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CRITICAL LOADS AND STABILITY OF AN OPEN ELASTIC-PLASTIC CYLINDRICAL SHELL WITH THE CORE OF VARIABLE STIFFNESS

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Abstract. *The objective of this work is the stability analysis of an open sandwich cylindrical shell with unsymmetrical faces under combined load basing on moderately large deflections (geometrically nonlinear theory), and elastic-plastic properties of the material of the faces are taken into considerations. The shell consists of two load-carrying faces made of isotropic, compressible, work hardening material and they are of different thickness and made of different materials. Kirchhoff-Love (*K-L*) hypotheses hold for the faces, and the active deformation processes are considered. The core is assumed to be elastic, incompressible in the normal* z *direction and it resists transverse shear only. The elastic constants* E_c, *and* G_c *of the core are taken to be variable, and the strength capacity of the shell is substantially influenced by these constants. Prebuckling stress state is taken to be membrane one and the virtual work principle is the basis to derive the strain energy expression. The resulting nonlinear stability equation is solved by Ritz method. An iterative algorithm of elastic-plastic analysis was elaborated to solve the stability equations and the final objective of the work is numerical analysis of the influence of geometrical and material parameters on critical loads and equilibrium paths.*

1 INTRODUCTION

Shell structures are very interesting from the design point of view and these are well recognized in the literature [1], [3], [4]. A very significant problem in linear and nonlinear analyses of shell structures is stability and associated phenomena. One can find here multilayered structures, which are widely used in the manufacturing of modern vehicles, planes, cisterns, tanks, and in civil engineering, as well. These are subjected to widely varying combinations of hydrostatic pressure and axial load; hence, stability problem for such structures is of great importance. The purpose of this study is investigation of large displacement stability loss of a sandwich cylindrical panel loaded by longitudinal forces and uniformly distributed external pressure. It is assumed that the shell under consideration is made of a compressible material with linear and exponential strain hardening. Thus, it is also assumed that the effective stress in prebuckling state of stress in the shell can exceed the yield limit of the shell material. To find a solution of the problem, the assumptions of geometrically nonlinear theory and elastic-plastic properties of the faces are taken into account and the core remains elastic. In Refs [5], [6] both, linear and nonlinear buckling analyses of elastic-plastic conical and cylindrical shells are presented. In Ref. [4] Vinson J. R. described and discussed the up-to-date methods for sandwich structures analysis, and included a large reference list there. Kim S. E. and Kim, S. C. [1], Pinna and Ronalds [2], and Siad [3] discuss stability problems of cylindrical shells under various external loadings, also with imperfections. The constitutive relations used in the elastic-plastic analysis follow the incremental J_2 Prandtl-Reuss plastic flow theory of plasticity with the Huber-Mises yield condition. The K-L (Kirchhoff-Love) hypotheses are accepted and the active deformation processes according to Shanley concept are considered. The system of stability equations expressed by the displacements does not have an exact solution. Any approximate solution, e.g. by Galerkin method is complicated because the appropriate calculations are time consuming. The necessity to satisfy the kinematic and static boundary conditions leads to the assumption of approximate functions in a very complicated form. Thus, the virtual work principle is used to derive the total strain energy in the shell, and the analysis is based on the strain energy minimization, where the total strain in the shell can be expressed in terms of displacement vector components. Ritz method is accepted to derive the stability equations for the considered shell. The final solution is a very compound function of the deflection function parameter, which makes it possible to trace the equilibrium paths for the shell. An iterative computer algorithm was elaborated to facilitate the numerical analysis for the shells in elastic, elastic-plastic, and in totally plastic prebuckling state of stress. The algorithm reflects a specific feature of the elastic-plastic shell stability problem, where the stability equation is a transcendental function, where the coefficients of this equation depend on the load acting the shell.

