEMATICAL FORMULATION

2.1 Shell equations

Consider a simply supported thin-walled circular cylindrical shell of radius R, length L, and thickness h. The shell is assumed to be made of an elastic, homogeneous and isotropic material with Young's modulus E, Poisson ratio ν, and mass density ρ_s. The axial, circumferential and radial co-ordinates are denoted by x, y and z, respectively, and the corresponding displacements on the shell surface are denoted by u, v and w, as shown in Fig. 1. In this work the mathematical formulation will follow that previously presented in references [10], [14], [15] and [17].

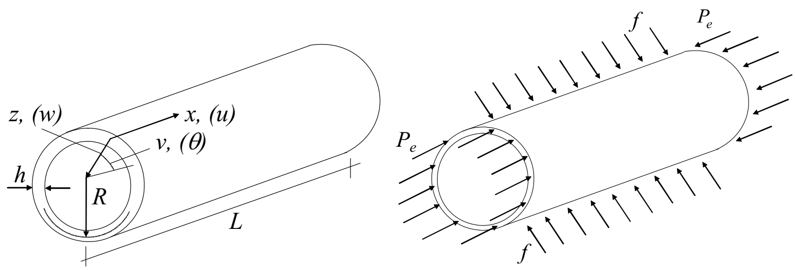

Figure 1. Shell geometry and loads

The shell is subjected to both a lateral pressure f and a distributed axial load $\tilde{N}_x$ along the edges x=0 and L given respectively by

$$f(t) = f_e + f_d \cos(\omega_L t); \qquad \tilde{N}_x(t) = -\frac{P_e}{2\pi R} - \frac{P_d}{2\pi R}\cos(\omega_L t) \tag{1}$$

where P_e is a compressive uniform static load, f_e is a uniform lateral static pressure and ω_L is the forcing frequency.

The nonlinear equation of motion, based on the von Kármán-Donnell shallow shell theory, in terms of a stress function F and the lateral displacement w, is given by

$$D\nabla^4 w + c\,h\,\dot{w} + \rho_s\,h\,\ddot{w} \;=\; f + \frac{1}{R}\frac{\partial^2 F}{\partial x^2} + \frac{1}{R}\left[\frac{\partial^2 F}{\partial \theta^2}\frac{\partial^2 w}{\partial x^2} - 2\frac{\partial^2 F}{\partial x\,\partial \theta}\frac{\partial^2 w}{\partial x\,\partial \theta} + \frac{\partial^2 F}{\partial x^2}\frac{\partial^2 w}{\partial \theta^2}\right], \tag{2}$$

where $D = Eh^3/[12(1-\nu^2)]$ is the flexural rigidity and c (kg/m^3 s) is the damping coefficient.

The compatibility equation is given by

$$\frac{1}{Eh}\nabla^4 F = -\frac{1}{R}\frac{\partial^2 w}{\partial x^2} - \frac{1}{R^2}\left[-\left(\frac{\partial w}{\partial x\,\partial\theta}\right)^2 + \frac{\partial^2 w}{\partial x^2}\frac{\partial^2 w}{\partial\theta^2}\right] \tag{3}$$

In Eqs. (2) and (3) the bi-harmonic operator is defined as $\nabla^4 = [\partial^2/\partial x^2 + \partial^2/(R^2\partial\theta^2)]^2$.

2.2 Solution expansion for the transverse displacement

The numerical model is developed by expanding the transverse displacement component w in series form in the circumferential and axial variables. From previous investigations on modal solutions for the nonlinear analysis of cylindrical shells under axial loads [10, 17] it is clear that, in order to obtain a consistent modeling with a limited number of modes, the sum of shape functions for the displacements must (i) express the nonlinear coupling between the modes and (ii) also describe consistently the unstable post-buckling response of the shell, as well as (iii) the correct frequency-amplitude relation. Here, the following modal expansion is adopted [10, 14]:

$$\begin{aligned} w(x,\theta,t) &= \xi_{1,1}(t)\,h\,\sin(q)\cos(n\theta) + \xi_{1,1c}(t)\,h\,\sin(q)\sin(n\theta) + \xi_{1,2}(t)\,h\,\sin(2q)\cos(n\theta) \\ &+ \xi_{1,2c}(t)\,h\,\sin(q)\sin(n\theta) + \xi_{0,1}(t)h\,\sin(q) + \xi_{0,3}(t)h\,\sin(3q) \\ &+ \xi_{0,5}(t)h\,\sin(5q) + \xi_{0,7}(t)h\,\sin(7q) \end{aligned} \tag{4}$$

where $\xi_{1,1}(t)$, $\xi_{1,1c}(t)$, $\xi_{1,2}(t)$, $\xi_{1,2c}(t)$, $\xi_{0,1}(t)$, $\xi_{0,3}(t)$, $\xi_{0,5}(t)$ and $\xi_{0,7}(t)$ are the time dependent modal amplitudes, $q = m\pi x/L$ and m and n are, respectively, the number of half-waves in the axial direction and the number of waves in the radial direction. This leads to an eight-degrees-of-freedom reduced order model. This model includes the basic vibration mode, the companion mode, symmetry-breaking modes in the axial direction and four axi-symmetric modes. These modes are enough to describe the basic nonlinear interactions responsible for the characteristic softening exhibited by cylindrical shells and the in-out asymmetry of the nonlinear displacement field.

2.3 Linear analysis

Substituting the fundamental mode in Eq. (3), obtaining the stress function, applying the Galerkin method and considering only one longitudinal half-wave (m=1), it is possible to obtain the expressions for the lowest natural frequency, axial critical load and lateral critical pressure in terms of two parameters. Using the circumferential wavelength parameter ($\bar{n}$) and the Batdorf's parameter (Z) given respectively by [18]

$$\bar{n} = \frac{nL}{\pi R}, \quad Z = \frac{L^2}{Rh}\left(1-\nu^2\right)^{1/2}, \tag{5}$$

and the following non-dimensional parameters

$$\Omega^2 = \frac{R^2\rho_s}{\pi^4 E}\omega^2, \quad \Gamma_0 = \frac{R}{2\pi^2 EL^2 h}P_e, \quad F_0 = \frac{L^2 R}{\pi^2 D}f, \quad \alpha = \frac{L}{R}, \quad \beta = \frac{R}{h} \tag{6}$$

the non-dimensional frequency, axial critical load and critical lateral pressure are obtained as

$$\Omega = \sqrt{\frac{\left(1+\bar{n}^2\right)^2}{12Z^2} + \frac{1}{\pi^2\left(1+\bar{n}^2\right)^2} - \frac{1}{\pi}\Gamma_o - \frac{\bar{n}^2}{12Z^2}F_0} \tag{7}$$

$$\Gamma_o = \frac{\pi\left(1+\bar{n}^2\right)^2}{12Z^2} + \frac{1}{\pi\left(1+\bar{n}^2\right)^2} - \frac{\bar{n}^2\pi}{12Z^2}F_0 \tag{8}$$

$$F_0 = \frac{\left(1+\bar{n}^2\right)^2}{\bar{n}^2} + \frac{12Z^2}{\pi^2\bar{n}^2\left(1+\bar{n}^2\right)^2} - \frac{12Z^2}{\bar{n}^2\pi}\Gamma_o \quad (9)$$

3 NUMERICAL RESULTS

3.1 Linear Analysis

Consider a simply supported cylindrical shell under both an axial load and a lateral pressure. As a first analysis, Fig. 2 shows the minimum values of the wave length parameter ($\bar{n}$) obtained for the lowest natural frequency, critical axial load and critical lateral pressure parameters as a function of the Batdorf's parameter (*Z*). As can be observed, the minimum wavelength parameter is the same for the lowest natural frequency and axial critical load but different for critical lateral pressure. It is also possible to see that, as the Batdorf's parameter increases, the wavelength parameter increases in a nonlinear manner.