2 BASIC ASSUMPTIONS AND GEOMETRICAL RELATIONS

The analyzed object is an open sandwich cylindrical shell, the element of which is presented in Figure 1. The shell consists of three layers: two thin face-layers, which are of different thickness h_1, h_2, and one core layer with thickness $H=2h$. The face layers can be made of different materials, which are compressible and isotropic ones.

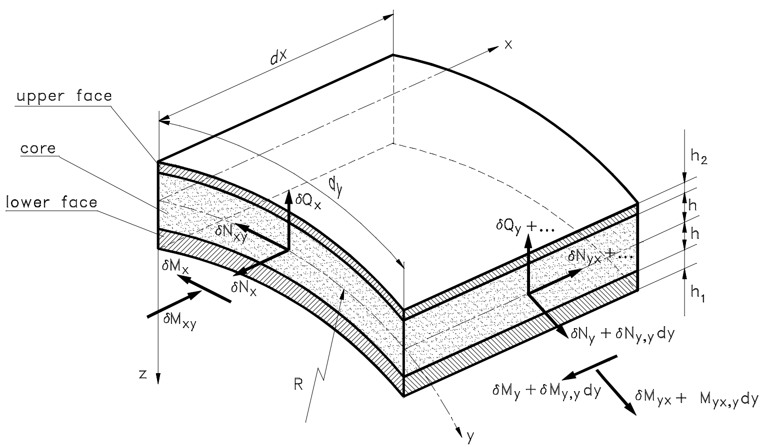

Figure 1: The element of a sandwich cylindrical shell with internal forces and moments

The core layer is assumed to be elastic, incompressible in the normal z direction and it resists transverse shear only. The middle surface of the core layer is taken as the reference surface of the shell. The main assumptions for the accepted model include that the shell is thin-layered and shallow one, and the post-buckling stress state is elastic or elastic-plastic. The following basic assumptions hold for the accepted model: (i) the shell is thin-layered, the core is elastic, incompressible in the z direction, the faces are of different thickness and they are made of different materials; (ii) the shell is shallow, the radii of curvatures of the layers are assumed to be equal; (iii) strains in the shell are described by nonlinear geometrical relations of the theory of moderately large deflections; (iv) the strains in post buckling stress state are elastic or elastic-plastic; (v) the displacements in normal direction do not depend on the z coordinate, and prebuckling stress state is the membrane one; (vi) constitutive relations in the analysis are those of the J_2 plastic flow theory of plasticity with the H-M-H (Huber-Mises-Hencky) yield condition.

If we accept the K-L hypotheses for the faces and include shear for the core we follow the so-called broken line approach (see Fig. 2) for the displacement scheme of the deformed shell.

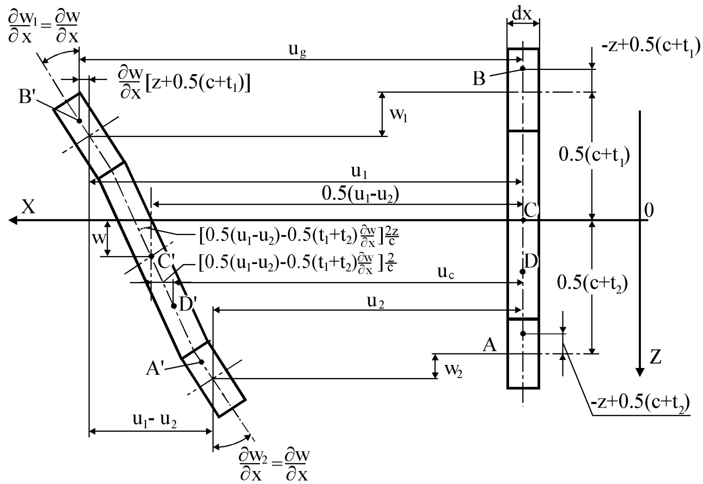

Figure 2: Geometrical configuration of deformed shell.