Figure 3 shows the influence of Batdorf (*Z*) parameter and the *L*/*R* and *R*/*h* ratios on the lowest natural frequency parameter Ω (Eq. 7). As can be observed in Fig. 3a, which is plotted considering the lowest value of the wavelength parameter $\bar{n}$, as the Batdorf's parameter increases there is a strong reduction of the natural frequency parameter, Ω. Batdorf's parameter includes the influence of both *L*/*R* and *R*/*h*. Figure 3b shows the influence of *L*/*R* and *R*/*h* ratios on the lowest natural frequency and the associated number of circumferential waves (*n*). The *L*/*R* and *R*/*h* ratios influence directly the natural frequencies values and the number of circumferential waves. Shells with the same *L*/*R* and *R*/*h* ratios have the same lowest natural frequency and wavelength numbers. This figure shows that most shell geometries can be analyzed using Donnell's shallow shell theory ($n \geq 5$).

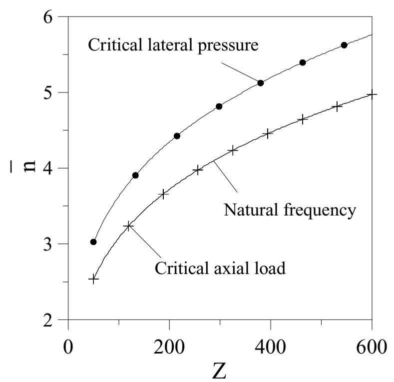

Figure 2. Critical values of wavelength parameter, $\bar{n}$.

3.2 Nonlinear Analysis

Now the influence of the shell geometry on the frequency-amplitude relations of the shell is investigated. Consider a thin-walled cylindrical shell with h=0.002 m, R=0.2 m, E=2.1x10^8 kN/m^2, ν=0.3 and ρ_S=7850 kg/m^3. For this shell, several geometries with increasing values of Batdorf's parameter (*Z*) and same *R*/*h* relation are considered. Table 1 shows the geometric characteristics, lowest natural frequency, associated circumferential wave number and *L*/*R* and *R/h* ratios for each Batdorf's parameter.

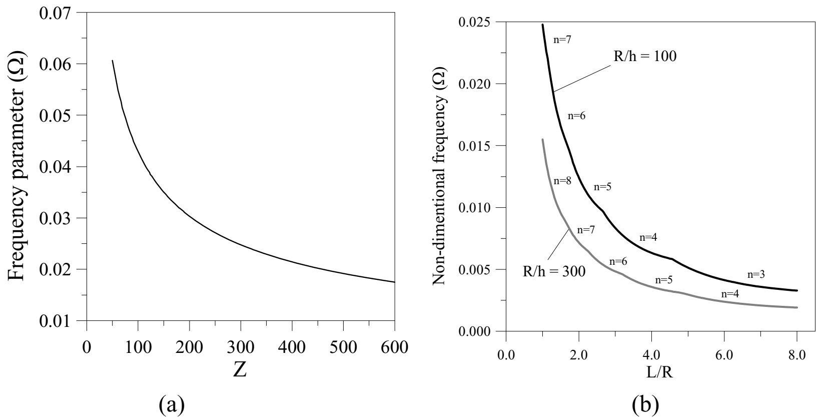

Figure 3. Influence of (a) Batdorf geometric parameter *Z* and (b) the *L*/*R* and *R*/*h* ratios on the lowest natural frequency parameter Ω.

Figure 4 displays the influence of Batdorf's parameter on the nonlinear frequency-amplitude relation for the shell geometries shown in Table 1. All shells display a softening behavior. The initial nonlinear softening behavior increases as the Batdorf's parameter decreases. On the other hand, the amplitude at which the bending back of the nonlinear response occurs increases with *Z*. These curves show the strong influence of Batdorf's parameter on the nonlinear behavior of the shell. The influence of the *L*/*R* and *R/h* ratios on the nonlinear frequency-amplitude relation is conducted considering a fixed value of Batdorf's parameter (*Z*=300). Table 2 shows three shell geometries with different natural frequencies (rad/s) and *L*/*R* and *R/h* ratios and Table 3 shows three shell geometries with the same natural frequency but different *L*/*R* and *R/h* ratios.