Following the broken line approach the relations between displacement vector components *u, v, w* of arbitrary point of the shell and the displacements of points situated on middle surface of the faces are as follows:

$$w^{\pm} = w_i, \qquad u^{\pm} = u_i + \left[z \pm \frac{1}{2}\left(c + t_i\right) \right] w_{i,x}, \qquad v^{\pm} = v_i + \left[z \pm \frac{1}{2}\left(c + t_i\right) \right] w_{i,\varphi} \tag{1}$$

Here:

i =1 – lower face - $0{,}5c \le z \le \left(0{,}5c + t_1\right)$

i =2 – upper face - $\left(0{,}5c + t_2\right) \le z \le -0{,}5c$, $c = 2h_1$, $t_i = h_i$

Superscripts „+" and „–" denote upper and lower face, respectively. We also introduce auxiliary relations between displacements of the faces, so called reduced displacements, in the following way:

$$u_\alpha = \frac{1}{2}\left(u_1 + u_2\right), \qquad v_\alpha = \frac{1}{2}\left(v_1 + v_2\right), \qquad u_\beta = \frac{1}{2}\left(u_1 - u_2\right), \qquad v_\beta = \frac{1}{2}\left(v_1 - v_2\right) \tag{2}$$

If we assume that the core does not deform in the normal z direction and the ratio h/R <<1, then we get

$$\begin{aligned} u_c &= u_\alpha + \frac{1}{4}\left(t_1 - t_2\right) w_{,x} + \frac{2z}{c}\left[u_\beta + \frac{1}{4}\left(t_1 + t_2\right) w_{,x} \right], \\ v_c &= v_\alpha + \frac{1}{4}\left(t_1 - t_2\right) w_{\varphi} + \frac{2z}{c}\left[v_\beta + \frac{1}{4}\left(t_1 + t_2\right) w_{\varphi} \right], \quad w_c = w^- = w^+ = w \end{aligned} \tag{3}$$

Non-linear geometric relations between strain tensor components and the components of displacement vector for particular layers of the shell are accepted in the following form

$$
\begin{aligned}
&\varepsilon_x^{\pm} = u,_x^{\pm} + \frac{1}{2}\left(w,_x\right)^2 && \varepsilon_\varphi^{\pm} = v,_\varphi^{\pm} - \frac{w}{R} + \frac{1}{2}\left(w,_\varphi\right)^2 \\
&\gamma_{xy}^{\pm} = v,_x^{\pm} + u,_\varphi^{\pm} + w,_x w,_\varphi && \gamma_{\varphi z}^{\pm} = w,_\varphi + v,_z^{\pm} \\
&\gamma_{zx}^{\pm} = w,_x + u,_z^{\pm} && \chi_x^{\pm} = w,_{xx} \\
&\chi_\varphi^{\pm} = \frac{1}{R^2} w + w,_{\varphi\varphi} && \chi_{x\varphi}^{\pm} = w,_{x\varphi} + \frac{1}{2R}\left(u,_\varphi - v,_x\right)
\end{aligned}
\tag{4}
$$

Here:

ε_x , ε_φ are strains along the coordinates x and y,

$\gamma_{x\varphi}$, $\gamma_{\varphi z}$, γ_{xz} are shear strains in the shell,

χ_x , χ_φ , $\chi_{x\varphi}$ are changes in curvature of the middle surface of the shell.

3 CONSTITUTIVE RELATIONS OF THE J_2 PLASTIC FLOW THEORY

We assume that effective stresses in the shell can be higher than yield stress of the shell material. Thus, constitutive relations are accepted according to Prandtl-Reuss plastic flow theory. Stresses and stress rates are related to strain rates or strain increments by a physical plasticity rule, which is flow rule, and Huber–Mises–Hencky (H-M-H) yield condition, generalized on the case of plastic stress hardening, is accepted. The following relations express Prandtl-Reuss plastic flow theory equations:

$$
d\varepsilon_{ij} = \frac{1}{2G}\left(d\sigma_{ij} - \delta_{ij}\frac{3\nu}{1+\nu}d\sigma_m\right) + 3d\lambda\left(\sigma_{ij} - \delta_{ij}\sigma_m\right), \quad \sigma_m = \frac{1}{3}\sigma_{kk}, \qquad d\lambda = \frac{1}{2}\frac{d\bar{\varepsilon}_i^p}{\sigma_i} \tag{5}
$$

Here, ε_i and σ_i are effective strain and effective stress, respectively. Parameter λ can be determined on the basis of plastic work increment:

$$
\delta W^p = \sigma_s \delta\varepsilon_s^p = \sigma_s\left(\frac{1}{E_t} - \frac{1}{E}\right)\delta\sigma_s \tag{6}
$$

That gives

$$
\lambda = \frac{3}{2}\left(\frac{1}{E_t} - \frac{1}{E}\right)\frac{\delta\sigma_i}{\sigma_i} \tag{7}
$$

The resultant middle surface forces and moments (see Figure 1) in shell faces are accepted in the following form:

$$
\begin{aligned}
\delta N_{\alpha\beta} &= \delta N_{\alpha\beta}^{+} + \delta N_{\alpha\beta}^{-} = \int_{-(h+h_2)}^{-h} \delta\sigma_{\alpha\beta} dz + \int_{h}^{h+h_1} \delta\sigma_{\alpha\beta} dz, \\
\delta M_{\alpha\beta} &= \delta M_{\alpha\beta}^{+} + \delta M_{\alpha\beta}^{-} = \int_{-(h+h_2)}^{-h} \delta\sigma_{\alpha\beta} z dz + \int_{h}^{h+h_1} \delta\sigma_{\alpha\beta} z dz,
\end{aligned}
\tag{8}
$$

If we substitute stress variations in equation (5) - using condition (7), by strain variations, and substitute them into (4) then, performing prescribed integration we obtain the following constitutive relations:

$$
\begin{aligned}
&\delta N_x = \left(B_{11}^{+}+B_{11}^{-}\right)\delta\varepsilon_{11}+\left(B_{12}^{+}+B_{12}^{-}\right)\delta\varepsilon_{22}-\left(B_{13}^{+}+b_{13}^{-}\right)\delta\gamma_{12},\\
&\delta N_y = \left(B_{21}^{+}+B_{21}^{-}\right)\delta\varepsilon_{11}+\left(B_{22}^{+}+B_{22}^{-}\right)\delta\varepsilon_{22}-\left(B_{23}^{+}+B_{23}^{-}\right)\delta\gamma_{12},\\
&\delta N_{xy} = -\left(B_{31}^{+}+B_{31}^{-}\right)\delta\varepsilon_{11}-\left(B_{32}^{+}+B_{32}^{-}\right)\delta\varepsilon_{22}+\left(B_{33}^{+}+B_{33}^{-}\right)\delta\gamma_{12},\\
&\delta M_x = -\left(D_{11}^{+}+D_{11}^{-}\right)\delta\kappa_1-\left(D_{12}^{+}+D_{12}^{-}\right)\delta\kappa_2+\left(D_{13}^{+}+D_{13}^{-}\right)\delta\kappa_{12},\\
&\delta M_y = -\left(D_{21}^{+}+D_{21}^{-}\right)\delta\kappa_1-\left(D_{22}^{+}+D_{22}^{-}\right)\delta\kappa_2+\left(D_{23}^{+}+D_{23}^{-}\right)\delta\kappa_{12},\\
&\delta M_{xy} = \left(D_{31}^{+}+D_{31}^{-}\right)\delta\kappa_1+\left(D_{32}^{+}+D_{32}^{-}\right)\delta\kappa_2-\left(D_{33}^{+}+D_{33}^{-}\right)\delta\kappa_{12}
\end{aligned}
\tag{9}
$$