Table 1: Geometric characteristics and natural frequencies for increasing values of *Z*.

Z	*L* (m)	*L*/*R*	*R*/*h*	*n*	ω_o (rad/sec)	Ω
100	0.20477	1.02	100	7	6186.47	0.02423
200	0.28959	1.44	100	6	4369.52	0.01712
300	0.35467	1.77	100	5	3628.90	0.01422
400	0.40954	2.05	100	5	3087.81	0.01210
500	0.45788	2.29	100	5	2776.22	0.01209
600	0.50159	2.51	100	5	2579.93	0.01010

Figure 5 displays the nonlinear frequency-amplitude relations obtained for the shell geometries presented in Tables 2 and 3. As shown in Fig. 5a, the curves display similar initial softening behavior but different bending back points. The bending back point of Case A (*R*/*h*=71.55) is lower than that of Case B (*R*/*h*=127.19) and Case 0 (*R*/*h*=100). This shows that shells with the same Batdorf's parameters but different *L*/*R* and *R/h* ratios do not have the same nonlinear behavior at large vibration amplitudes. Figure 5b shows the nonlinear frequency-amplitude relations obtained for shell geometries in Table 3. The shells do not have the same behavior even though they have the same natural frequency ω_0 and parameter *Z*.

Table 2: Geometric characteristics for different natural frequencies, *L*/*R* and *R/h* ratios and the same *Z*.

Case	*h* (m)	*R* (m)	*L* (m)	*L*/*R*	*R*/*h*	*Z*	n	ω_o (rad/sec)	Ω
0	0.002	0.2	0.35467	1.77	100	300	5	3628.90	0.01421
A	0.004	0.28618	0.6	2.09	71.55	300	5	2560.52	0.01435
B	0.006	0.76315	1.2	1.57	127.1	300	6	934.59	0.01397

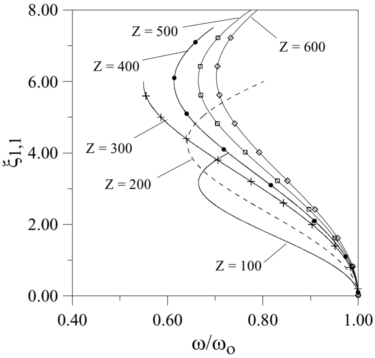

Figure 4: Frequency-amplitude relations for increasing values of *Z*.

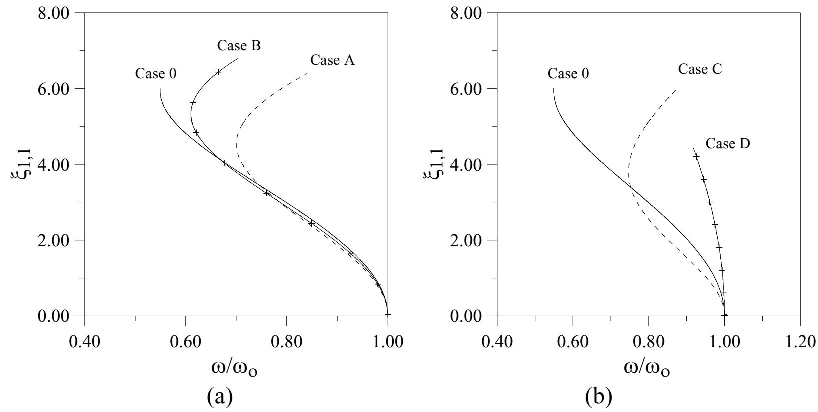

Figure 5: Frequency-amplitude relations: a) different natural frequencies and different *L*/*R* and *R/h* ratios but same *Z*, b) same natural frequencies and *Z* but different *L*/*R* and *R/h* ratios

Table 3: Geometric characteristics: same natural frequencies and *Z* but different *L*/*R* and *R/h* ratios.