Here B_{ij} and D_{ij} are coefficients of the local stiffness matrix given below:

$$
B_{11}=\frac{12}{t_i^2}D_{11}=\psi_o\left\{2(1+\nu)+\psi_t\left[\frac{1+\nu}{2}\left(2\bar{\sigma}_\varphi-\bar{\sigma}_x\right)^2+9\bar{\tau}_{x\varphi}^{2}\right]\right\},
$$

$$
B_{12}=B_{21}=\frac{12}{t_i^2}D_{12}=\frac{12}{t_i^2}D_{21}=\psi_o\left\{2\nu(1+\nu)-\psi_t\left[\frac{1+\nu}{2}\left(2\bar{\sigma}_x-\bar{\sigma}_\varphi\right)\left(2\bar{\sigma}_\varphi-\bar{\sigma}_x\right)^2+9\nu\bar{\tau}_{x\varphi}^{2}\right]\right\},
$$

... (10)

$$
B_{33}=\frac{6}{t_i^2}D_{33}=\psi_o\left\{\left(1-\nu^2\right)+\frac{1}{4}\psi_t\left[(5-4\nu)\left(\bar{\sigma}_x^2+\bar{\sigma}_\varphi^2\right)-2(4-5\nu)\bar{\sigma}_x\bar{\sigma}_\varphi\right]\right\}
$$

$$
\psi_o=\frac{Et_i}{1+\nu}\left\{2\left(1-\nu^2\right)+\frac{1}{2}\psi_t\left[(5-4\nu)\left(\bar{\sigma}_x^2+\bar{\sigma}_\varphi^2\right)-2(4-5\nu)\bar{\sigma}_x\bar{\sigma}_\varphi+18(1-\nu)\bar{\tau}_{x\varphi}^2\right]\right\}^{-1},\quad \psi_t=\frac{E}{E_t}-1,
$$

$$
\bar{\sigma}_x=\frac{\sigma_x}{\sigma_i},\qquad \bar{\sigma}_y=\frac{\sigma_y}{\sigma_i},\qquad \bar{\tau}_{x\varphi}=\frac{\tau_{x\varphi}}{\sigma_i},\qquad \sigma_i=\sqrt{\sigma_x^2-\sigma_x\sigma_x+\sigma_\varphi^2+3\tau_{x\varphi}^2},
$$

The above relations show that constitutive relations of the plastic flow theory are independent of secant modulus E_s. If, moreover, a bilinear stress-strain material model is accepted in the analysis, then tangent modulus E_t and B_{ij} and D_{ij} coefficients are constant in plastic range.

4 STABILITY EQUATIONS OF SANDWICH CYLINDRICAL SHELL

Stability equations are derived from the virtual work principle and the strain energy methods. In order to obtain the stability equations from the variational relations, the principle of the stationary potential energy will be invoked, with the sandwich cylindrical shell considered to be in a state of neutral equilibrium. Since the principle of the stationary potential energy states that the necessary condition of the equilibrium of any given state is that the variation of the total potential energy of the considered system is equal to zero, we have the following relation

$$
\delta\Pi_T=\delta(U-L)=0 \tag{11}
$$

We conclude from Eqs. (11) that if the shell is given the small virtual displacements, the equilibrium still persists if an increment of the total potential energy of the system $\delta\Pi_T$ is equal to zero. Relation (11) is the basis to derive the variational equation of equilibrium of a shell. For cylindrical sandwich shell the total potential energy of internal forces is equal to the energy of the specified layers. Here U is the strain energy accumulated in the shell and represented by strain components

$$U = U^{+} + U^{-} + U_c \tag{12}$$

The terms in Eqn (12) and the work of external forces L are as follows

$$U^{\pm} = \frac{1}{2}\int_0^l \int_0^{\beta R} \left(\varepsilon_x \delta N_x^{\pm} + \varepsilon_y \delta N_y^{\pm} + \gamma_{xy} \delta N_{xy}^{\pm}\right) dxdy \tag{13}$$

$$U^{\pm} = \frac{1}{2}\int_V \left[\frac{E_c}{1-\nu_c^2}\left(\delta\varepsilon_{xc}^2 + \delta\varepsilon_{yc}^2 + 2\nu_c \delta\varepsilon_{xc}\delta\varepsilon_{yc} + \frac{1-\nu_c}{2}\gamma_{xyc}^2\right) + G_c\left(\delta\gamma_{yzc}^2 + \delta\gamma_{xzc}^2\right)\right] dV \tag{14}$$

$$L = \int_0^l \int_0^{\beta R} qwdxdy + \left\{\int_0^{\beta R}\left[N_x^o\left(u_\alpha + \frac{h_1 - h_2}{H}u_\beta + \frac{h(h_1-h_2)}{2H}w_{,x}\right) + N_{xy}^o\left(v_\alpha + \frac{h_1-h_2}{H}v_\beta + \frac{h(h_1-h_2)}{2H}w_{,y}\right)\right]dy\right\}_0^l \tag{15}$$

The particular terms in the above equation are related with three layers: upper layer U^+, lower layer U^-, and core U_c. L represents the potential of external loads. Equation (11) with its nature has a form of equilibrium equation in variational sense, and it is correct both for the pre- and postcritical deformation state. Instead of exact expressions for the displacements u_i we introduce approximate functions with coefficients A_i. These coefficients must be chosen in such a way that they correspond as far as possible to real displacements.

As it results from the assumptions, the elastic material constants E_c, and G_c of the core are variable. It is a well-known fact that E_c, and G_c influence substantially the strength capacity of shell structures. These parameters have their highest values at the faces and change nonlinearly toward shell middle surface according to formula

$$E_{zc} = E_c \cosh(kz) \tag{16}$$

Here k is a parameter that depends on the properties of the core material. The variable properties can be reached by changing the core material density in the technological process. It also results from experimental data that Kirchhoff modulus G_c is a quadratic form of the material (plastic foam) density. Very good effects can be obtained when the cores are used with variable stiffness along middle surface coordinates. This stiffness can be varied continuously, or with sudden jumps. In this work it is assumed that the modules E_c, and G_c depend on spatial variable z.

Ritz method will be used to solve the equations. The equation

$$\delta\Pi_T = \int_0^l \int_0^{\beta R} \frac{\partial L}{\partial u_i}\delta u_i dxdy = 0 \tag{17}$$

is satisfied for an arbitrary value of the variations of parameters δA_i, where i=1,2,...,k. Thus we have

$$\delta\Pi = \sum_{i=1}^{5}\left(\frac{\partial \Pi}{\partial A_i}\right)\delta A_i = 0, \quad i=1,2,..,5, \quad \text{hence} \quad \frac{\partial \Pi_T}{\partial A_i} = 0 \tag{18}$$

In order to solve the considered problem by Ritz method, we use variational equation (11) with taking into account nonlinear geometrical relations (4), and we accept the displacement basis functions w, u and v in the following form

$$\begin{aligned} &w(x,\varphi) = A_1 \sin kx \sin p(\varphi + \gamma x), \\ &u_\alpha(x,\varphi) = A_2 \cos kx \sin p(\varphi + \gamma x), \quad u_\beta(x,\varphi) = A_3 \cos kx \sin p(\varphi + \gamma x), \\ &v_\alpha(x,\varphi) = A_4 \sin kx \cos p(\varphi + \gamma x), \quad v_\beta(x,\varphi) = A_5 \sin kx \cos p(\varphi + \gamma x) \end{aligned} \tag{19}$$

Here $$k = \frac{m\pi}{l}, \quad 0 \le x \le 1, \qquad p = \frac{n}{R}, \qquad 0 \le \varphi \le \beta, \tag{20}$$

γ is angle of inclination of buckle waves with respect to shell generatrix.