Case	*h* (m)	*R* (m)	*L* (m)	*L*/*R*	*R*/*h*	*Z*	n	ω_o (rad/sec)	Ω
0	0.002	0.2	0.35467	1.77	100	300	5	3628.90	0.01421
C	0.007766	0.3	0.8559	2.85	38.63	300	5	3628.90	0.02132
D	0.0013	0.4	0.40439	1.01	307.69	300	5	3628.90	0.02844

Finally, Table 4 displays three shell geometries with same natural frequencies and same *L*/*R* and *R/h* ratios (consequently the same *Z* and Ω). The associated nonlinear frequency-amplitude relations are displayed in Fig. 6. All shells display the same nonlinear behavior. This shows that the nonlinearity is basically governed by the *L*/*R* and *R/h* ratios.

Table 4: Geometric characteristics same natural frequencies and same *L*/*R* and *R/h* ratios.

Case	*h* (m)	*R* (m)	*L* (m)	*L*/*R*	*R*/*h*	*Z*	n	ω_o (rad/sec)	Ω
0	0.002	0.2	0.35467	1.77	100	300	5	3628.90	0.0142
E	0.0028195	0.28195	0.5	1.77	100	300	5	2574.12	0.0142
F	0.0045112	0.45112	0.8	1.77	100	300	5	1608.83	0.0142

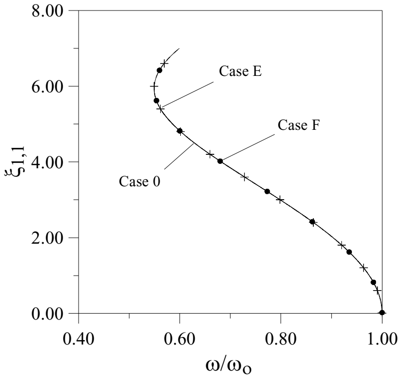

Figure 6: Frequency-amplitude relations for same natural frequencies and same *L*/*R* and *R/h* ratios.

The nonlinear frequency-amplitude relation governs the bifurcations and jumps observed in cylindrical shells under both lateral pressure and axial loads, as illustrated in Figure 7 where the resonance curve for a shell with *L*/*R*=1.0 and *R/h*=100 and subjected to a lateral pressure is shown. A detailed explanation of the influence of the frequency-amplitude relation on the instabilities of cylindrical shells can be found in [14] and [17].

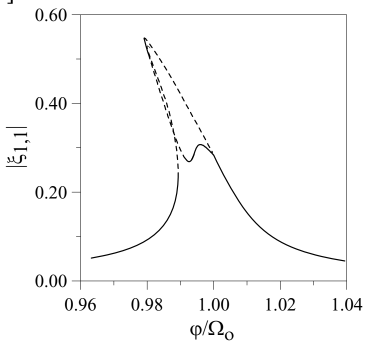

Figure 7: Resonance curve for a shell under lateral pressure. *L*/*R*=1.0 and *R/h*=100.

4 CONCLUSION

In this work, the influence of geometric characteristics on the natural frequencies, critical loads, critical modes and nonlinear frequency-amplitude relations of a simply supported cylindrical shell subjected to both axial and lateral pressure loads is analyzed. To model the shell the Donnell shallow shell theory is used together with an expansion of eight degrees of freedom to describe the lateral displacements of the shell. As observed, the nonlinear frequency-amplitude relation of the shell is basically governed by the *L*/*R* and *R/h* ratios and not by the Batdorf's parameter and shells with same *L*/*R* and *R/h* ratios display similar nonlinear behavior. The nonlinear frequency-amplitude relation governs the bifurcations and instabilities of the shell under external forcing. These results could serve as a design basis for engineers interested in choosing optimal geometries of cylindrical shells.

Acknowledgements

This work was made possible by the support of the Brazilian Ministry of Education – CAPES, CNPq and FAPERJ-CNE.

REFERENCES

[1] Amabili, M. and Païdoussis, M.P. "Review of studies on geometrically nonlinear vibrations and dynamics of circular cylindrical shells and panels, with and without fluid-structure interaction", *Applied Mechanics Reviews*, 56, 349 – 381, 2003.

[2] Karagiosis, K.N."Experiments and theory on the nonlinear dynamics and stability of clamped shells subjected to axial fluid flow or harmonic excitation", *PhD thesis*, McGill University, Montreal, Canada, 2005.

[3] Païdoussis, M.P. *Fluid Structure Interactions. Slender Structures and Axial Flow, Vol. 2*, Elsevier Academic Press, London, 2004.

[4] Amabili, M. *Nonlinear Vibrations and Stability of Shells and Plates*, Cambridge University Press, Cambridge, UK, 2008.

[5] Evensen, D.A. "Nonlinear flexural vibrations of thin-walled circular cylinders". *NASA TN D-4090*, 1967.

[6] Dowell, E.H. and Ventres, C.S. "Modal equations for the nonlinear flexural vibrations of a cylindrical shell", *International Journal of Solids and Structures*, 4, 975–991, 1968.

[7] Ginsberg, J.H. "Large amplitude forced vibrations of simply supported thin cylindrical shells", *Journal of Applied Mechanics*, 40, 471–477, 1973.

[8] Chen, J.C. and Babcock, C.D. "Nonlinear vibration of cylindrical shells", *American Institute of Aeronautics and Astronautics Journal*, 13, 868–876, 1975.

[9] Gonçalves, P.B. and Batista, R.C. "Nonlinear vibration analysis of fluid-filled cylindrical shells", *Journal of Sound and Vibration*, 127, 133–143, 1988.

[10] Amabili, M., Pellicano, F., Païdoussis, M.P. "Nonlinear dynamics and stability of circular cylindrical shells containing flowing fluid. Part I: stability", *Journal of Sound and Vibration*, 225, 655–699, 1999a.

[11] Amabili, M., Pellicano, F., Païdoussis, M.P. "Nonlinear dynamics and stability of circular cylindrical shells containing flowing fluid. Part II: large-amplitude vibrations without flow", *Journal of Sound and Vibration*, 228, 1103–1124, 1999b.

[12] Amabili, M., Pellicano, F., Païdoussis, M.P. "Nonlinear dynamics and stability of circular cylindrical shells containing flowing fluid. Part III: truncation effect without flow and experiments", *Journal of Sound and Vibration*, 237, 617–640, 2000a.

[13] Amabili, M., Pellicano, F., Païdoussis, M.P. "Nonlinear dynamics and stability of circular cylindrical shells containing flowing fluid. Part IV: large-amplitude vibrations with flow", *Journal of Sound and Vibration*, 237, 641–666, 2000b.

[14] Del Prado, Z.J.G.N., Gonçalves, P.B., Païdoussis, M.P. "Nonlinear vibrations and imperfection sensitivity of a cylindrical shell containing axial fluid flow", *Journal of Sound and Vibration*, 327, 211-230, , 2009

[15] Pellicano, F. and Amabili, M. "Dynamic instability and chaos of empty and fluid-filled circular cylindrical shells under periodic axial loads", *Journal of Sound and Vibration*, 293, 227–252, 2006.

[16] Catellani, G., Pellicano, F., Dall'Asta, D., Amabili, M. "Parametric instability of a circular cylindrical shell with geometric imperfections", *Computers & Structures*, 82, 2635–2645, 2004.

[17] Gonçalves, P.B. and Del Prado, Z.J.G.N. "Nonlinear oscillations and stability of parametrically excited cylindrical shells", *Meccanica*, 37, 569–597, 2002.

[18] Brush, D.O. and Almroth, B.O. *Buckling of Bars, Plates and Shells*. McGraw Hill Book Company, New York, 1975.