Free parameters A_i in (19) are determined in the solution process. m and $2n$ are parameters related with the number of halfwaves developed in the longitudinal and circumferential direction, respectively. The term γx in (19) represents the influence of the external shear forces on the deformation modes of the shell. The accepted basis functions satisfy kinematic boundary conditions of a free-support of shell edges [6]. According to Eqs. (18), we calculate derivatives of the total potential energy of the shell with respect to free parameters A_i (i = 1, 2, 3, 4, 5) of the basis functions, and, finally, we get a non-homogeneous and non-linear set of algebraic equations, which are the stability given here in a very concise form:

$$\begin{aligned} &a_{11}A_1 + a_{12}A_2 + a_{13}A_3 + a_{14}A_4 + a_{15}A_5 = b_{11}A_1^3 + b_{12}A_1^2 + b_{13}A_1A_2 + b_{14}A_1A_3 + b_{15}A_1A_4 + b_{16}A_1A_5 + b_{17}, \\ &a_{21}A_1 + a_{22}A_2 + a_{23}A_3 + a_{24}A_4 + a_{25}A_5 = b_{21}A_1^2 + b_{22}, \quad a_{31}A_1 + a_{32}A_2 + a_{33}A_3 + a_{34}A_4 + a_{35}A_5 = b_{31}A_1^2 + b_{32}, \\ &a_{41}A_1 + a_{42}A_2 + a_{43}A_3 + a_{44}A_4 + a_{45}A_5 = b_{41}A_1^2 + b_{42}, \quad a_{51}A_1 + a_{52}A_2 + a_{53}A_3 + a_{54}A_4 + a_{55}A_5 = b_{51}A_1^2 + b_{52} \end{aligned} \tag{21}$$

Coefficients b_{17}, b_{22}, b_{32}, b_{42}, b_{52} include longitudinal, surface, and shear external loads. The other coefficients depend exclusively on geometrical and material parameters of the shell and on number of buckling halfwaves m, n of the shell middle surface, and on parameter γ. We present, for example:

$$\begin{aligned} a_{11} = &\frac{1}{R^2}B_{22}F_{35} + \left(B_{11}'' - D_{11}\right)F_{79} + \frac{2}{R}D_{12}F_{80} + 2\left(B_{12}'' - D_{12}\right)F_{81} - \left(3D_{31} - 4B_{13}''\right)F_{82} + \left(4B_{33}'' - D_{33}\right)F_{87} + \\ &- D_{22}F_{103} + \left(B_{22}'' - D_{22}\right)F_{118} + 3D_{22}F_{104} + \left(3D_{32} - 4B_{23}''\right)F_{119} + \frac{G_3}{c}\frac{\left(t_1 + t_2 + 2c\right)^2}{4}F_{113}, \\ a_{12} = &-\frac{1}{R}B_{12}F_8 + \frac{1}{R}B_{23}F_{48} - \frac{3}{4R}D_{31}F_{83} - \frac{1}{2R}D_{33}F_{88} + \frac{3}{2}D_{32}F_{105}, \ldots \end{aligned}$$

5 SOLUTION OF NONLINEAR EQUATIONS AND THE RESULTS

The set of equations (21) allows us to determine the nonlinear equilibrium paths for the considered shell. We eliminate parameters A_2-A_5 from set of Eqs. (21), and if appropriate transformations and simplifications are made, we obtain the final solution in the form of the following non-linear algebraical equation:

$$q = \frac{e_1A_1 + e_2A_1^2 + e_3A_1^3}{A_1e_4\kappa + e_5}, \qquad \kappa = \frac{N_x^0}{qR} \tag{22}$$

Here: e_i are coefficients of the stability equation that have a very complicated form and they depend on geometrical parameters, material properties, buckling form, and external loading acting the shell. We elaborate a special numerical iterative algorithm where the basis is stability equation (22). The computer program makes it possible to determine the equilibrium paths and critical loads for cylindrical sandwich shells being in elastic, plastic, or elastic-plastic state of stress. Thus, lateral pressure q, and longitudinal force N_x can be determined as the functions of deflection w of the shell.

Solution algorithm and program of numerical calculation take into consideration a specific feature of elastic-plastic stability of shells. Stability equation (22) is a transcendental function, where the coefficients of local stiffness matrix depend on parameters of external loads. So, we have to use some iterative techniques to build up equilibrium paths, and to determine upper and lower critical loads.

A cylindrical sandwich panel with the basic dimensions l = 0.9m (shell length), r_s = 1.2m (mean radius), β = 0.6 rad (shell radius), h_1 = 0.0012m, h_2 = 0.0012m, $2h$ = 0.01m shell (thickness of the upper, lower and core layers, respectively) was accepted in numerical calculations. The face material was structural carbon steel St1, St4, St5 (Polish grade) with E_t = 32000 MPa; yield limit σ_{pl} = 240 MPa; elastic modulus E = 210000 MPa; shear modulus of the core is G_3 = 24 MPa.

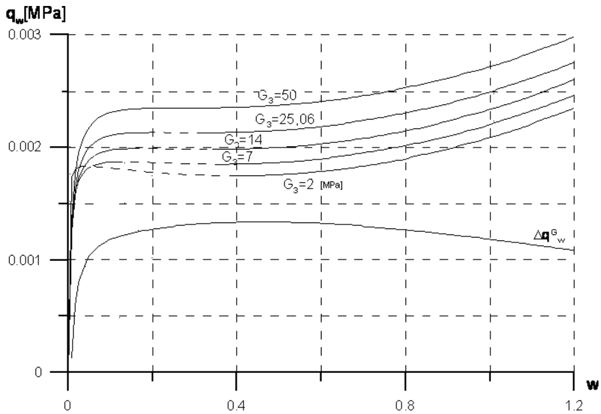

Figure 3: Equilibrium paths; the influence of shear modulus of the core on critical loads.

Fig. 3 shows an example of the results of numerical calculations. The curves in the diagram represent lateral pressure versus shell deflection $q = q(w)$. One can notice that the increase of shear modulus causes increasing both upper and lower critical loads. The lower curve in the diagram shows non-relative changes in critical loads Δq_w^G for different value of shear modulus $G_c = G_3$. The other numerical calculations were also carried out to analyze the postcritical equilibrium paths for arbitrary combinations of the external loads and geometrical and material parameters of the shell.

5 CONCLUSION

The analysis presented in this work shows that the accepted method of the stability analysis of elastic-plastic sandwich shells with the core of variable stiffness was appropriately chosen; the results are new and valuable. The elaborated algorithm of iterative numerical calculations can trace the equilibrium paths, is versatile one and can be used for the shells in elastic, elastic-plastic or in totally plastic state of stress.

REFERENCES

[1] Kim, S. E. and Kim, S. C., "Buckling strength of the cylindrical shell and tank subjected to axially compressed loads", *Thin-Walled structures*, **40**(4), 329-353, 2002.

[2] Pinna, R. and Ronalds, B. F., "Buckling and post buckling of cylindrical shells with one end pinned and the other end free", *Thin-Walled Structures*, **41**(6), 507-527, 2003.

[3] Siad, L., "Buckling of thin-walled cylindrical shells under uniform external pressure", *Thin-Walled Structures*, **35**(2), 101-115, 1999.

[4] Vinson, J. R., "Sandwich Structures", *Applied Mechanics Reviews*, **54**(3), 201-214, 2001.

[5] Zielnica, J., "Elastic Plastic Buckling of Sandwich Conical Shells under Axial Compression and Lateral Pressure". *Bull. Acad. Pol. Sci.*, XXIX, 239-251, 1981.

[6] Zielnica, J., *Stability of elastic-plastic shells* [in Polish]. PUT Edition, Poznan, pp. 243, 2001.